EE211
PRINCIPLES OF MICROECONOMICS

Topic 6:

The Theory of Consumer Choice

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The Theory of Consumer Choice

Topics

- Utility Theory (Cardinal Approach)
  - Utility
  - Marginal utility
  - Law of diminishing marginal utility
- Indifference Curve Theory (Ordinal Approach) Rank
  - Indifference curve
  - Marginal rate of substitution
  - Budget line
  - Consumer equilibrium
- Demand curve derivation
- Applications

Part I: Utility Theory (Cardinal Approach)

- **Utility** is the *satisfaction* or *well-being* that a consumer receives from consuming some good or service.
  - Economists assume that, in making their choices, consumers are motivated to maximize their utility.

- **Total utility** is the total satisfaction resulting from consumption of a given commodity by a consumer.

(MU)

- **Marginal utility** is the *additional satisfaction* obtained by a consumer from consuming one *additional unit* of a commodity.
  - $MU_x = \frac{\Delta U}{\Delta X}$ $\approx \frac{du}{dx}$ (derivative of u w.r.t. x)

Example: Utility Schedules

MU's are +ve.

Given

Number of Coffee Tom Drinks per day	Tom's (TU) Total Utility	Tom's (MU) Marginal Utility
0	0	
1	30	ΔU = 30
2	50	ΔU = 20
3	65	ΔU = 15
4	75	ΔU = 10
5	83	ΔU = 8
6	89	ΔU = 6

$\Delta X = 1$

$\Delta X = 1$

Big

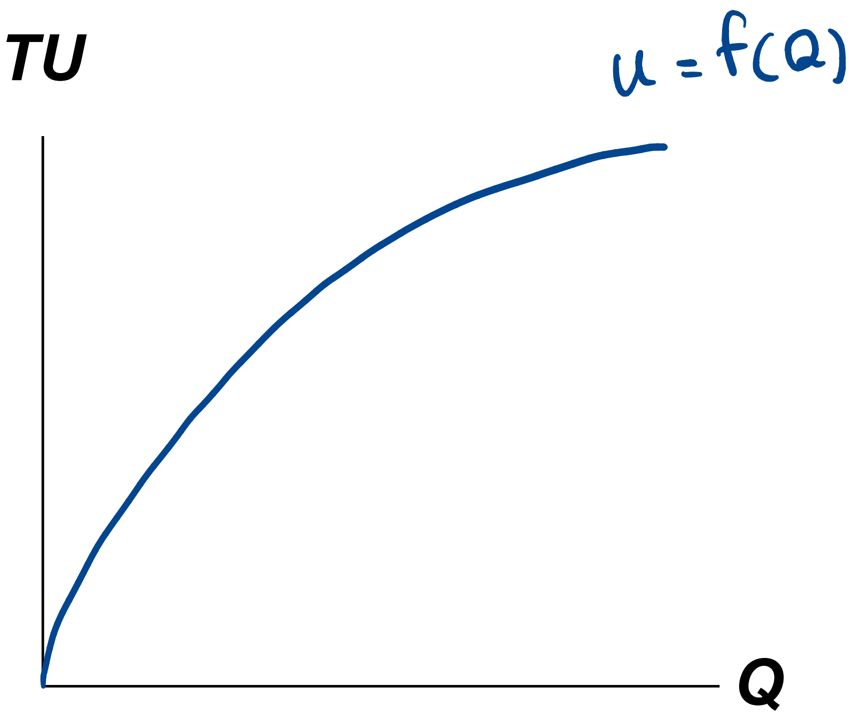
small

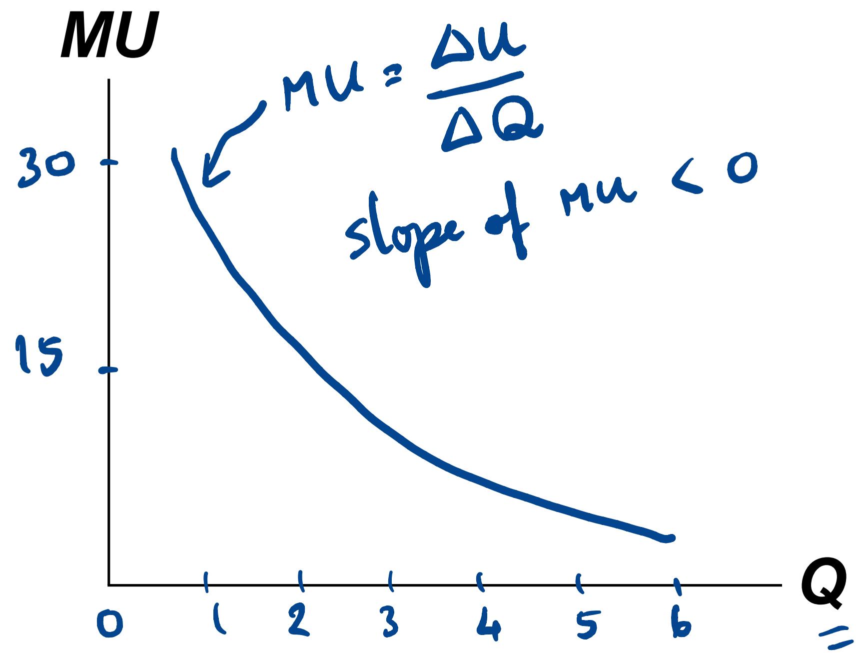
Q: Can MU of any good be negative?

→ Yes.

Example: Utility Graphs

- Total Utility
- Marginal Utility





Total utility rises, but marginal utility decreases, as consumption rises.

Diminishing Marginal Utility

- ***Law of diminishing marginal utility***

"The utility that any consumer derives from *successive* units of a particular product consumed over some period of time *diminishes* as total consumption of the product increases, if the consumption of all other products is unchanged."

Maximizing Utility

Given P_x and P_Y

- The consumer's decision: (in Case there are 2 consumption goods : X and Y)

A utility-maximizing consumer allocates expenditures so that the utility obtained from the last dollars spent on each product is equal.

optimal

Consumer's utility maximization :

$$\max_{X,Y} U(X,Y) \quad \text{subject to} \quad P_X \cdot X + P_Y \cdot Y = I$$

income

Mathematically,

$$\boxed{\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}}$$

$$\Updownarrow$$

Alternatively,

$$\boxed{\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}}$$

$\rightarrow (X^*, Y^*)$

optimal bundle of X & Y.

Example: Utility Maximization

- Suppose you are buying Good X for \$1 each and Good Y for \$3 each, and the marginal utility of each good is the following.

P_x $= P_Y$

$$\frac{MU_x}{P_x} = \frac{MU_Y}{P_Y}$$

Quantity of X

# of X	MU_x	$MU_x/P_x = \frac{MU_x}{1}$
1	30	30
2	21	21
3	15	$\boxed{15}$

# of Y	MU_y	$MU_y/P_y = \frac{MU_Y}{3}$
1	60	20
2	51	17
3	45	$\boxed{15}$

$\Rightarrow (X^*, Y^*) = (3,3)$

How many units of goods X and Y should you buy in order to maximize your utility?

What if $\overset{\uparrow x}{\frac{MU_x}{P_x}} > \overset{\downarrow Y}{\frac{MU_Y}{P_Y}}$? ← not in eqm

Should you consume more of x or more of Y?

Goal:

$$\frac{MU_x}{P_x} = \frac{MU_Y}{P_Y}$$

If $\frac{MU_x}{P_x}$ is higher,

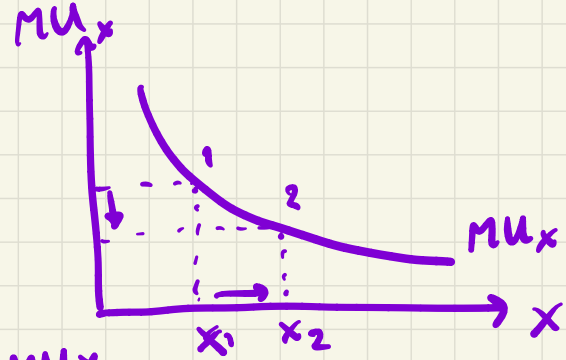
⇒ want to consume more of x

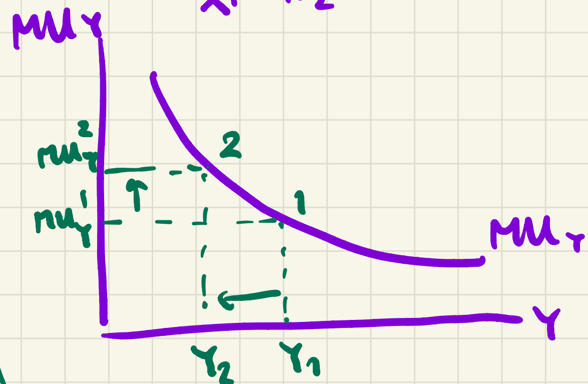
⇒ $MU_x \downarrow$

If $\frac{MU_Y}{P_Y}$ is lower,

⇒ want to consume less of Y

⇒ $MU_Y \uparrow$





$$\boxed{\frac{MU_x}{P_x} = \frac{MU_Y}{P_Y}}$$ ← eqm ✓✓

Part II. Indifference Curve Theory (Ordinal Approach)

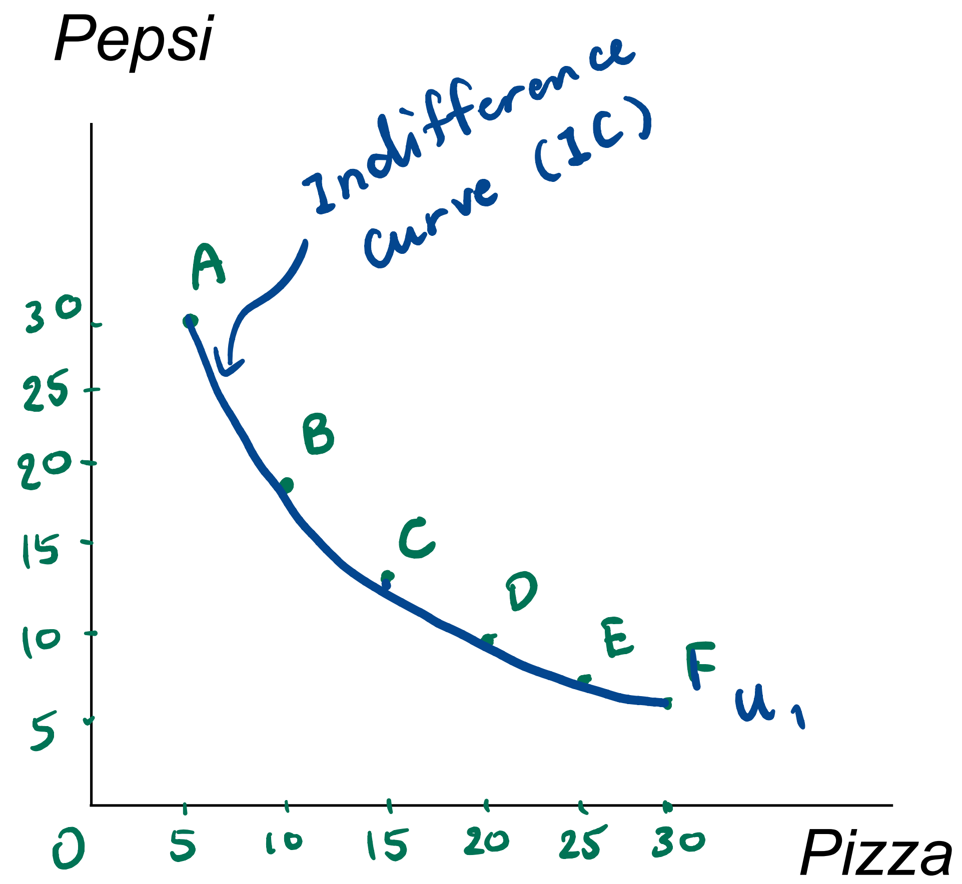
- There is **no** numerical value attached to consumer's utility.
- Instead, consumer is asked which bundle is preferred to which.
- **Indifference curve** shows consumption bundles that give the consumer the same level of satisfaction.
- With 2 products X and Y, a bundle is represented by a point in diagram.

Example: Indifference Curve

units

Bundle	Pepsi	Pizza
A	30	5
B	18	10
C	13	15
D	10	20
E	8	25
F	7	30

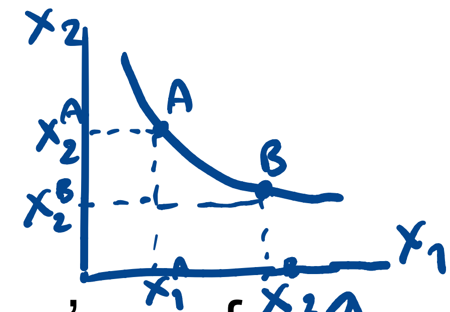
same utility level : u_1



Assumptions

A (x_1^A, x_2^A)

B (x_1^B, x_2^B)



- Given any 2 bundles A & B, the consumer's preference is assumed to be exactly one of the followings:

✓ 1. A is preferred to B ($A \succ B$). , OR

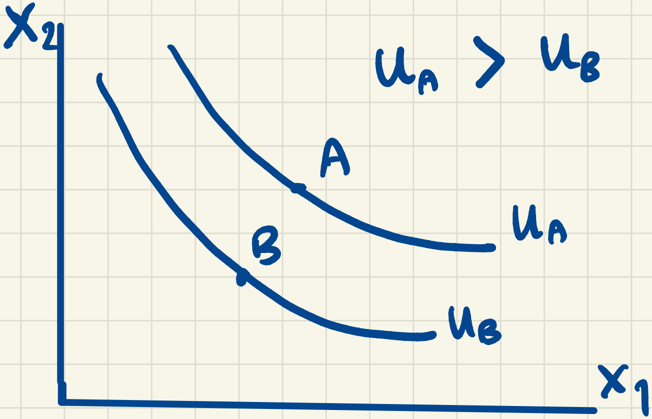
✓ 2. B is preferred to A ($A \succ B$). , OR

✓ 3. A & B are indifferent ($A \sim B$).

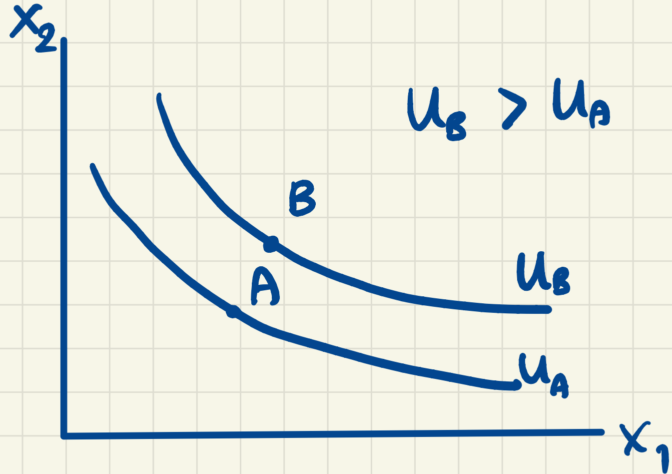
- Consumer is assumed to be rational.
  - More is preferred to less.
  - If $A \succ B$ and $B \succ C$, then $A \succ C$.

