2. A zoo veterinarian can purchase animal food of three different types: A, B and C. Each food comes in the same size bag and the number of grams of each of three nutrients (N1, N2, N3) in each bag is summarised in the following table

	A	B	C
N1	5	5	10
N2	10	5	30
N3	15	15	10

For one animal, the veterinarian determines that she needs to combine the food types to get 10,000g of N1, 20,000 of N2 and 20,000g of N3

(a) Write down the matrix equation ($\underline{\mathbf{A}}\underline{\mathbf{x}} = \underline{\mathbf{b}}$) representing the problem.

(b) Use **Gauss-Jordan** method to obtain the inverse of the matrix $\underline{\mathbf{A}}$ in part (a)

(c) Use the answer from (b) to determine how many bags of each type of food should she order?

(d) If she would like to change the number of grams for each nutrients to be 8000g of N1, 18,400 of N2 and 19,000 of N3, determine how many bags of each type of food that she need for her new order.

(8 marks)

Solution - 2014

(2) (a)

$$\begin{bmatrix} 5 & 5 & 10 \\ 10 & 5 & 30 \\ 15 & 15 & 10 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 10{,}000 \\ 20{,}000 \\ 20{,}000 \end{bmatrix}$$

(b)

$$\left[\begin{array}{ccc|ccc} 5 & 5 & 10 & 1 & 0 & 0 \\ 10 & 5 & 30 & 0 & 1 & 0 \\ 15 & 15 & 10 & 0 & 0 & 1 \end{array}\right] \underset{\substack{R_2-2R_1\to R_2 \\ R_3-3R_1\to R_3}}{\sim} \left[\begin{array}{ccc|ccc} 5 & 5 & 10 & 1 & 0 & 0 \\ 0 & -5 & 10 & -2 & 1 & 0 \\ 0 & 0 & -20 & -3 & 0 & 1 \end{array}\right] \frac{R_3}{-2}$$

$$\left[\begin{array}{ccc|ccc} 5 & 5 & 10 & 1 & 0 & 0 \\ 0 & -5 & 10 & -2 & 1 & 0 \\ 0 & 0 & 10 & \frac{3}{2} & 0 & -\frac{1}{2} \end{array}\right] \underset{\substack{R_1-R_3\to R_1 \\ R_2-R_3\to R_2}}{\sim} \left[\begin{array}{ccc|ccc} 5 & 5 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & -5 & 0 & -\frac{7}{2} & 1 & \frac{1}{2} \\ 0 & 0 & 10 & \frac{3}{2} & 0 & -\frac{1}{2} \end{array}\right]$$

$$\underset{R_1+R_2\to R_1}{\sim} \left[\begin{array}{ccc|ccc} 5 & 0 & 0 & -4 & 1 & 1 \\ 0 & -5 & 0 & -\frac{7}{2} & 1 & \frac{1}{2} \\ 0 & 0 & 10 & \frac{3}{2} & 0 & -\frac{1}{2} \end{array}\right] \xrightarrow{\substack{R_1/5 \\ R_2/-5 \\ R_3/10}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{4}{5} & \frac{1}{5} & \frac{1}{5} \\ 0 & 1 & 0 & \frac{7}{10} & -\frac{1}{5} & -\frac{1}{10} \\ 0 & 0 & 1 & \frac{3}{20} & 0 & -\frac{1}{20} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -0.8 & 0.2 & 0.2 \\ 0.7 & -0.2 & -0.1 \\ 0.15 & 0 & -0.05 \end{bmatrix}$$

(c)

$Ax = b$

$\therefore \; \underline{x} = \underline{A}^{-1} b$

$$\underline{x} = \begin{bmatrix} -0.8 & 0.2 & 0.2 \\ 0.7 & -0.2 & -0.1 \\ 0.15 & 0 & -0.05 \end{bmatrix} \begin{bmatrix} 10000 \\ 20{,}000 \\ 20000 \end{bmatrix}$$

$$\underline{x} = \begin{bmatrix} 0 \\ 1000 \\ 500 \end{bmatrix}$$

a)

