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1.1 Use OS to find estimator of β_1 and β_2

$$\bar{x} = \frac{63+72+78+81+87+75+75+90}{8} = 77.625$$

$$\bar{y} = \frac{2.8+3.4+3+3.5+3.6+3+2.7+3.7}{8} = 3.2125$$

$$\hat{\beta}_1 = \bar{y} - \beta_2 \bar{x}$$

$$= 3.2125 - \beta_2'(77.625) \longrightarrow ①$$

$$\hat{\beta}_2 = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2}$$

$$= \frac{8(2012.4) - (621)(25.7)}{8(48717) - (621)^2}$$

$$= \frac{16099.2 - 15959.7}{389736 - 385641} = 0.0341$$

$$\therefore \hat{\beta}_2 = 0.0341 \ \#$$

$$\hat{\beta}_1 = 3.2125 - (0.0341)(77.625)$$

$$= 0.5655 \ \#$$

1.2. $\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 x_i + u_i$

$\hat{y}_i = 0.44905 + 0.0356\, x_i + u_i$

$\hat{y}_i = \beta_1 + \beta_2 x_i \longrightarrow$ use to find $\hat{y}_i$ | from $y_i = \hat{u}_i + \hat{y}_i \rightarrow \hat{u}_i = y_i - \hat{y}_i$

$i=1$,	$0.5655 + 0.0341(63) = 2.7138$	$2.8 - 2.7138 = 0.0862$
$i=2$,	$0.5655 + 0.0341(72) = 3.0207$	$3.4 - 3.0207 = 0.3793$
$i=3$,	" " $(78) = 3.2253$	$3.0 - 3.2253 = -0.2253$
$i=4$,	" " $(81) = 3.3276$	$3.5 - 3.3276 = 0.1724$
$i=5$,	" " $(87) = 3.5322$	$3.6 - 3.5322 = 0.0678$
$i=6$,	" " $(75) = 3.123$	$3.0 - 3.123 = -0.123$
$i=7$,	" " $(75) = 3.123$	$2.7 - 3.123 = -0.423$
$i=8$,	" " $(90) = 3.6345$	$3.7 - 3.6345 = 0.0655$

$$\sum_{i=0}^{n} \hat{u}_i = 0.0862 + 0.3793 + (-0.2253) + (0.1724) + (0.0678) + (-0.123) + (-0.423) + 0.0655 = 0$$

1.3.) $\operatorname{var}(\hat{u}_i) = \dfrac{\sum \hat{u}_i^2}{n-2} = \dfrac{0.4347}{8-2} = 0.0725$

$$\operatorname{var}(\hat{\beta}_1) = \frac{\sum x_i^2}{n \sum x_i^2} \sigma^2 = \frac{48717\,(0.0725)}{8\,(48717)} = 0.0091$$

$$\operatorname{var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum_{i=1}^{n} x_i^2} = \frac{(0.0725)}{(48717)} = 0.00000149$$

2.1.) $\bar{x} = \dfrac{(10+12+14+16+18+22+24+26+28+30)}{10} = 20$

$$\bar{y} = \frac{(0+2+5+6+7+10+10+15+16+20)}{10} = 9.1$$

$$\sum x_i^2 = 10^2 + 12^2 + \ldots + 30^2 = 4440$$

$$\sum x_i y_i = 10(0) + 12(2) + 14(5) + \ldots + 30(30) = 2214$$

$$\hat{\beta}_2 = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{\sum x_i^2 - n\bar{x}^2} = \frac{2214 - 10(20)(9.1)}{4440 - 4000} = 0.895$$

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x} = 9.1 - (0.895)(20) = -8.81$$

2.2.)

$\hat{y}_i = \hat{\beta}_2 + \hat{\beta}_1 x_i$

- $i=1$, $y_i = -8.81 + 0.895(10) = 0.1455$
- $i=2$, " (12) = 1.9364
- $i=3$, " (14) = 3.7233
- $i=4$, " (16) = 5.5182
- $i=5$, " (18) = 7.3091
- $i=6$, " (22) = 10.8909
- $i=7$, " (24) = 12.6818
- $i=8$, " (26) = 14.4723
- $i=9$, " (28) = 16.2636
- $i=10$, " (30) = 18.0549

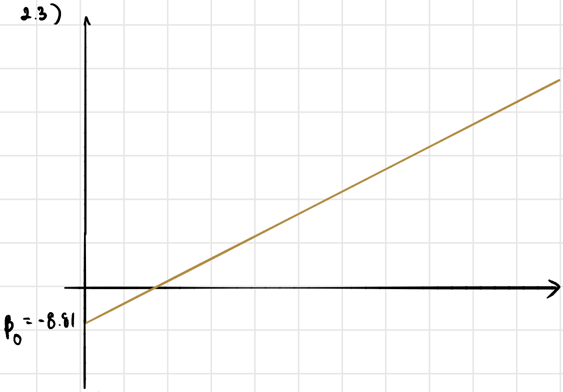
$\hat{u}_i \mid y_i = \hat{u}_i + \hat{y}_i \rightarrow \hat{u}_i = y_i - \hat{y}_i$

- $i=1$, $\hat{u}_1 = -0.145$
- $i=2$, $\hat{u}_2 = 0.064$
- $i=3$, $\hat{u}_3 = 1.273$
- $i=4$, $\hat{u}_4 = 0.482$
- $i=5$, $\hat{u}_5 = -0.3091$
- $i=6$, $\hat{u}_6 = -0.891$
- $i=7$, $\hat{u}_7 = -2.682$
- $i=8$, $\hat{u}_8 = 0.527$
- $i=9$, $\hat{u}_9 = -0.264$
- $i=10$, $\hat{u}_{10} = 1.945$

$$\therefore \sum_{i=1}^{10} \hat{u}_i = -0.145 + 0.064 + 1.273 + 0.482 - 0.3091 - 0.891 - 2.682 + 0.527 - 0.264 + 1.945$$

$$= 0$$

2.3)



Find the intersection with $(\bar{x}, \bar{y})$

$\bar{y} = -8.81 + 0.895(20)$

$9.1 = 9.1$ #

2.4) $y_i = \beta_0 + \beta_1 x_i$

$= -8.81 + 0.895(16)$

$= 5.51$

2.5) $Var(\hat{u}_i) = \sigma^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{14.091}{8} = 1.7614$

$Var(\hat{\beta}_i) = \frac{\sigma^2}{\sum(x_i - \bar{x})^2} = \frac{1.7614}{440} = 0.0040$

$Var(\hat{\beta}_2) = \frac{\sum x_i^2 (\sigma^2)}{n \sum (x_i - \bar{x})^2} = \frac{440(1.7614)}{10(440)} = 1.7774$

3.7) $\hat{\beta}_1 = \frac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i}{n \sum x_i^2 - (\sum x_i)^2} = \bar{y} - \hat{\beta}_2 \bar{x}$

$\hat{\beta}_1$ is an unbiased estimator

$E(\hat{\beta}_1) = E(\bar{y} - \beta_2 \bar{x})$

$= E(\bar{y}) - \beta_2 E(\bar{x})$

$= \beta_1 + \cancel{\beta_2 \bar{x}} - \cancel{\beta_2 \bar{x}}$

$E(\hat{\beta}_1) = \beta_1$