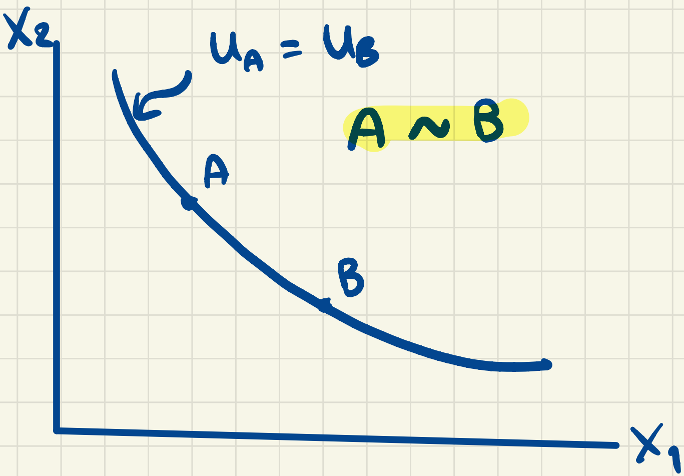
preferred to

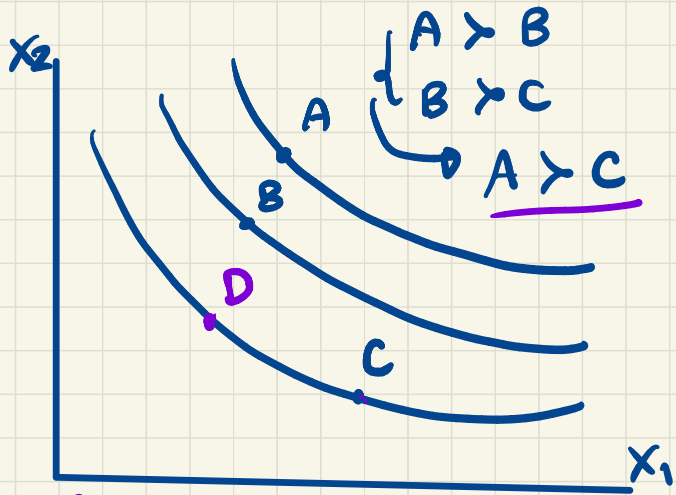
$A \succ B$



$B \succ A$







If $C \sim D$, $A \succ C$

Marginal Rate of Substitution

- **Marginal rate of substitution (MRS)** is the rate at which a consumer is willing to trade one good for another.
- Also, it is the slope of the indifference curve.

$MRS = -\frac{MU_X}{MU_Y} = \frac{\Delta Y}{\Delta X}$

- Basic assumptions of indifference curve theory:

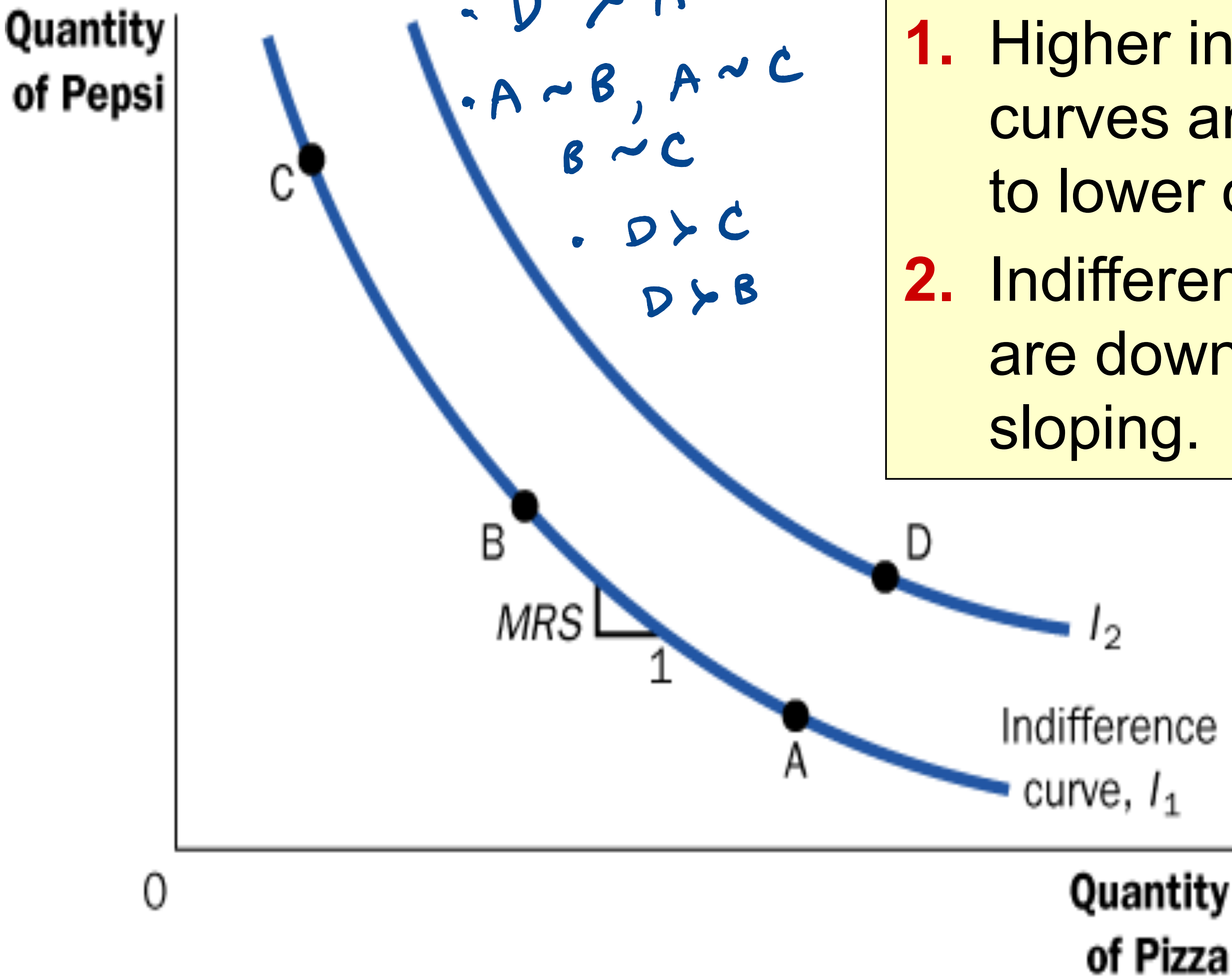
1. The algebraic value of the MRS between two goods is always negative.
2. The MRS is diminishing (i.e. any indifference curve becomes flatter as the consumer moves downward and to the right along the curve).

Example: Indifference Curve& MRS

	Y	X
Bundle	**Pepsi**	**Pizza**
A	Y_A 30	5
B	Y_B 18	10
C	13	15
D	10	20
E	8	25
F	7	30

	ΔY	ΔX	$= \frac{\Delta Y}{\Delta X}$
Change	**Change in Pepsi**	**Change in Pizza**	**MRS**
A→B	-12	5	-2.4
B→C	-5	5	-1
C→D	-3	5	-0.6
D→E	-2	5	-0.4
E→F	-1	5	-0.2

Properties of Indifference Curve



- D ≻ A
- A ~ B, A ~ C, B ~ C
- D ≻ C, D ≻ B

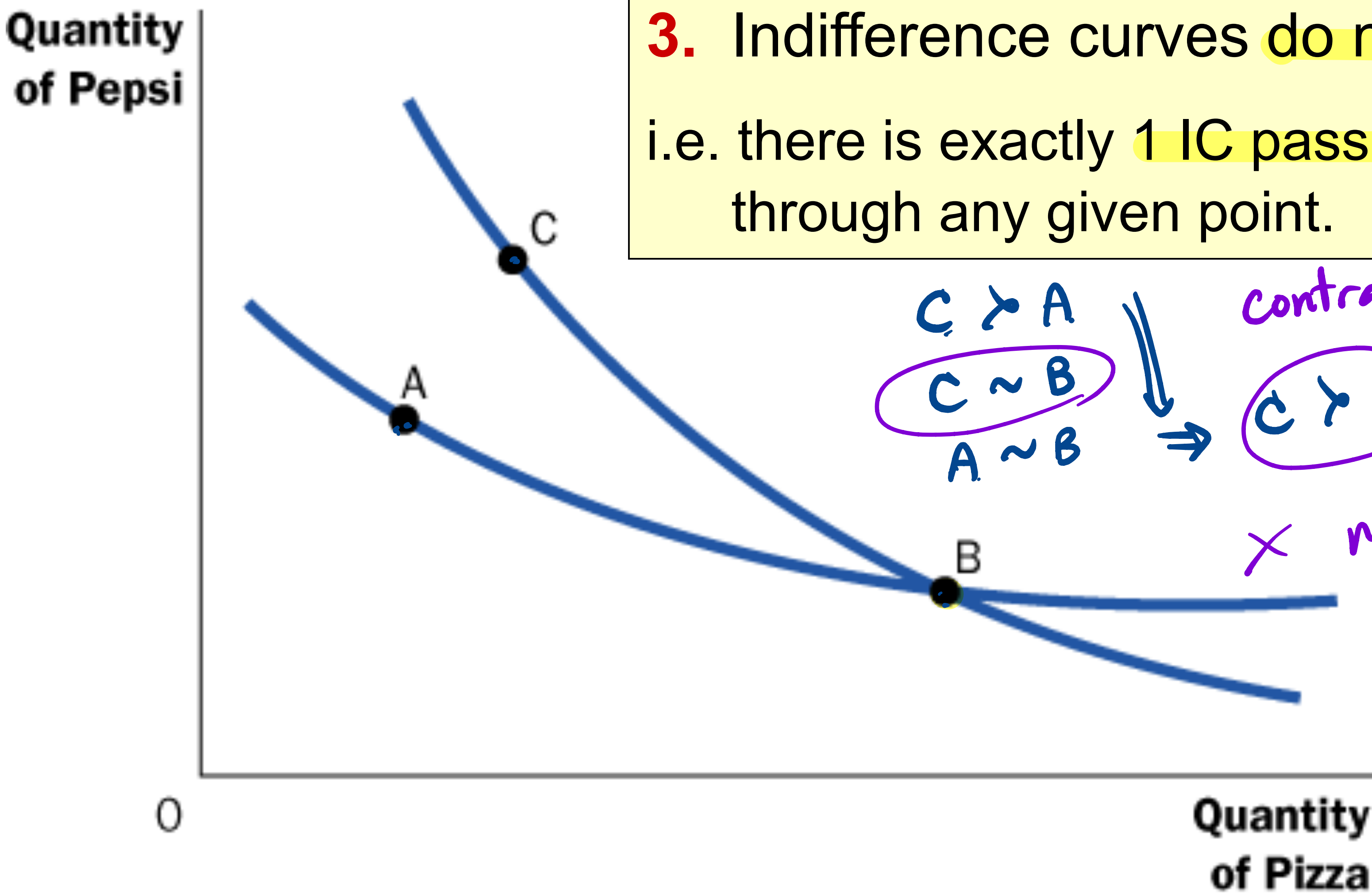
1. Higher indifference curves are preferred to lower ones.
2. Indifference curves are downward sloping.

MRS < 0

Properties of Indifference Curve

3. Indifference curves do not cross.

i.e. there is exactly 1 IC passing through any given point.



$C \succ A$

$C \sim B$

$A \sim B$

$\Rightarrow C \succ B$

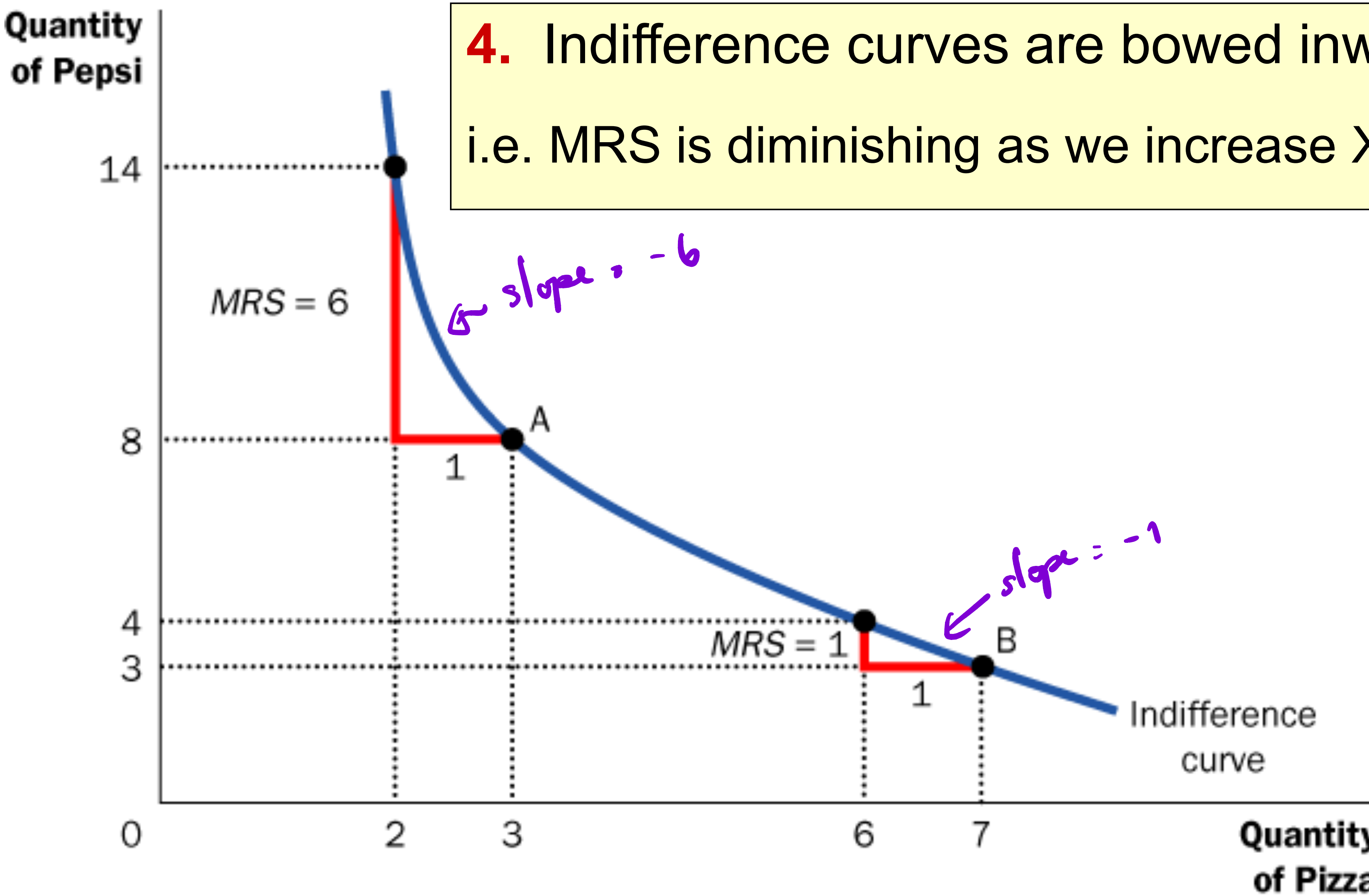
contradiction!