$$\underline{x} = \begin{bmatrix} 1080 \\ 20 \\ 250 \end{bmatrix}$$

Q2 1/2

Q − 3 ~~1~~. Given $\underline{\mathbf{A}} = \begin{bmatrix} a^3 & a^2 & a & 1 \\ a & 1 & a & a^2 \\ a^2 & a & 1 & a \\ 1 & a & a^2 & a^3 \end{bmatrix}$; $\underline{\mathbf{B}} = \begin{bmatrix} 4 & 0 & 0 & 3 \\ 0 & 0 & 1 & -9 \\ 0 & 0 & 0 & 3 \\ 0 & 2 & 0 & 0 \end{bmatrix}$ and $\underline{\mathbf{C}} = \begin{bmatrix} 2 & 4 & -8 \\ -1 & -1 & 3 \\ -1 & -2 & 5 \end{bmatrix}$

a) If $\det(-144\underline{\mathbf{A}}) = \det(\underline{\mathbf{B}}^{-1}\underline{\mathbf{D}}^3)\det\underline{\mathbf{C}}$, find the determinant of $\underline{\mathbf{D}}$.

b) Find adjoint of $\underline{\mathbf{C}}$ (*adj* $\underline{\mathbf{C}}$) show that $\underline{\mathbf{C}}^{-1} = \begin{bmatrix} \frac{1}{2} & -2 & 2 \\ 1 & 1 & 1 \\ \frac{1}{2} & 0 & 1 \end{bmatrix}$.

c) Find the complete solution to $\underline{\mathbf{C}}^T\underline{\mathbf{x}} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$.

(10 points)

~~9 marks~~

1)

$$|A| = \begin{vmatrix} a^3 & a^2 & a & 1 \\ a & 1 & a & a^2 \\ a^2 & a & 1 & a \\ 1 & a & a^2 & a^3 \end{vmatrix} \underset{R_4 \leftrightarrow R_1}{=} - \begin{vmatrix} 1 & a & a^2 & a^3 \\ a & 1 & a & a^2 \\ a^2 & a & 1 & a \\ a^3 & a^2 & a & 1 \end{vmatrix} \begin{matrix} R_2 - (R_1)a \\ R_3 - a^2R_1 \\ R_4 - a^3R_1 \end{matrix} =$$

$$- \begin{vmatrix} 1 & a & a^2 & a^3 \\ 0 & 1-a^2 & a-a^3 & a^2-a^4 \\ 0 & a-a^3 & 1-a^4 & a-a^5 \\ 0 & a^2-a^4 & a-a^5 & 1-a^6 \end{vmatrix} \begin{matrix} R_3 - aR_2 \\ R_4 - a^2R_2 \end{matrix} = - \begin{vmatrix} 1 & a & a^2 & a^3 \\ 0 & 1-a^2 & a-a^3 & a^2-a^4 \\ 0 & 0 & 1-a^2 & a-a^3 \\ 0 & 0 & a-a^3 & 1-a^4 \end{vmatrix}$$

$$R_4 - aR_3 = - \begin{vmatrix} 1 & a & a^2 & a^3 \\ 0 & 1-a^2 & a-a^3 & a^2-a^4 \\ 0 & 0 & 1-a^2 & a-a^3 \\ 0 & 0 & 0 & 1-a^2 \end{vmatrix} = -(1)(1-a^2)^3$$
$$= -(1-a^2)^3$$

Q3 1/3 PTO

Continue

$$|B| = \begin{vmatrix} 4 & 0 & 0 & 3 \\ 0 & 0 & 1 & -9 \\ 0 & 0 & 0 & 3 \\ 0 & 2 & 0 & 0 \end{vmatrix} \underset{\text{interchange twice}}{=} \begin{vmatrix} 4 & 0 & 0 & 3 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & -9 \\ 0 & 0 & 0 & 3 \end{vmatrix} = 4 \times 2 \times 1 \times 3 = 24$$

$$|C| = \begin{vmatrix} 2 & 4 & -8 \\ -1 & -1 & 3 \\ -1 & -2 & 5 \end{vmatrix} = 2 \begin{vmatrix} 1 & 2 & -4 \\ -1 & -1 & 3 \\ -1 & -2 & 5 \end{vmatrix} \underset{R_3+R_1}{\overset{R_2+R_1}{=}} 2 \begin{vmatrix} 1 & 2 & -4 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\therefore \boxed{|C| = 2}$$

$|-A| = (-1)^m |A|$

$|KA| = K^m |A|$

Note

$$\det(-18A) = \det(B^{-1}D^3)\det C$$

$$\det(-18A) = \det B^{-1} \det D^3 \det C$$

$$|-18A| = \frac{1}{|B|} \times |D|^3 \times |C|$$

$$18^4 \cdot |-A|_4 = \frac{1}{\cancel{24}\ 12} |D|^3 \times \cancel{2}$$

$$12 \times 18^4 (-1)(1-a^2)^3 = |D|^3$$

$$6 \cdot 2 \cdot 6 \cdot 3 \cdot 18^3 (-1)(1-a^2)^3 = |D|^3$$

$$6 \cdot 18 \cdot (1-a^2)(-1) = |D|$$

$$|D| = -(6 \cdot 18)(1-a^2) \boxed{= (-108)(1-a^2)}$$

(Q3) 2/3

$$\underline{C} = \begin{bmatrix} 2 & 4 & -8 \\ -1 & -1 & 3 \\ -1 & -2 & 5 \end{bmatrix}$$

(b) Find adj $\underline{C}$ and show that $\underline{C}^{-1} = ?$

adj $\underline{C}$ = cofactor of $\underline{C}$ transpose $= C^T(\underline{C})$

$$C_{11} = + \begin{vmatrix} -1 & 3 \\ -2 & 5 \end{vmatrix} = 1 \qquad C_{12} = - \begin{vmatrix} -1 & 3 \\ -1 & 5 \end{vmatrix} = +2 \qquad C_{13} = \begin{vmatrix} -1 & -1 \\ -1 & -2 \end{vmatrix} = 1$$

$$C_{21} = - \begin{vmatrix} 4 & -8 \\ -2 & 5 \end{vmatrix} = -4 \qquad C_{22} = \begin{vmatrix} 2 & -8 \\ -1 & 5 \end{vmatrix} = 2 \qquad C_{23} = - \begin{vmatrix} 2 & 4 \\ -1 & -2 \end{vmatrix} = 0$$

$$C_{31} = + \begin{vmatrix} 4 & -8 \\ -1 & 3 \end{vmatrix} = 4 \qquad C_{32} = - \begin{vmatrix} 2 & -8 \\ -1 & 3 \end{vmatrix} = 2 \qquad C_{33} = + \begin{vmatrix} 2 & 4 \\ -1 & -1 \end{vmatrix} = 2$$

cofactor of $\underline{C} = \underline{C}(\underline{C}) = \begin{bmatrix} 1 & 2 & 1 \\ -4 & 2 & 0 \\ 4 & 2 & 2 \end{bmatrix}$

$$C^T(\underline{C}) = \begin{bmatrix} 1 & -4 & 4 \\ 2 & 2 & 2 \\ 1 & 0 & 2 \end{bmatrix} \doteq \text{adj}(\underline{C})$$

$|C| = 2$; $\underline{C}^{-1} = \frac{1}{|C|} \text{adj}(C)$

$$\underline{C}^{-1} = \frac{1}{2} \begin{bmatrix} 1 & -4 & 4 \\ 2 & 2 & 2 \\ 1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1/2 & -2 & 2 \\ 1 & 1 & 1 \\ 1/2 & 0 & 1 \end{bmatrix}$$

(c) $\underline{C}^T \underline{x} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$

$\underline{x} = (\underline{C}^T)^{-1} \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = (\underline{C}^{-1})^T \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$

$\underline{x} = \begin{bmatrix} 1/2 & 1 & 1/2 \\ -2 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$

Q3 3/3.

4. Given a matrix equation

$$\underline{A}\underline{x} = \underline{b} \quad ; \quad \underline{A} \text{ is a 4 by 3 matrix}$$

If $\underline{x} = \begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix}$ then $\underline{b} = \begin{bmatrix} -4 \\ -3 \\ -7 \\ -1 \end{bmatrix}$. However, if $\underline{x} = c\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$, where $C \in \Re$, then $\underline{b} = 0$

a) What is the 3rd column if $\underline{A}$?
b) What is the 2nd column of $\underline{A}$?