× not possible.

Properties of Indifference Curve

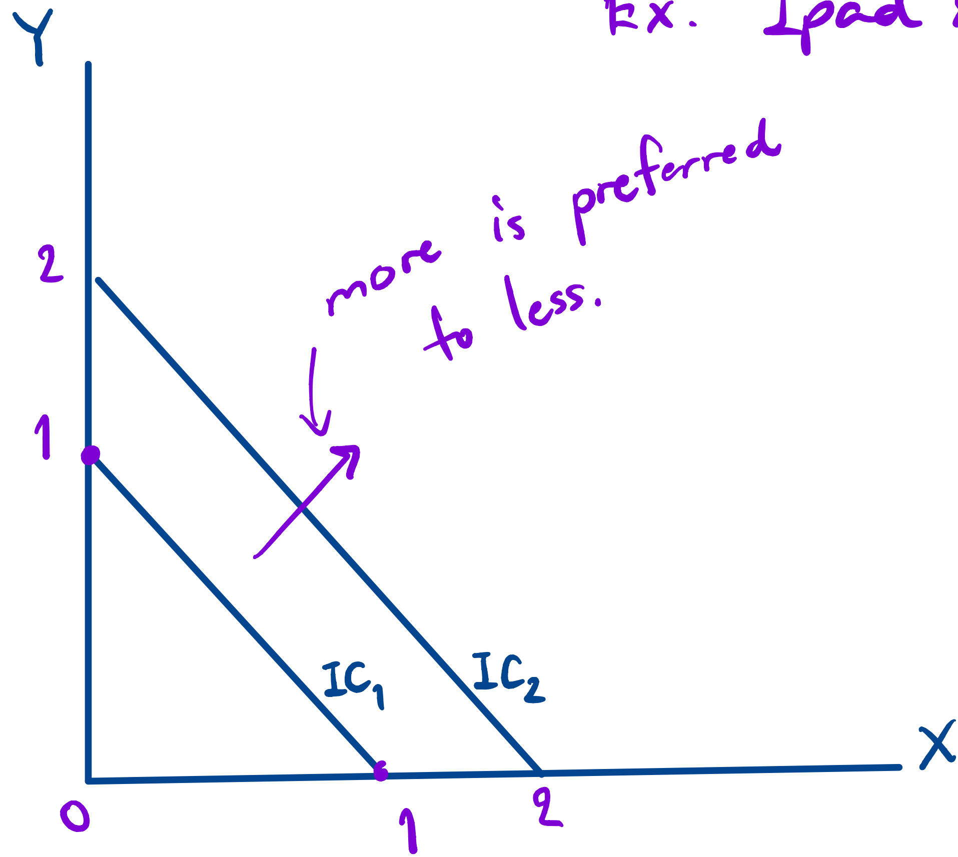
4. Indifference curves are bowed inward.

i.e. MRS is diminishing as we increase X.

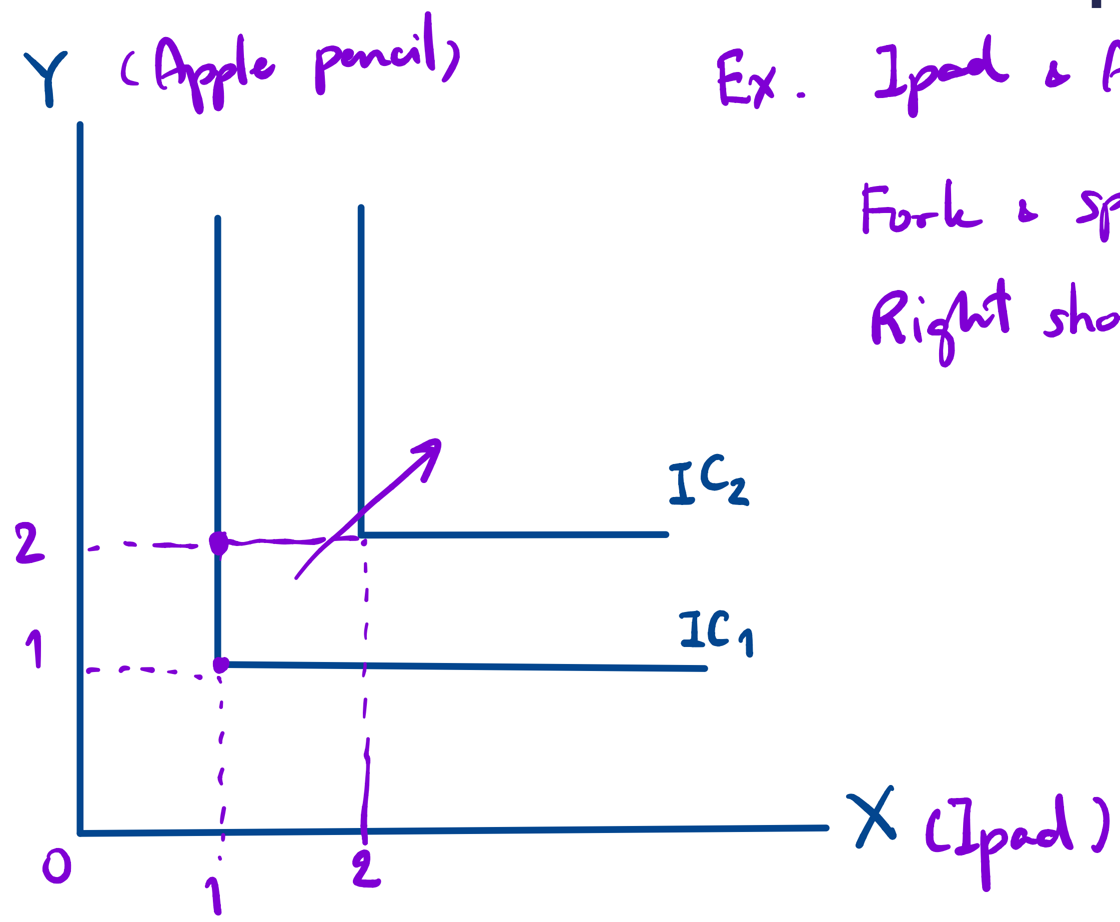


Extreme Case: Perfect Substitutes

Ex. Ipad & Galaxy Note



Extreme Case: Perfect Complements



Ex. Ipad & Apple pencil.

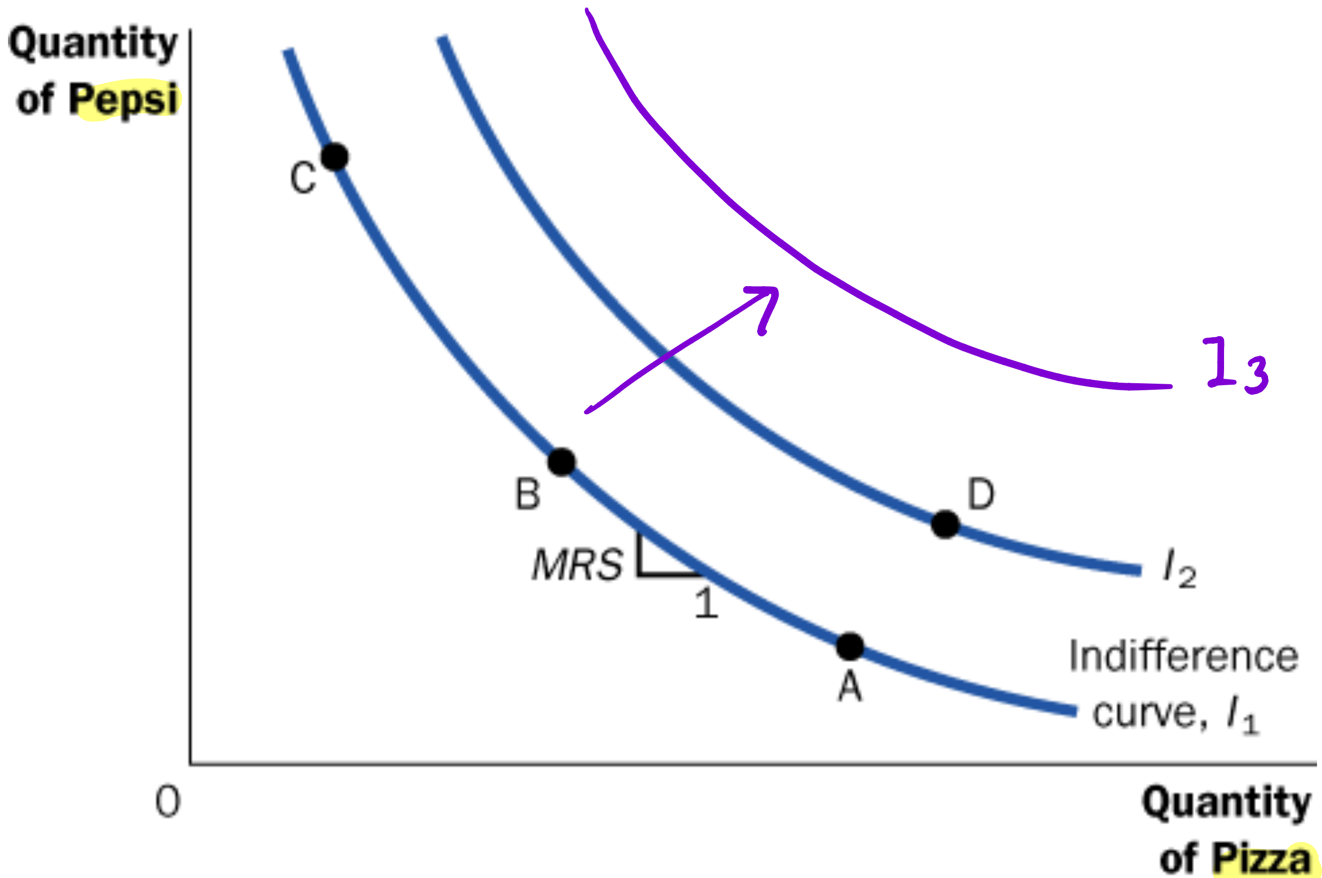
Fork & spoon.

Right shoe & Left shoe.

Part III. Consumer's Equilibrium

① IC
② Budget constraint

- Recall: **Indifference curve** shows consumption bundles that give the consumer the same level of satisfaction.



Extra Notes on Indifference Curve (1)

- MRS is the slope of the indifference curve, and MRS is diminishing (in absolute value) as X increases.
- At any point on indifference curve, the slope is $\frac{\Delta Y}{\Delta X}$.
- As the consumer consumes more X and reduces consumption of Y, her utility changes by:

$U(X, Y)$

$$\Delta X > 0 \rightarrow \Delta U_x \approx MU_x \times \Delta X$$

$$\Delta U = \Delta U_X + \Delta U_Y$$

$$\Delta Y < 0 \rightarrow \Delta U_y \approx MU_y \times \Delta Y$$

- But on the an indifference curve, the utility is the same:

$$\Delta U = \rightarrow \Delta U_x + \Delta U_y = 0$$

Extra Notes on Indifference Curve (2)

$\Delta U = \Delta U_X + \Delta U_Y = 0 \iff MU_X \cdot \Delta X + MU_Y \cdot \Delta Y = 0$

- So,

$$\Delta U_x = -\Delta U_y$$

$$\mathrm{MU_x}\Delta X \approx -\mathrm{MU_y}\Delta Y$$

$$\rightarrow \quad \frac{\Delta Y}{\Delta X} \approx -\frac{MU_x}{MU_y}$$

(Discrete change)

$$\rightarrow \quad \frac{dY}{dX} = -\frac{MU_x}{MU_y}$$: MRS

= slope of IC.

Continuous change

Budget Constraint

- **Budget constraint (or budget line)**: the limit on the consumption bundles that a consumer can afford.
  - ➢ It shows all combinations (bundles) of the two goods that the consumer can afford to buy.
- Consider the case of 2 goods: Pizza (X) and Pepsi (Y). Suppose P_x = \$10, P_y = \$2, and budget (B) = \$1000. (Given)
- The budget line can be written as:

Assume consumer spends all the budget.

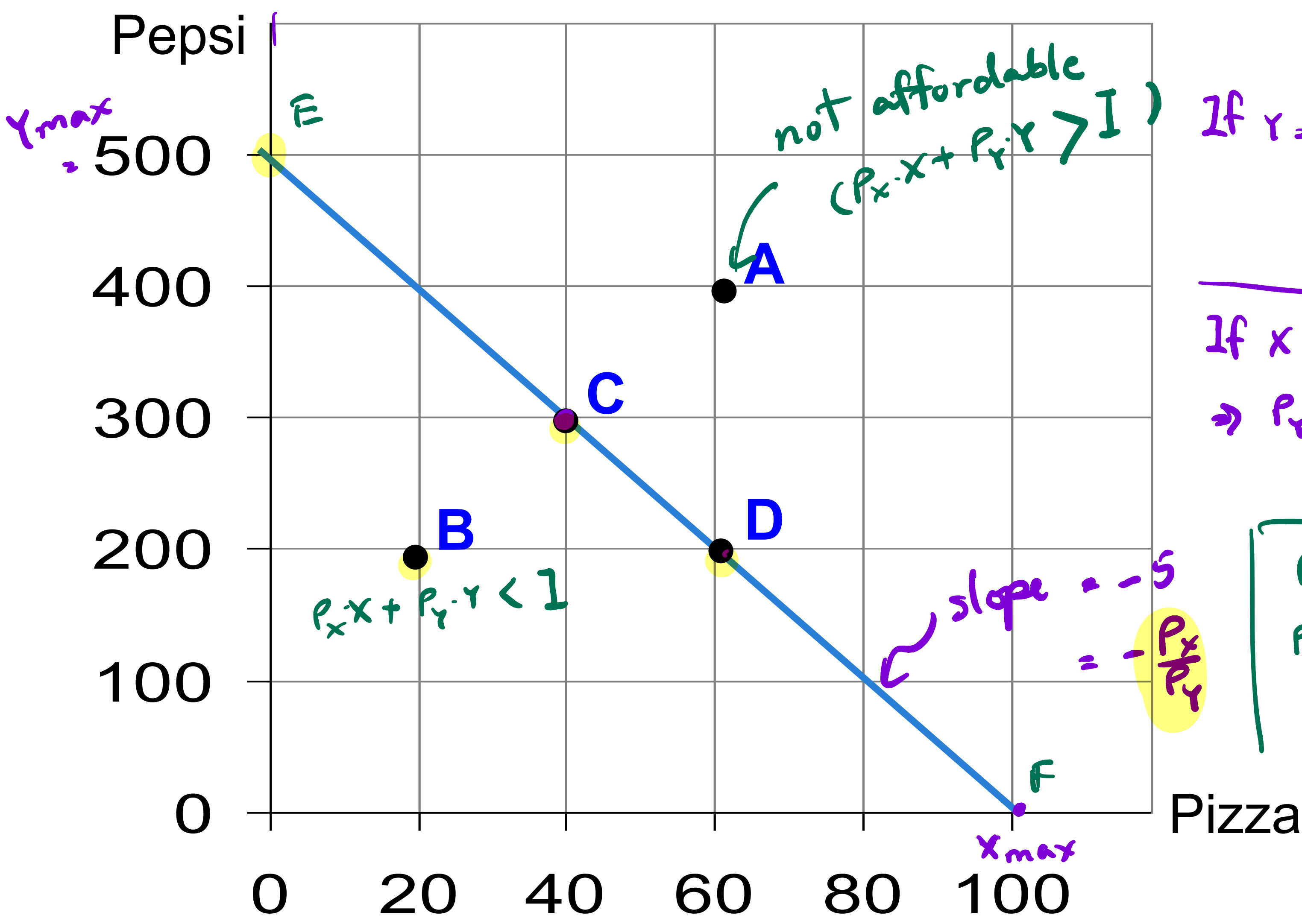
$$P_x X + P_y Y = B$$

total expenditures

total income (no savings, no borrowing).