6 points

$\underline{A}\underline{x} = \underline{b} \Rightarrow$ let $\underline{A} = [\underline{a}_1 \ \underline{a}_2 \ \underline{a}_3]$

$$[\underline{a}_1 \ \underline{a}_2 \ \underline{a}_3]\begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix} = \underline{b}$$

$$-2\underline{a}_2 + 3\underline{a}_3 = \underline{b} \quad \text{— (1)}$$

$$[\underline{a}_1 \ \underline{a}_2 \ \underline{a}_3]\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}\cancel{c} = 0 \quad \text{—}$$

$$2\underline{a}_2 - \underline{a}_3 = 0 \quad \text{— (2)}$$

(1) + (2)

$$2\underline{a}_3 = \underline{b}$$

$$\underline{a}_3 = \frac{1}{2}\underline{b} = \begin{bmatrix} -2 \\ -3/2 \\ -7/2 \\ -1/2 \end{bmatrix}$$

Sub into (1)

$$-2\underline{a}_2 + 3 \cdot \frac{1}{2}\underline{b} = \underline{b}$$

$$+2\underline{a}_2 = \frac{3}{2}\underline{b} + \underline{b} = \frac{5}{2}\underline{b}$$

$$\underline{a}_2 = \frac{5}{4}\underline{b}$$

$$\underline{a}_2 = \begin{bmatrix} -4 \\ -3 \\ -7 \\ -1 \end{bmatrix} \cdot \frac{5}{4} = \begin{bmatrix} -5 \\ -15/4 \\ -7/4 \\ -1/4 \end{bmatrix}$$

Column 3rd $\underline{a}_3$

$$\begin{bmatrix} -2 \\ -3/2 \\ -7/2 \\ -1/2 \end{bmatrix}$$

$$\underline{a}_2 = \begin{bmatrix} -5 \\ -15/4 \\ -7/4 \\ -1/4 \end{bmatrix}$$

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let $\underline{A} = [\underline{a}_1 \ \underline{a}_2 \ \underline{a}_3]$

$[\underline{a}_1 \ \underline{a}_2 \ \underline{a}_3]\begin{bmatrix}0\\-2\\3\end{bmatrix} = \underline{b}$ $\quad -2\underline{a}_2 + 3\underline{a}_3 = \underline{b} = \begin{bmatrix}-4\\-3\\-7\\1\end{bmatrix}$ — ①

$[\underline{a}_1 \ \underline{a}_2 \ \underline{a}_3]\begin{bmatrix}0\\2\\-1\end{bmatrix} = \underline{0}$ $\quad 2\underline{a}_2 - \underline{a}_3 = \underline{0}$ — ②

① + ② $\quad 2\underline{a}_3 = \begin{bmatrix}-4\\-3\\-7\\-1\end{bmatrix} \rightarrow \therefore \underline{a}_3 = \begin{bmatrix}-2\\-3/2\\-7/2\\-1/2\end{bmatrix}$

From ②; $\quad \underline{a}_2 = \frac{1}{2}\underline{a}_3 \rightarrow \therefore \underline{a}_2 = \begin{bmatrix}-1\\-3/4\\-7/4\\-1/4\end{bmatrix}$

Can also do Elemental way but take more work.

$\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & \cdots & \cdots\\ a_{31} & \cdots & \cdots\\ a_{41} & \cdots & a_{43}\end{bmatrix}\begin{bmatrix}0\\-2\\3\end{bmatrix} = \begin{bmatrix}-4\\-3\\-7\\-1\end{bmatrix}$

$\begin{bmatrix}a_{11} & \cdots & a_{13}\\ \vdots & \ddots & \vdots\\ a_{41} & \cdots & a_{43}\end{bmatrix}\begin{bmatrix}0\\2\\-1\end{bmatrix} = \underline{0}$

$-2a_{12} + 3a_{13} = -4$ — ①
$-2a_{22} + 3a_{23} = -3$ — ②
$-2a_{32} + 3a_{33} = -7$ — ③
$-2a_{42} + 3a_{43} = -1$ — ④
$2a_{12} - a_{13} = 0$ — ⑤
$2a_{21} - a_{23} = 0$ — ⑥
$2a_{31} - a_{33} = 0$ — ⑦
$2a_{41} - a_{43} = 0$ — ⑧

①+⑤; $2a_{13} = -4 \rightarrow a_{13} = -2$
②+⑥; $2a_{23} = -3 \rightarrow a_{23} = -3/2$
③+⑦; $2a_{33} = -7 \rightarrow a_{33} = -7/2$
④+⑧; $2a_{43} = -1 \rightarrow a_{43} = -1/2$