Graph: Budget Constraint

$$\overset{10}{P_x} \cdot X + \overset{2}{P_Y} \cdot Y = 1000$$



If $X = 0$, $Y_{max} = \frac{1000}{2} = 500$

If $Y = 0$, $X_{max} = \frac{1000}{10} = 100$

If $X = 40$, $P_x \cdot X = 400$

$\Rightarrow P_Y \cdot Y = 600$

$Y = 300$

$P_x \cdot X + P_Y \cdot Y = B$

$P_Y \cdot Y = B - P_x \cdot X$

$Y = \frac{B}{P_Y} - \frac{P_x}{P_Y} \cdot X$

Slope of the Budget Constraint

- The slope of the budget constraint equals:
  - the rate at which the consumer can trade Pepsi for pizza (Given or determined by the market).
  - the opportunity cost of pizza in terms of Pepsi
  - the relative price of pizza.

That is:

$$\boxed{Slope\ of\ Budget\ line = -\frac{P_x}{P_y}}$$

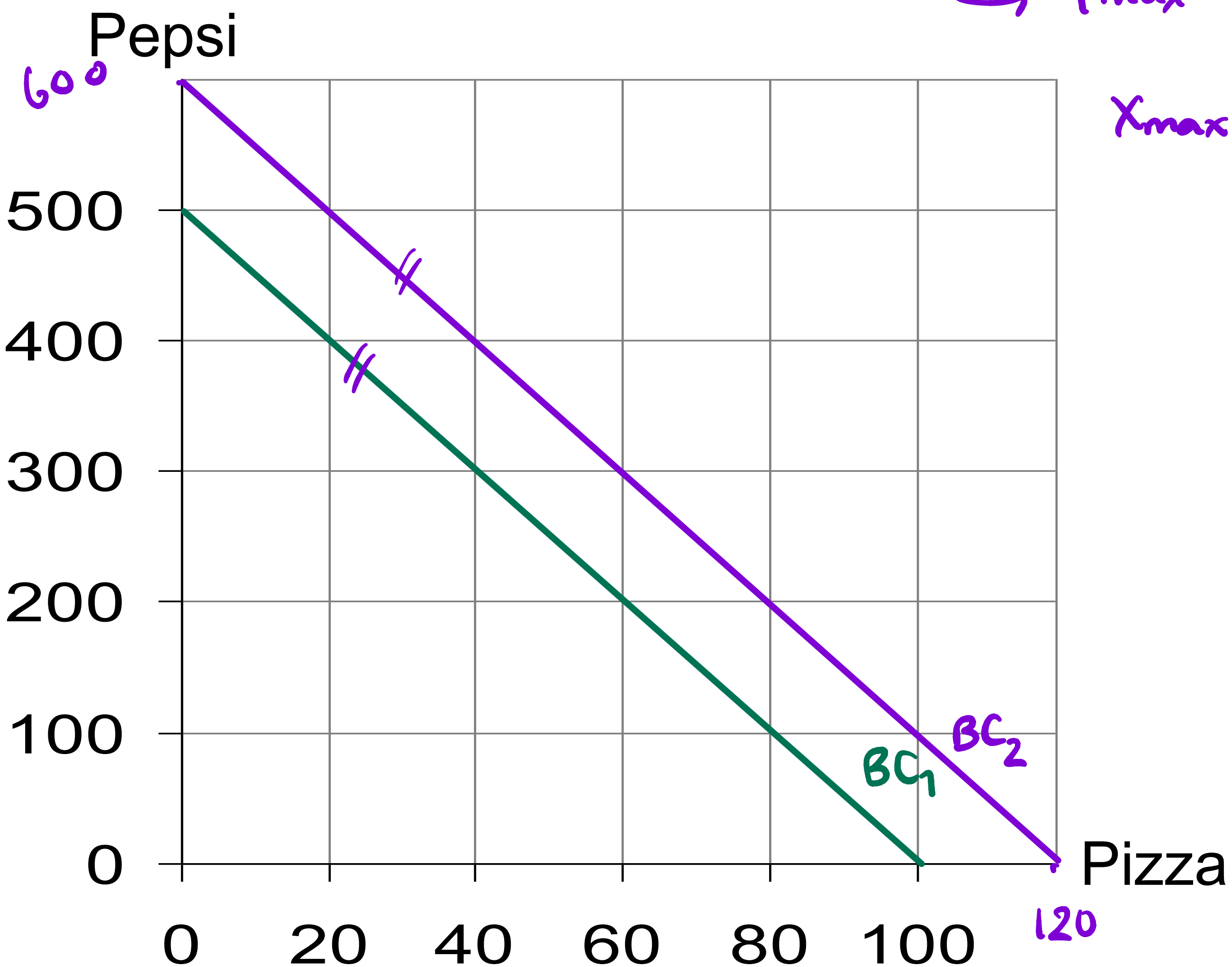
Change in Budget Constraint: Higher Income (I)

- Suppose budget increases to $1200. ($I_2$)

$I_1 = 1000$

$\rightarrow Y_{max} = \frac{1200}{2} = 600$

$X_{max} = \frac{1200}{10} = 120$



Consumer's Problem: Optimization

$\max_{X,Y} U(X, Y)$ subject to $P_X \cdot X + P_Y \cdot Y = \text{Income}$ $\Rightarrow X^*, Y^*$

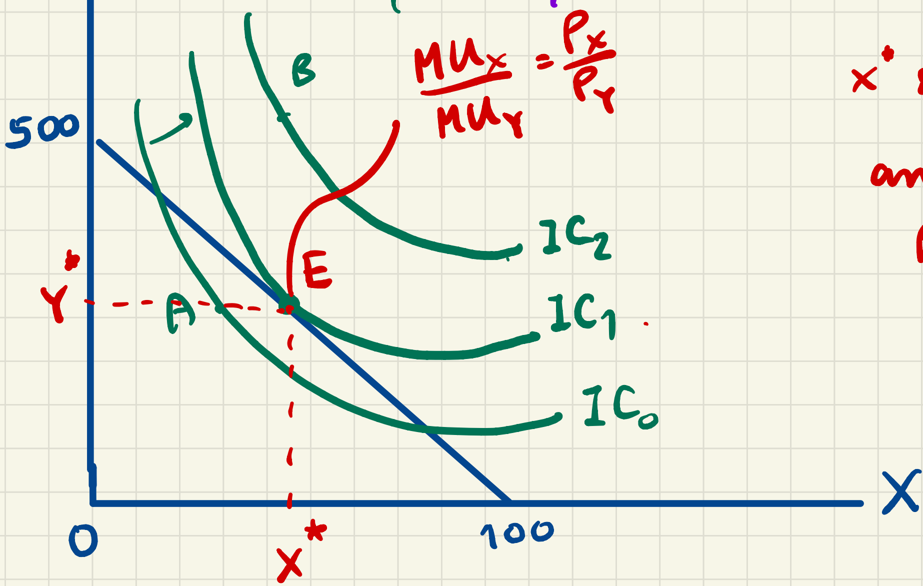
- Consumer's problem is to maximize his/her utility (i.e. satisfaction) under the budget constraint.
- The **optimal bundle** is at the point where the budget constraint touches the highest indifference curve.
  - i.e., the indifference curve and budget constraint have the same slope. $= -\frac{MU_X}{MU_Y}$ $\quad -\frac{P_X}{P_Y}$
- Since the slope of IC is the MRS and the slope of the budget constraint is the relative price, the optimal bundle is where: $\boxed{\frac{P_x}{P_y} = \frac{MU_x}{MU_y}}$ $\Leftrightarrow$ $\boxed{\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}}$ ☆☆☆

Utility Maximization

$P_x = \$10$, $P_Y = \$2$

Budget $= \$1{,}000$

$$\overbrace{\frac{MU_x}{MU_Y}}^{\text{slope of IC}} = \overbrace{\frac{P_x}{P_Y}}^{\text{slope of BC}}$$



X^* & Y^* maximize U

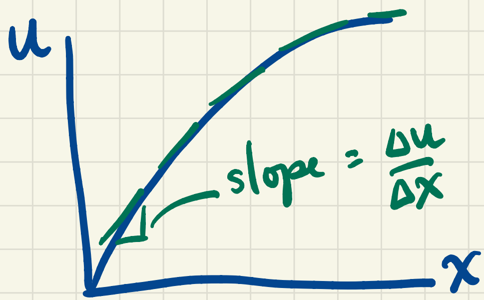
and

$P_x \cdot X^* + P_Y \cdot Y^* = 1000$

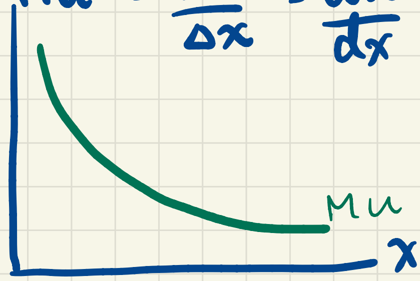
Midterm

Utility – consumer's satisfaction

- 1 good → $U = U(x) = f(x)$ — marginal utility

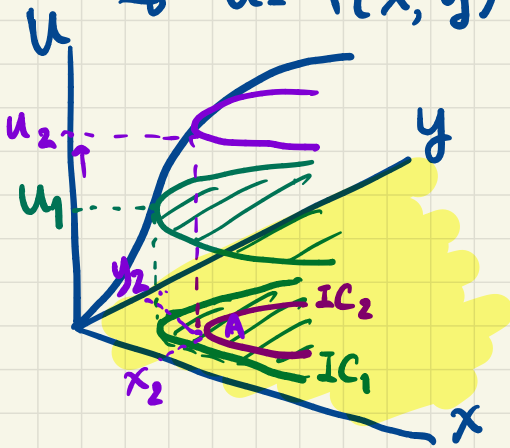


$MU = \frac{\Delta U}{\Delta x} = \frac{dU}{dx}$

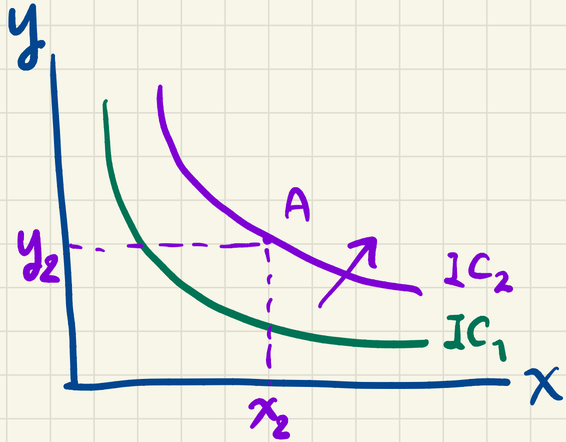


U increases at a decreasing rate!

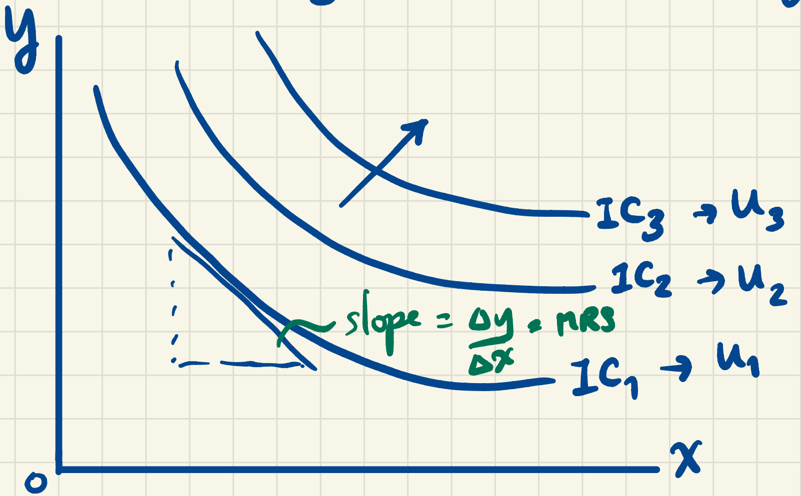
- 2 good → $U = f(x, y)$



⇒



① Utility

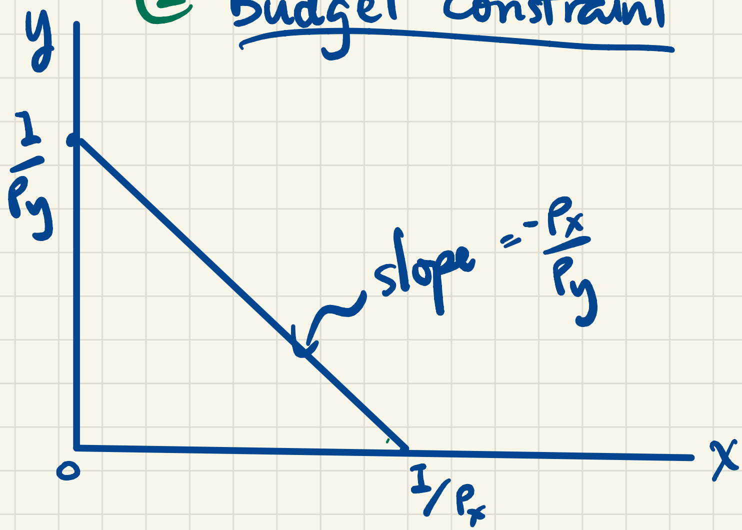


$$u = f(x, y)$$

$$\text{slope of IC} = \frac{\Delta y}{\Delta x}$$

$$\text{MRS} = \frac{-MU_x}{MU_y}$$

② Budget constraint



Suppose income (I) is fixed.