Column 3 of $\underline{A} = \begin{bmatrix}-2\\-3/2\\-7/2\\-1/2\end{bmatrix}$

⑤; $a_{12} = \frac{a_{13}}{2} = -1$
⑥; $a_{22} = \frac{a_{23}}{2} = -3/4$
⑦; $a_{32} = \frac{a_{33}}{2} = -7/4$
⑧; $a_{42} = \frac{a_{43}}{2} = -1/4$

column 2 of $\underline{A} = \begin{bmatrix}-1\\-3/4\\-7/4\\-1/4\end{bmatrix}$

5. (a) Given

$$\underline{\mathbf{A}}\begin{bmatrix} x \\ y \\ z \\ w \\ v \end{bmatrix} = \begin{bmatrix} -3y-6z+4w+9v \\ -x-2y-z+3w+v \\ -2x-3y+3w+Dv \\ x+4y+5z-9w-7v \end{bmatrix}$$, where D is a constant.

And $\underline{\mathbf{C}}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$. , where $\underline{\mathbf{C}} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ -\frac{1}{2} & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix}$

Determine matrix $\underline{\mathbf{A}}$ and $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$. Show that $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$ is a summation (combination) of multiple of column of $\underline{\mathbf{C}}$.

(b) Obtain echelon form of matrix $\underline{\mathbf{A}}$. What is the maximum number of pivots that this matrix can have? What is the value of D that will make this matrix a rank 4 matrix?

(c) If $\underline{\mathbf{A}}\begin{bmatrix} x \\ y \\ z \\ w \\ v \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$, can this linear system have unique solution? Explain why?

(d) If $\underline{\mathbf{A}}\begin{bmatrix} x \\ y \\ z \\ w \\ v \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$, what value of D that will make the matrix $\underline{\mathbf{A}}$ a rank 3 matrix? Find the solution and give the solution in a vector form.

(f) If $\underline{\mathbf{A}}\begin{bmatrix} x \\ y \\ z \\ w \\ v \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ -2 \\ 4 \end{bmatrix}$ where $\underline{\mathbf{A}}$ is a rank 3 matrix, is this linear system consistent? If not, explain why? If so, give the solution in a vector form?

(13 marks)

5) #014

$$A=\begin{bmatrix} 0 & -3 & -6 & 4 & 9 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & D \\ 1 & 4 & 5 & -9 & -7 \end{bmatrix} \; ; \qquad \underline{b}=\begin{bmatrix} 3 \\ 1 \\ 1 \\ -3 \end{bmatrix}$$

$$\underline{b} = \underline{a}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ -1/2 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 1 \\ -1/2 \\ -1 \end{bmatrix} + (-1)\begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} + (1)\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad -(1)$$

as equation (1) show that $\underline{b}$ is a combination of multiple of columns of $\underline{c}$

$$\boxed{\underline{b} = \begin{bmatrix} 3 \\ 1 \\ 1 \\ -3 \end{bmatrix}} \; *$$

b)

$$\left[\begin{array}{ccccc|c} 0 & -3 & -6 & 4 & 9 & 3 \\ -1 & -2 & -1 & 3 & 1 & 1 \\ -2 & -3 & 0 & 3 & D & 1 \\ 1 & 4 & 5 & -9 & -7 & -3 \end{array}\right] \underset{\sim}{R_4 \leftrightarrow R_1} \left[\begin{array}{ccccc|c} 1 & 4 & 5 & -9 & -7 & -3 \\ -1 & -2 & -1 & 3 & 1 & 1 \\ -2 & -3 & 0 & 3 & D & 1 \\ 0 & -3 & -6 & 4 & 9 & 3 \end{array}\right] \begin{array}{c} \sim \\ R_2+R_1 \to R_2 \\ R_3+2R_1 \to R_3 \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 4 & 5 & -9 & -7 & -3 \\ 0 & 2 & 4 & -6 & -6 & -2 \\ 0 & 5 & +10 & -15 & D-14 & -5 \\ 0 & -3 & -6 & 4 & 9 & 3 \end{array}\right] \begin{array}{c} R_2/2 \\ R_3/5 \\ R_4/(-3) \end{array} \left[\begin{array}{ccccc|c} 1 & 4 & 5 & -9 & -7 & -3 \\ 0 & 1 & 2 & -3 & -3 & -1 \\ 0 & 1 & 2 & -3 & \frac{D-14}{5} & -1 \\ 0 & 1 & 2 & \frac{4}{-3} & -3 & -1 \end{array}\right] \begin{array}{c} \sim \\ R_3-R_2 \to R_3 \\ R_4-R_2 \to R_4 \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 4 & 5 & -9 & -7 & -3 \\ 0 & 1 & 2 & -3 & -3 & -1 \\ 0 & 0 & 0 & 0 & \frac{D-14}{5}+3 & 0 \\ 0 & 0 & 0 & -\frac{5}{3} & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccccc|c} \boxed{1} & 4 & 5 & -9 & -7 & -3 \\ 0 & \boxed{1} & 2 & -3 & -3 & -1 \\ 0 & 0 & 0 & \boxed{-\frac{5}{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \boxed{\frac{D+1}{5}} & 0 \end{array}\right]$$