$$P_x \cdot X + P_y \cdot y = I$$

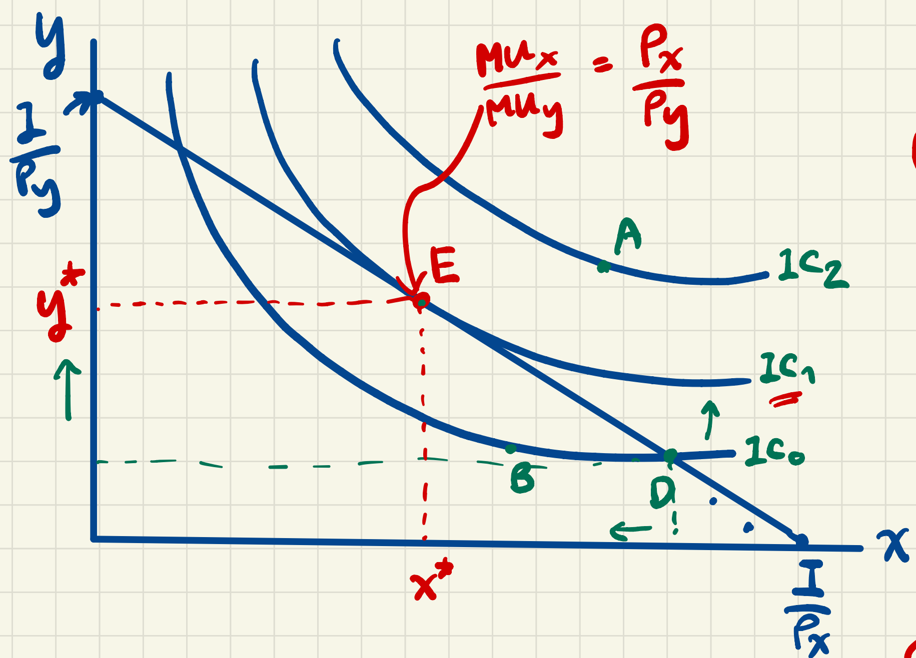
$$\text{slope of BC} = -\frac{P_x}{P_y}$$

$$\left(y = \frac{I}{P_y} - \frac{P_x}{P_y} x \right)$$

Consumer's Utility Maximization

**

$$\max_{x,y} \; U(x,y) \quad \text{subject to} \quad P_x \cdot x + P_y \cdot y = I$$



At eqm,

① slope of IC = slope of BC

ie. $MRS = \frac{-P_x}{P_y}$

$$\frac{MU_x}{MU_y} = \frac{P_x}{P_y}$$

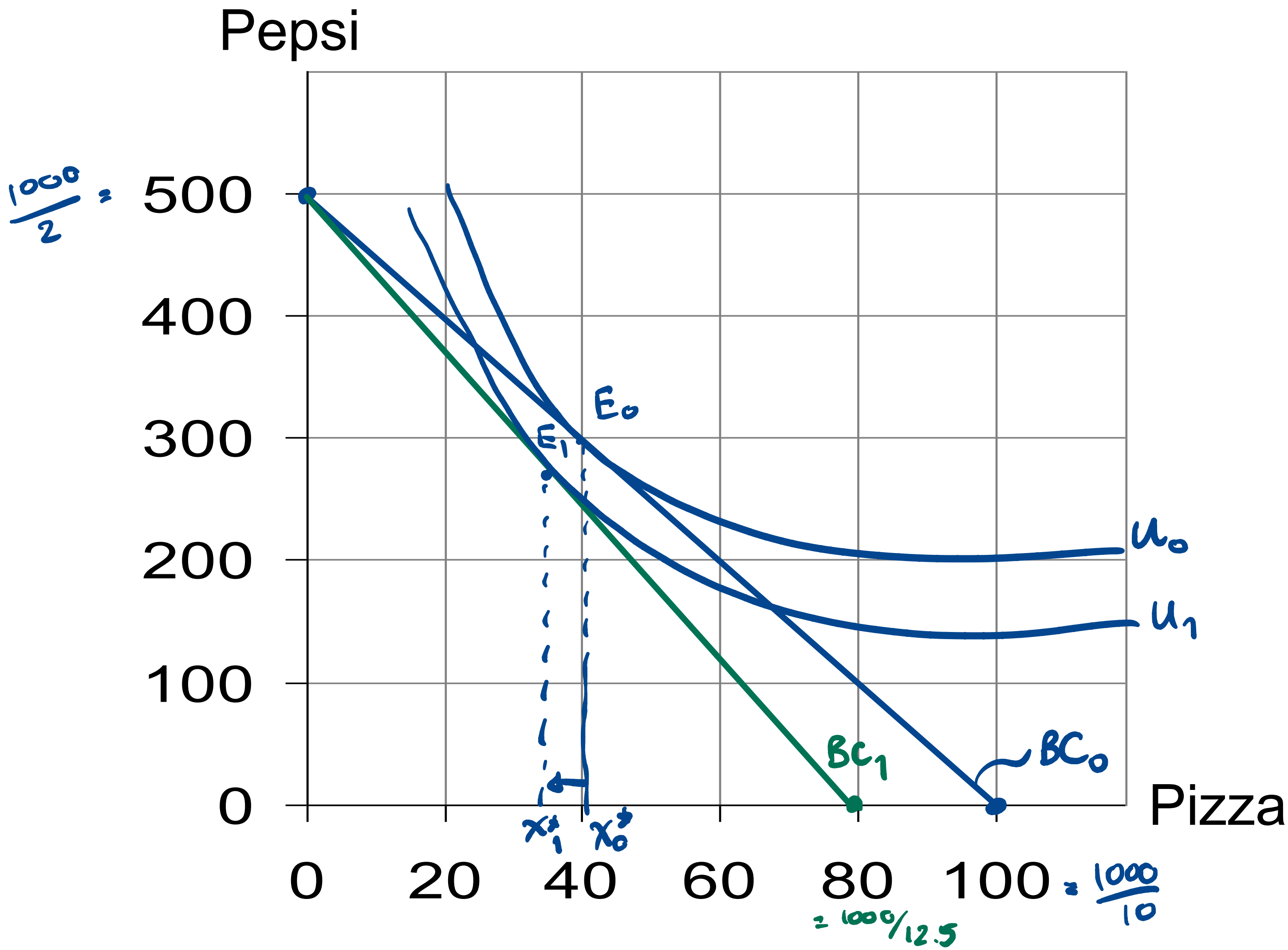
② $P_x \cdot x^* + P_y \cdot y^* = I$

Change in Budget Constraint: Px changes.

P_x = \$10, P_y = \$2, and budget (B) = \$1000.

- Suppose P_x increases from \$10 to \$12.5.

New $x_{max} = \frac{1000}{12.5} = 80$



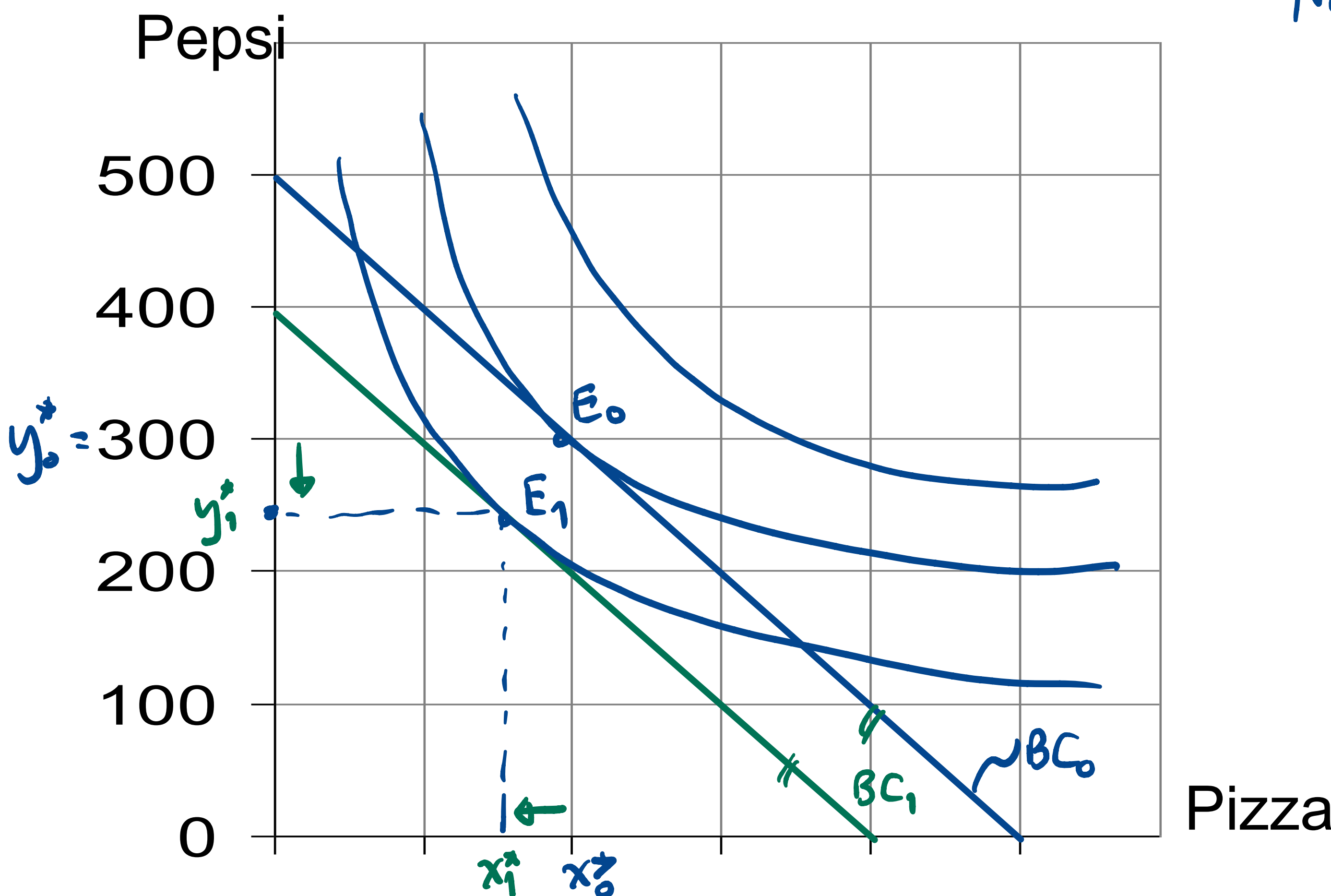
Change in Budget Constraint: Px & Py change by the same proportion.

- Suppose P'_x = \$12.5 and P'_y = \$2.5.

$I = 1000 \rightarrow$ New $X_{max} = \frac{1000}{12.5}$

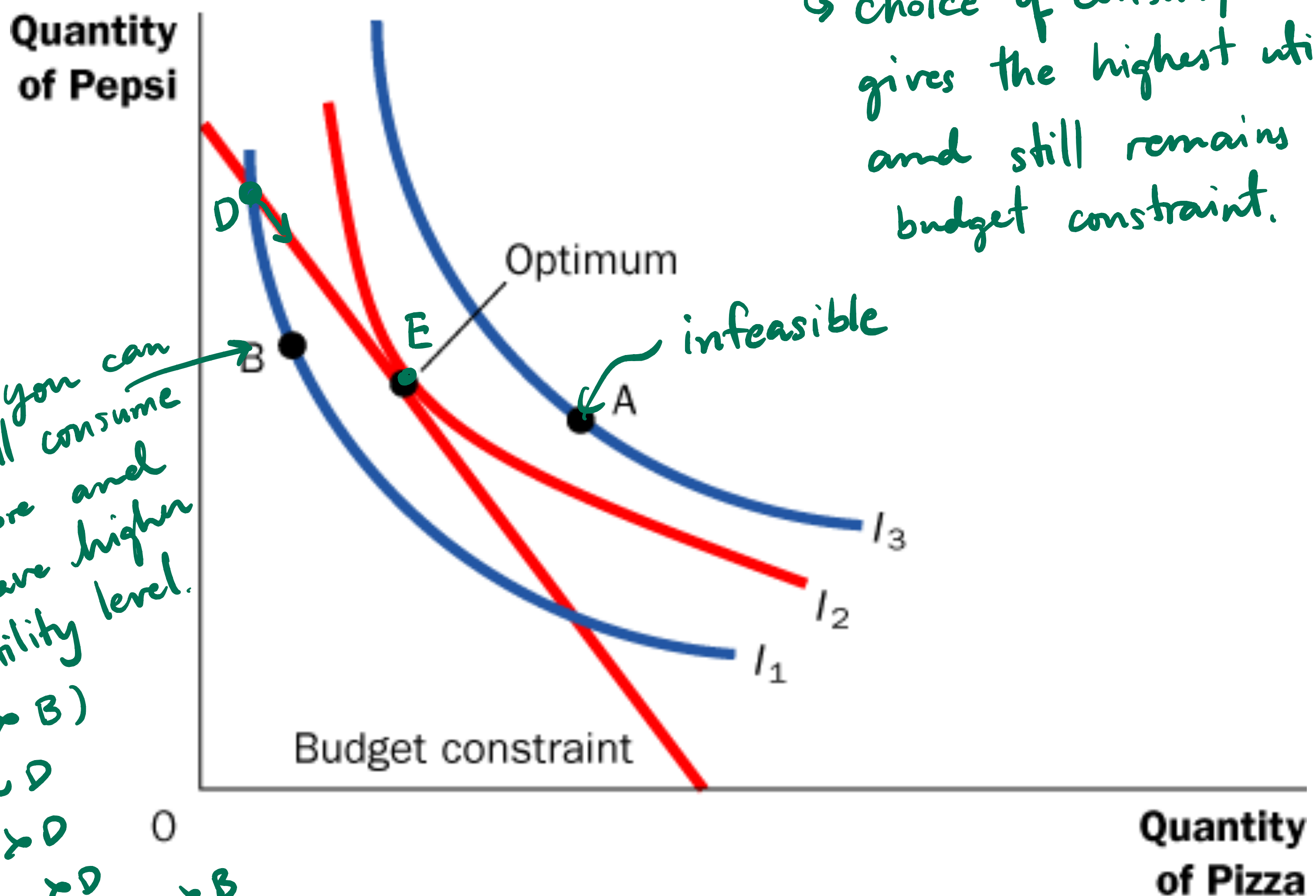
$= 80$

New $Y_{max} = \frac{1000}{2.5} = 400$



Graph: Consumer's Optimal Choice

↳ choice of consumption that gives the highest utility and still remains on the budget constraint.



still you can consume more and have higher utility level.