the max number of pivots is [illegible] for this matrix to have rank 4

$\frac{D+1}{5} \neq 0 \quad \therefore \boxed{D \neq -1}$

Q5-1/2

$\therefore m = 4$

5 (c) No, this linear system cannot have a unique solution since

$\underline{m < n}$ the maximum number of pivot is $m = 4$ $\underline{\text{but } n = 5}$

there must be at least 1 free variable

(d) From b for ~~A~~ to have rank 3 $\Rightarrow \frac{D+1}{5} = 0$.

$\therefore \boxed{D = -1}$ and free variables are $\underline{z \text{ and } v}$

$\therefore$ $w = 0$ — ①

$y + 2z - 3\cancel{w}^{0} - 3v = -1$ — ②

$\therefore \underline{y = -1 - 2z + 3v}$ (z, v are free variables)

$x + 4y + 5z - 9\cancel{w}^{0} - 7v = -3$

$x = -3 - 4(-1 - 2z + 3v) - 5z + 7v = -3 + 4 + 8z - 12v - 5z + 7v$

$x = \underline{1 + 3z - 5v}$

$$\therefore \underline{x} = \begin{bmatrix} x \\ y \\ z \\ w \\ v \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + z \begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + v \begin{bmatrix} -5 \\ 3 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$; z, v are free variables

check $\Longrightarrow$ answer again

$$\underline{x} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + C_3 \begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + C_5 \begin{bmatrix} -5 \\ 3 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$; $C_3, C_5 \in \mathbb{R}$

(f) $$\begin{bmatrix} 3 \\ 1 \\ -2 \\ 4 \end{bmatrix} \overset{R_4 \leftrightarrow R_1}{\sim} \begin{bmatrix} 4 \\ 1 \\ -2 \\ 3 \end{bmatrix} \underset{R_3 + 2R_1}{\overset{R_2 + R_1}{\sim}} \begin{bmatrix} 3 \\ 4 \\ 6 \\ 4 \end{bmatrix} \underset{\substack{R_2/2 \\ R_3/5 \\ R_4/3}}{\sim} \begin{bmatrix} 3 \\ 2 \\ 6/5 \\ -4/3 \end{bmatrix} \underset{R_4 - R_2}{\overset{R_3 - R_2}{\sim}} \begin{bmatrix} 3 \\ 2 \\ -4/5 \\ -10/3 \end{bmatrix} \Rightarrow \left[\begin{array}{ccccc|c} 1 & 4 & 5 & -9 & -7 & 3 \\ 0 & 1 & 2 & -3 & -3 & 2 \\ 0 & 0 & 0 & -5/13 & 0 & -4/5 \\ 0 & 0 & 0 & 0 & 0 & -10/3 \end{array}\right]$$

It is not possible this system is inconsistent. [illegible] X [illegible]

$$\left[\begin{array}{ccccc|c} 1 & 0 & -3 & 0 & 5 & 1 \\ 0 & 1 & 2 & 0 & -3 & 7 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$ ✓

Q5.