(E ≻ B)

∵ B ~ D

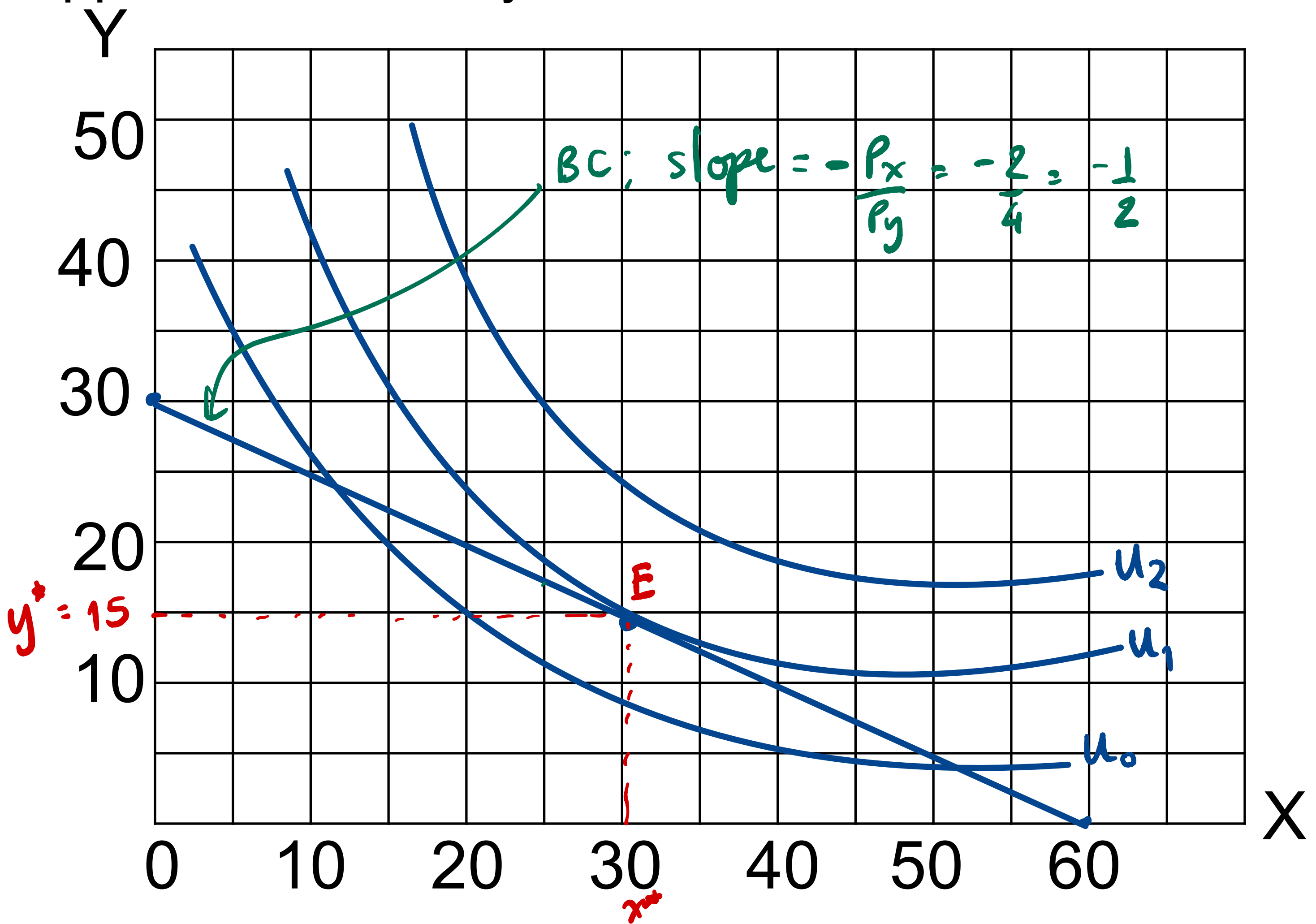
E ≻ D

⇒ E ≻ D ⇒ E ≻ B

Example: Optimization

- Suppose Px = \$2, Py = \$4, and B = 120.

$X_{max} = ?$
$Y_{max} = ?$



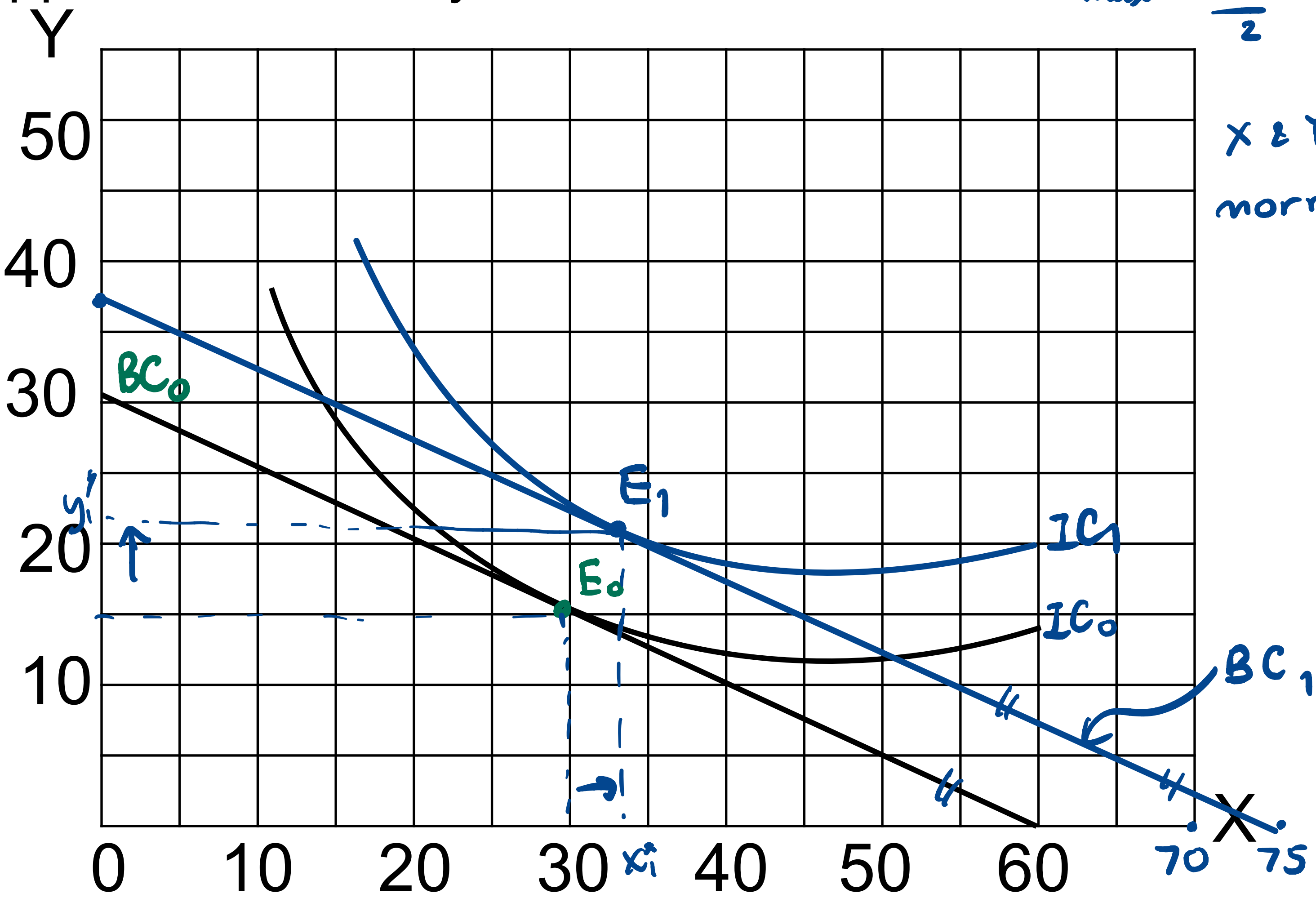
Example: Effect of Income Increase

$B_0 = 120$

$Y'_{max} = \frac{150}{4} = 37.5$

$X'_{max} = \frac{150}{2} = 75$

- Suppose Px = \$2, Py = \$4, and $B_1 = 150$. ($\underbrace{\text{Px = \$2, Py = \$4}}_{\text{constant}}$)



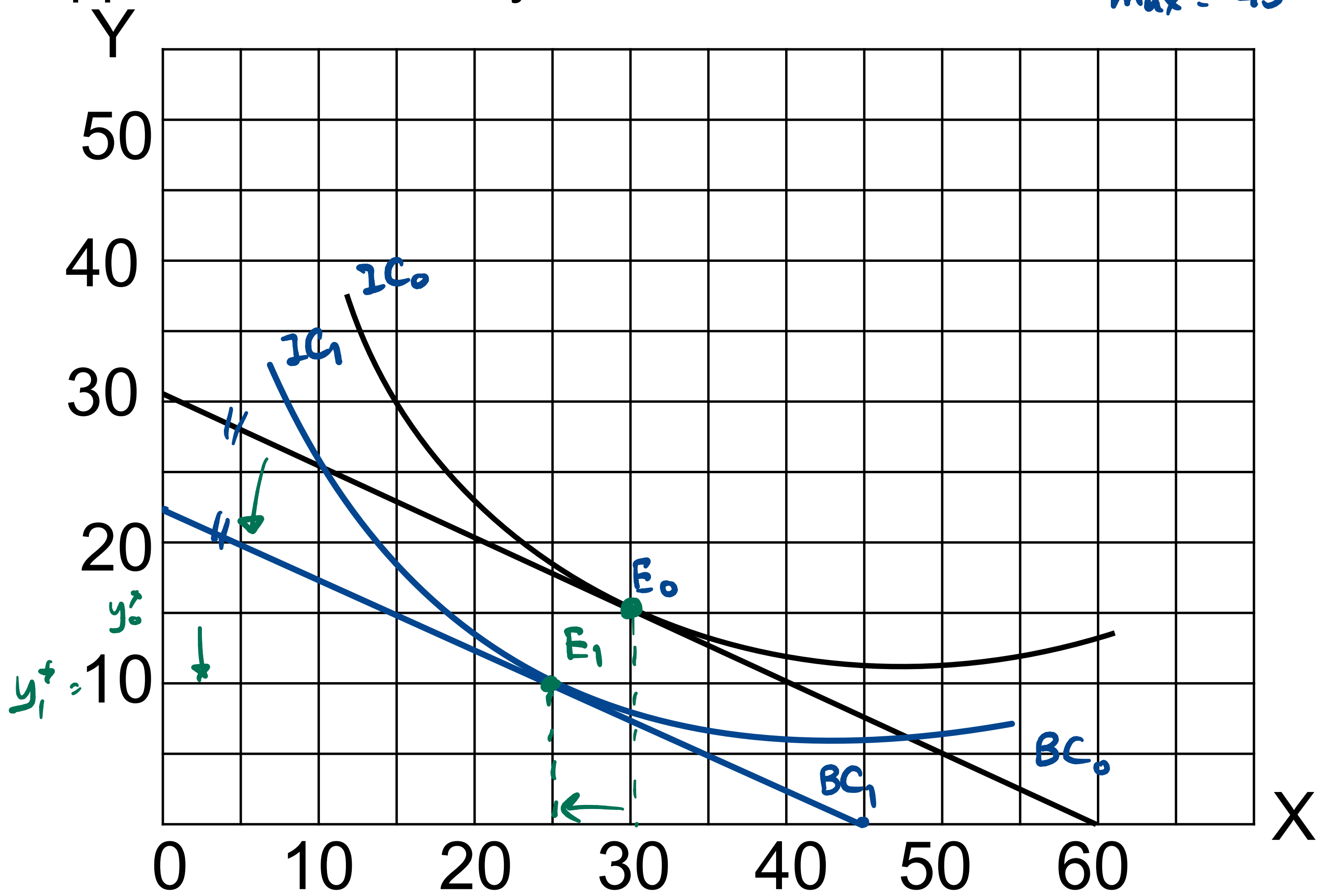
X & Y are normal goods.

Example: Effect of Income Reduction

- Suppose Px = \$2, Py = \$4, and B = 90.

$Y'_{max} = \frac{90}{4} = 22.5$

$X'_{max} = 45$

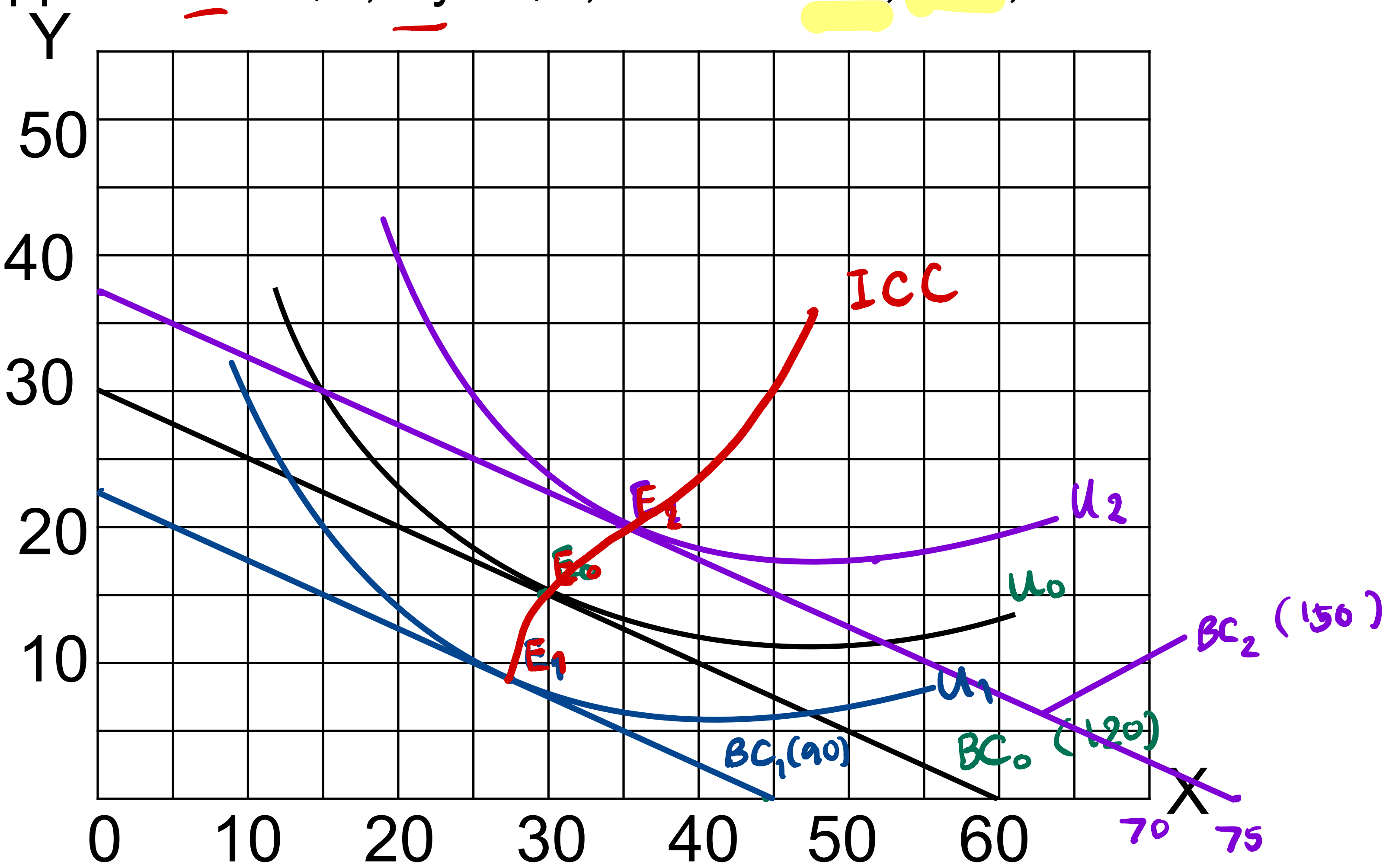


Income Consumption Curve (ICC)

- A change in income, *ceteris paribus*, will shift the consumer's budget constraint.
- For each level of income, there will be a utility maximizing points where IC is tangent to the relevant budget line.
- **Income Consumption Curve (ICC)** is the line that connects all the utility-maximizing points for different levels of income, given prices Px and Py constant.
- I.e. , ICC shows how the consumer's purchases react to a change in money income with relative prices being held constant.