From MATLAB

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(a)

$$\underline{A} = \begin{bmatrix} 0 & -3 & -6 & 4 & 9 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & D \\ 1 & 4 & 5 & -9 & -7 \end{bmatrix}$$

$$\underline{C}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ -1/2 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \\ -3 \end{bmatrix} = \underline{b}$$

$$\underline{b} = 2\begin{bmatrix} 1 \\ 1 \\ -1/2 \\ -1 \end{bmatrix} + (-1)\begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} + (1)\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \\ -3 \end{bmatrix}$$

$\underline{b}$ is the combination of multiple of column of $\underline{C}$

(b)

$$\begin{bmatrix} 0 & -3 & -6 & 4 & 9 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & D \\ 1 & 4 & 5 & -9 & -7 \end{bmatrix} \quad \overset{(d)}{\begin{bmatrix} 3 \\ 1 \\ 1 \\ -3 \end{bmatrix}} \quad \overset{(f)}{\begin{bmatrix} 3 \\ 1 \\ -2 \\ 4 \end{bmatrix}}$$

$R_1 \leftrightarrow R_4$

$$\sim \begin{bmatrix} 1 & 4 & 5 & -9 & -7 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & D \\ 0 & -3 & -6 & 4 & 9 \end{bmatrix} \quad \begin{bmatrix} -3 \\ 1 \\ 1 \\ 3 \end{bmatrix} \quad \begin{bmatrix} 4 \\ 1 \\ -2 \\ 3 \end{bmatrix}$$

$R_1 + R_2 \to R_2$; $2R_1 + R_2 \to R_2$

$$\sim \begin{bmatrix} 1 & 4 & 5 & -9 & -7 \\ 0 & 2 & 4 & -6 & -6 \\ 0 & 5 & 10 & -15 & D-14 \\ 0 & -3 & -6 & 4 & 9 \end{bmatrix} \quad \begin{bmatrix} -3 \\ -2 \\ -5 \\ 3 \end{bmatrix} \quad \begin{bmatrix} 4 \\ 5 \\ 6 \\ 3 \end{bmatrix}$$

$\frac{1}{2}R_2 \to R_2$

$$\sim \begin{bmatrix} 1 & 4 & 5 & -9 & -7 \\ 0 & 1 & 2 & -3 & -3 \\ 0 & 5 & 10 & -15 & D-14 \\ 0 & -3 & -6 & 4 & 9 \end{bmatrix} \quad \begin{bmatrix} -3 \\ -1 \\ -5 \\ 3 \end{bmatrix} \quad \begin{bmatrix} 4 \\ 5/2 \\ 6 \\ 3 \end{bmatrix}$$

$-5R_2 + R_3 \to R_3$; $3R_2 + R_4 \to R_4$

$$\sim \begin{bmatrix} 1 & 4 & 5 & -9 & -7 \\ 0 & 1 & 2 & -3 & -3 \\ 0 & 0 & 0 & 0 & D+1 \\ 0 & 0 & 0 & -5 & 0 \end{bmatrix} \quad \begin{bmatrix} -3 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 4 \\ 5/2 \\ -13/2 \\ 21/2 \end{bmatrix}$$

$R_3 \leftrightarrow R_4$

$$\sim \underline{A}_{\text{echelon form}} = \begin{bmatrix} \boxed{1} & 4 & 5 & -9 & -7 \\ 0 & \boxed{1} & 2 & -3 & -3 \\ 0 & 0 & 0 & \boxed{-5} & 0 \\ 0 & 0 & 0 & 0 & \boxed{D+1} \end{bmatrix} \quad \begin{bmatrix} -3 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 4 \\ 5/2 \\ 21/2 \\ -13/2 \end{bmatrix}$$

Max. no. of piots = 4.

For rank = 4 ; $D+1 \neq 0 \to D = -1$

EBA Final 2013 - 2014 Q5 continue...

(c) No, this linear system cannot ~~be~~ have unique solution.

Because no. of maximum pivots (r) = 4, no. of unknowns (n) = 5

no. of max pivots (r) < no. of unknowns and from (b) the system is consistent. Hence, this system has many solutions.

(d) Rank of 3; $D+1=0 \Rightarrow \therefore \boxed{D=-1}$

$$\begin{array}{c} \begin{array}{ccccc} x & y & z & w & v \end{array} \\ \left[\begin{array}{ccccc|c} 1 & 4 & 5 & -9 & -7 & -3 \\ 0 & 1 & 2 & -3 & -3 & -1 \\ 0 & 0 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

Row 3: $-5w = 0 \rightarrow w = 0$

$-4R_2 + R_1 \rightarrow R_1$

$$\sim \begin{array}{c} \begin{array}{ccccc} x & y & z & w & v \end{array} \\ \left[\begin{array}{ccccc|c} 1 & 0 & -3 & 3 & 5 & 1 \\ 0 & 1 & 2 & -3 & -3 & -1 \\ 0 & 0 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