Graph: ICC

- Suppose Px = $2, Py = $4, and B = 90, 120, 150.



Exercise: Inferior good

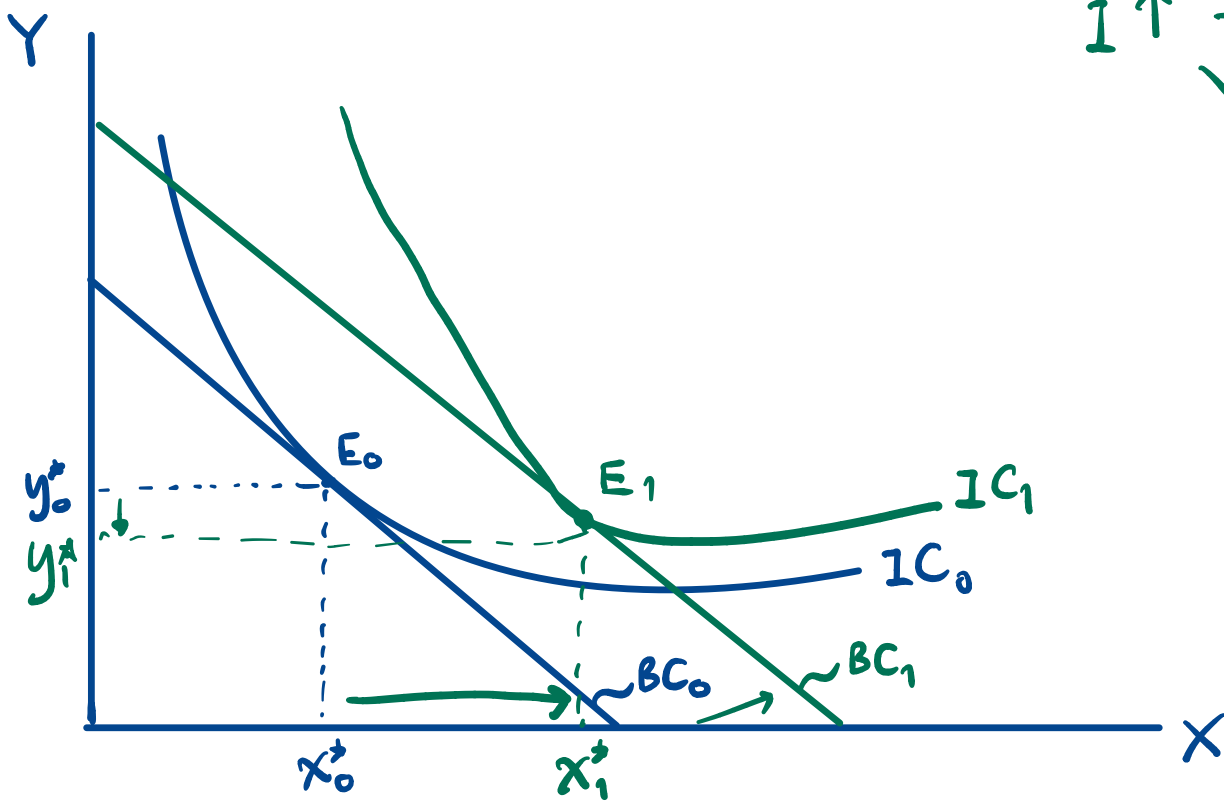
→ income ↑, cons ↓

$\varepsilon_I < 0$; $\varepsilon_I = \frac{\%\Delta Q_d}{\%\Delta I} < 0$

- Suppose X is a normal good but Y is an inferior good.
- Use a diagram to show the effects of an increase in income on the consumer's optimal bundle of X and Y.

$I \uparrow \rightarrow Q_x^* \uparrow$

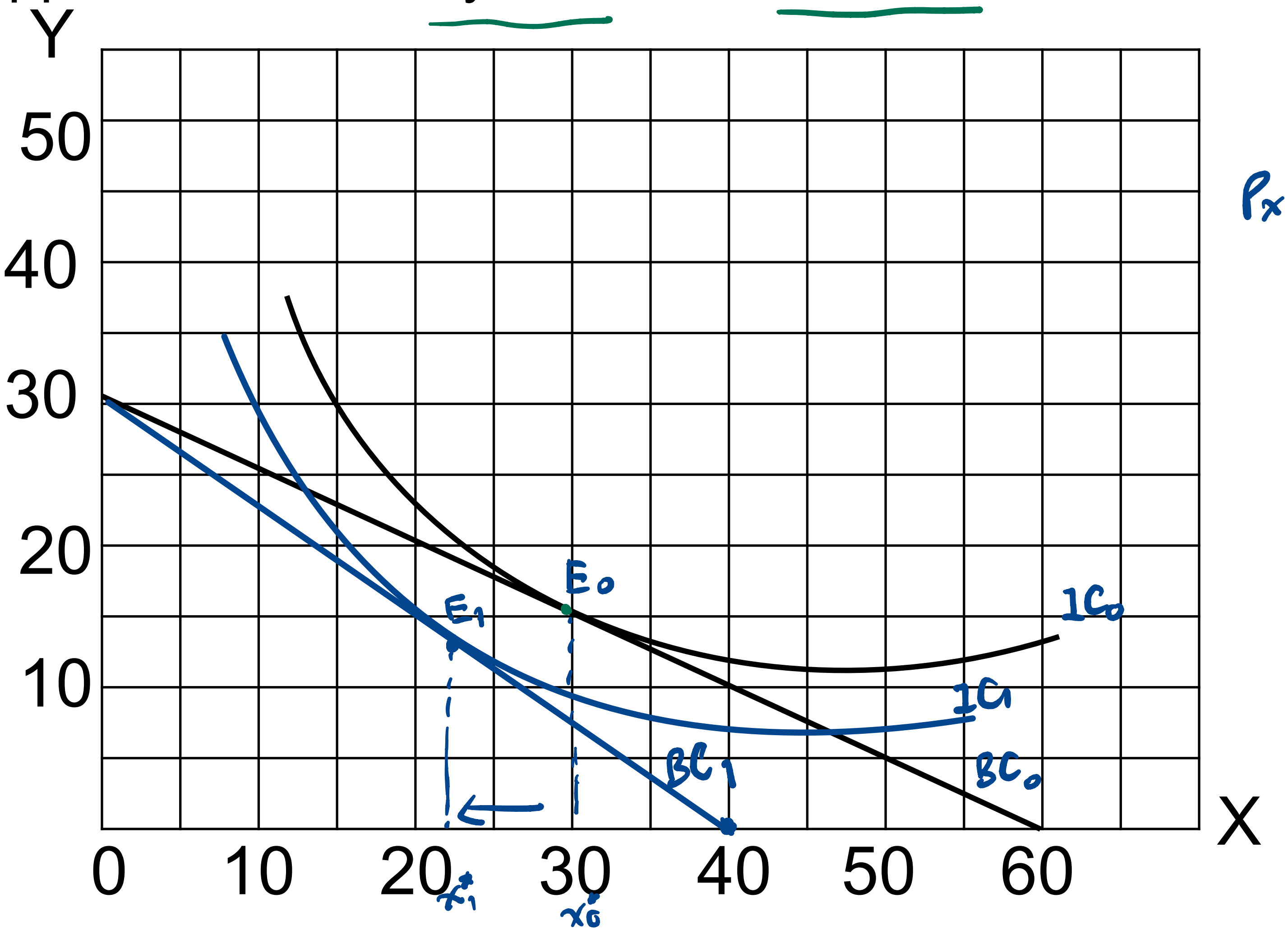
$\searrow Q_y^* \downarrow$



Example: Effect of Price Change (1)

$P_x = \$2$

- Suppose P’x = \$3, Py = \$4, and B = 120.

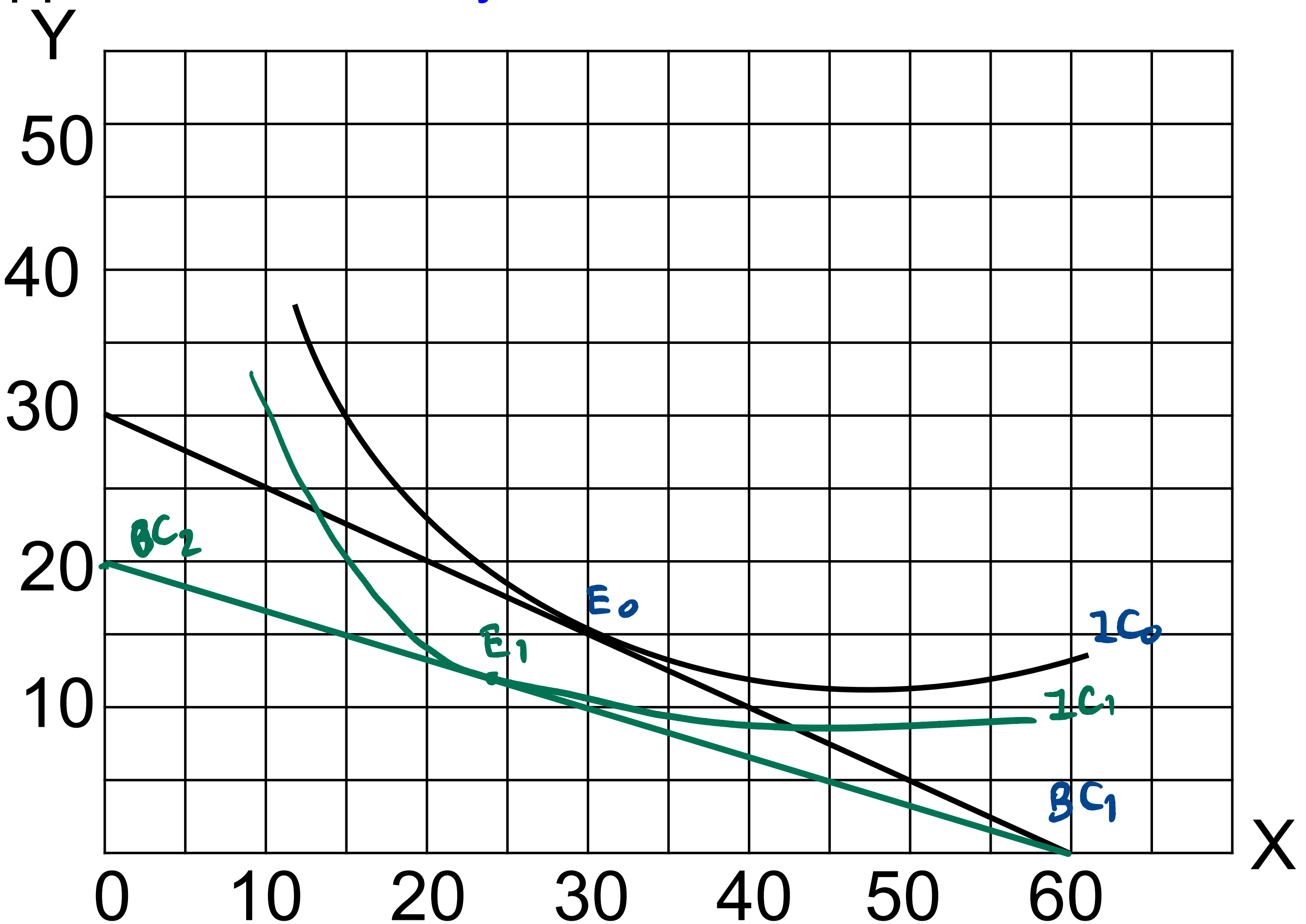


$P_x \uparrow \Rightarrow x^* \downarrow$

Example: Effect of Price Change (2)

$P_y = \$4$

- Suppose Px = $2, P'y = $6, and B = 120.



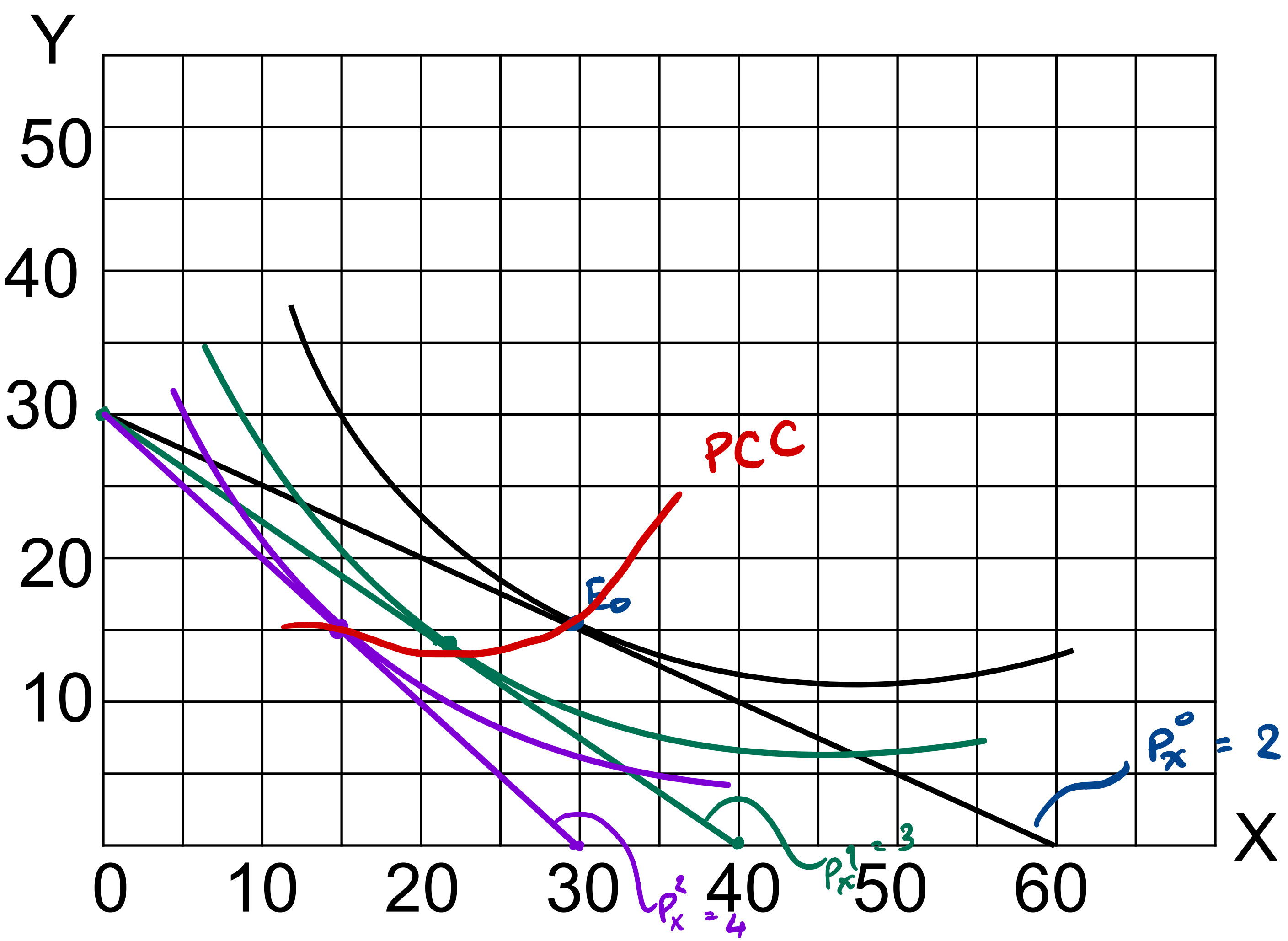
Price Consumption Curve

- A change in relative prices of two goods changes the slope of the budget constraint.
- Suppose Px changes while Py is constant. For each Px, there is a different utility-maximizing consumption bundle.
- **Price Consumption Curve (PCC)** is the line that connects all the utility-maximizing points for different Px's, given income and Py constant.
- I.e. , PCC shows how the consumer's purchases react to a change in one price with income and other prices being held constant.