Row 2: $y + 2z - 3\cancel{w}^{0} - 3v = -1$

$\therefore y = -1 - 2z + 3v$

Row 1; $x - 3z + 3\cancel{w} + 5v = 1$

$\therefore x = 1 + 3z - 5v$

$$\therefore \begin{bmatrix} x \\ y \\ z \\ w \\ v \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -5 \\ 3 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \text{where } c_1, c_2 \in \mathbb{R}$$

(f)
$$\left[\begin{array}{ccccc|c} 1 & 4 & 5 & -9 & -7 & 4 \\ 0 & 1 & 2 & -3 & -3 & 5/2 \\ 0 & 0 & 0 & -5 & 0 & 21/2 \\ 0 & 0 & 0 & 0 & 0 & -13/2 \end{array}\right]$$

Row 4: $0 = -13/2$ inconsistent.

No, the linear system is inconsistent.

6. Suppose **A** is an m by n matrix of rank r โจทย์ไม่ชัด ∴ ยอมรับ 2 คำตอบ

   a. Under what conditions on m, n and r does $\underline{\mathbf{A}}\underline{\mathbf{x}} = \underline{\mathbf{b}}$ have infinitely many solution for every $\underline{b}$? $\rightarrow r = m < n$ OR $r < n$ and $r \le m$.

   b. If the only solution to $\underline{\mathbf{A}}\underline{\mathbf{x}} = 0$ is $\underline{\mathbf{x}} = 0$, what is the rank of matrix $\underline{\mathbf{A}}$? Explain why? rank $r = n$ since it means there is pivot in every column. (No free var

   c. If $\underline{\mathbf{A}}\underline{\mathbf{x}} = \underline{\mathbf{b}}$ always has at least one solution, show that the only solution to $\underline{\mathbf{A}}^T \underline{\mathbf{y}} = \underline{\mathbf{b}}$ is $\underline{\mathbf{y}} = 0$. (hint: what is the rank?)

$\rightarrow$ there is no zero row in the echelon form of $\underline{A}$

$\therefore r = m$ (full row rank) $(m \le n)$

number of pivots is m which become number of column of A^T

$\therefore \underline{A}^T$ will have pivot in every column. No free variable s.

$\therefore \underline{A}^T \underline{y} = 0$ is $y = 0$

4 points

F-2016

10. State whether each of the following statement is **True or False**. Explain your answer briefly. (*answer without explanation will not have any score*)

F a) If rank of $\underline{\mathbf{B}}_{3x2}$ is 2 then there is no solution to $\underline{\mathbf{B}^T\mathbf{x}} = [1\ 1]^T$. / infinitely many solution or unique

False since B has pivot in both all column and B^T will have pivot in every row (2)

F b) Any system of n linear equations in n variables has at most n solutions. It always has

~~Not~~ "infinitely many solutions" could be greater than n solutions. ? solutions (consi

F c) If a system of linear equations has no free variables, then it has a unique solution.

It is not true for all cases.. If $m > n$, there will be ~~exist~~ zero row which depen on r/hs side whether will al s

It can have no solution too

T d) If $\underline{\mathbf{A}}$ is an m by n matrix and the equation $\underline{\mathbf{Ax}} = \underline{\mathbf{b}}$ is consistent for every $\underline{\mathbf{b}}$, then $\underline{\mathbf{A}}$ has m pivot columns.

Full row rank always yield consistent sol

the echelon form of A contains pivot in every row (no zero row)

$\therefore \boxed{r = m}$

F e) If $\underline{\mathbf{C}}$ is 3 by 4 matrix and det $\underline{\mathbf{C}} = 0$ then echelon form of A will have a zero row.

False since C is not a square matrix $|C|$ is undefined

T f) If $\underline{\mathbf{D}}$ is 5 by 5 matrix, it is invertible provided that det $\underline{\mathbf{D}}$ is a non-zero quantity.

If $|D| \neq 0$ it is invertible (It is a non-singular matrix) 3 points

~~3marks~~

3Pts.

To as $\frac{1}{2}$ mark

→ No explanation 0

→ Explain not Yo'ansurlal (main point) as $\frac{1}{4}$

97-80