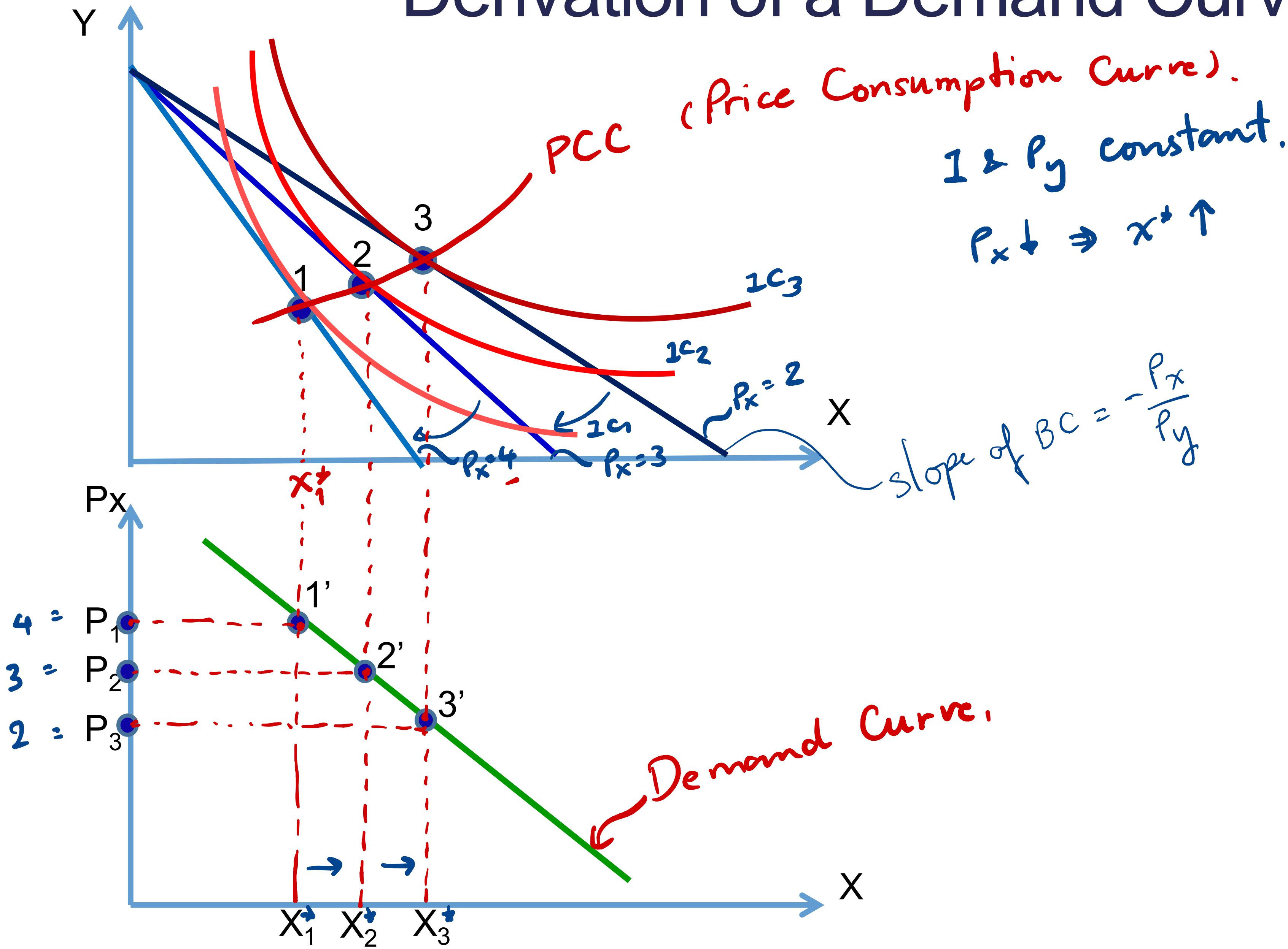
Graph: PCC

P_x^0 P_x^1 P_x^2

- Suppose Px = \$2, \$3, \$4, Py = \$4, and B = 120.



Derivation of a Demand Curve



Income and Substitution Effects

- A fall in Px has two effects: ⇒ $x^* <$ SE ("price effect") / IE (purchasing power)

Substitution effect (SE)

- Change in X due to change in relative price with *real income* unchanged
- A fall in Px makes Y more expensive relative to X, causing consumer to buy more X and less Y

(Note: Real income is kept unchanged by staying on the original IC.) → "Hicksian" Approach (John Hicks)

Income effect

- Change in X due to change in real income
- A fall in Px increases the consumer's purchasing power, allowing him to reach a higher IC

$P_x \downarrow$

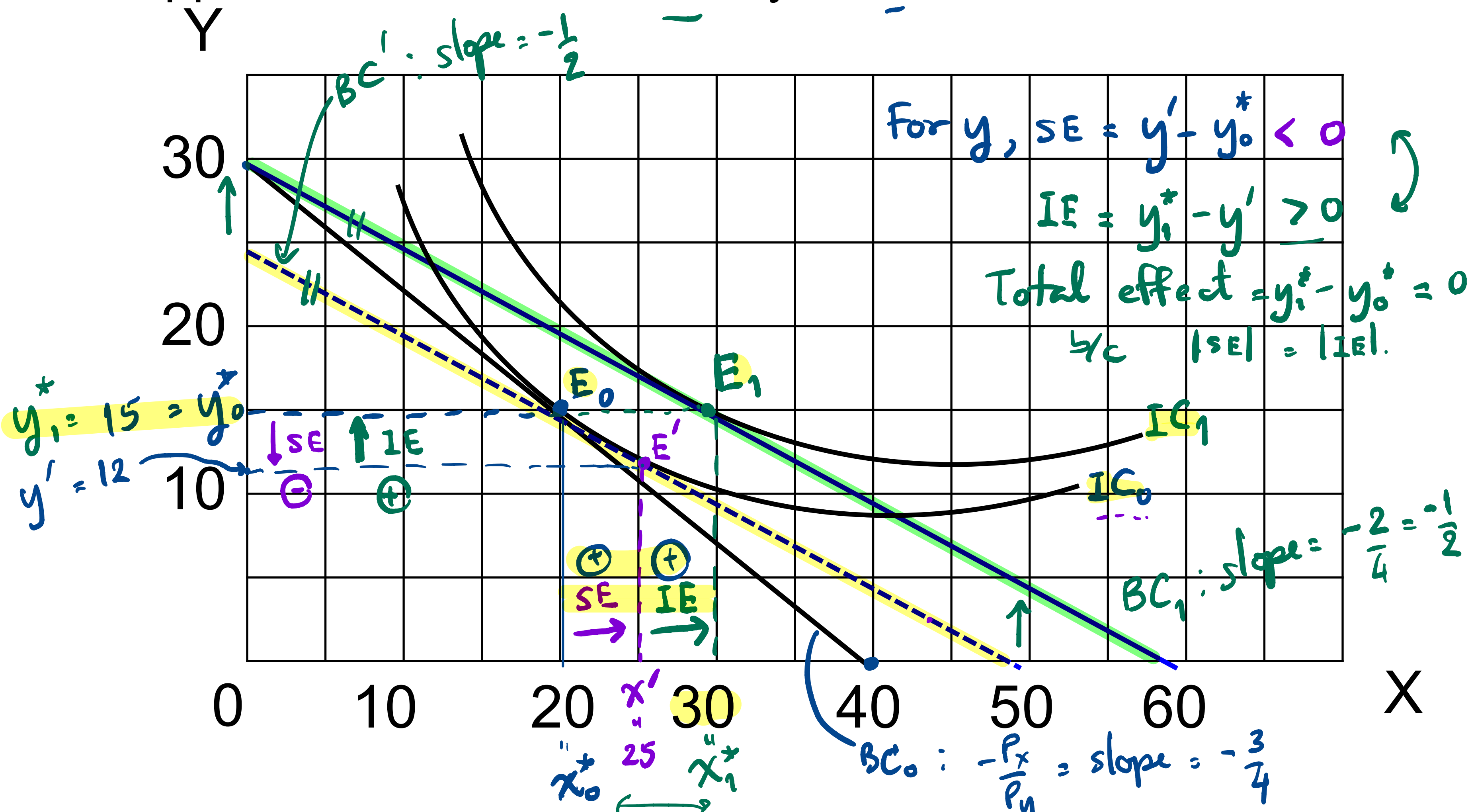
$SE = x' - x_0^* = 25 - 20 = 5 > 0$

$IE = x_1^* - x' = 30 - 25 = 5 > 0$

Total effect : $x_1^* - x_0^* = \Delta x^* = 10$

Graph

- Suppose Px = \$3, P'x = \$2, Py = \$4, and B = 120.



Steps to identify SE and IE

1. Start with the original tangency of IC_0 and BC_0.
   → called E_0^* (x_0^*, y_0^*)

2. Draw the new budget constraint (BC_1) from the given price change (eg. $P_x \downarrow$). Then, draw the new IC_1 that is tangent to BC_1. This optimum is called E_1^* (x_1^*, y_1^*)

3. Draw a hypothetical budget constraint that has the same slope as BC_1 BUT tangent to IC_0. call the new optimum E' (x', y').

④ Identify the effects on x and y:

$SE \text{ on } x = x' - x_0^*$

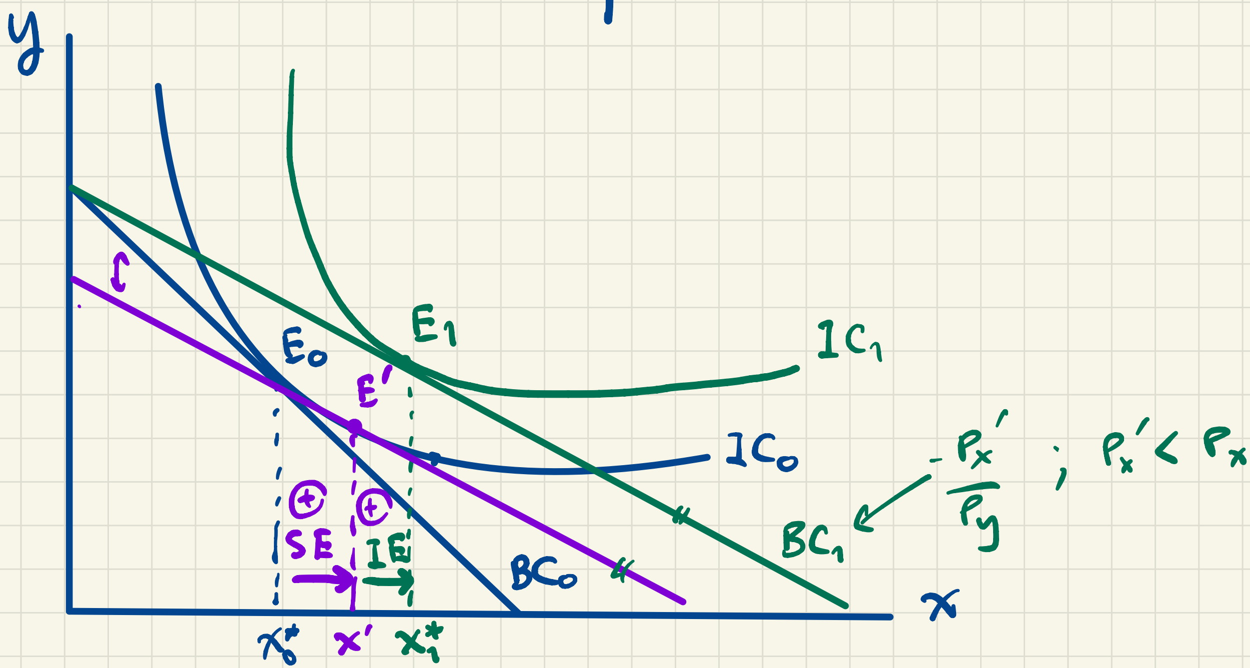
$IE \text{ on } x = x_1^* - x'$

Total effect $= x_1^* - x_0^*$

$SE \text{ on } y = y' - y_0^*$

$IE \text{ on } y = y_1^* - y'$

Total effect $= y_1^* - y_0^*$



Applications of Utility Maximization Problem

- Giffen good
- Subsidy
- Vouchers
- Work & leisure
- Intertemporal consumption

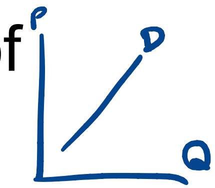
Application 1: Giffen Good

$\varepsilon_I < 0 = \frac{\% \Delta Q_d}{\% \Delta I}$

higher income, lower Q_D.

- Giffen good is a special case of *inferior good*.
- It is a good at which quantity demanded decreases when its price is lower, which is not consistent with the law of demand.

$P \downarrow \Rightarrow Q_d \downarrow$



- This is possible when income effect (negative) is greater than substitution effect (positive).
- Example: Suppose there are two goods – potatoes (X) and meat (Y).

Giffen good

normal good,

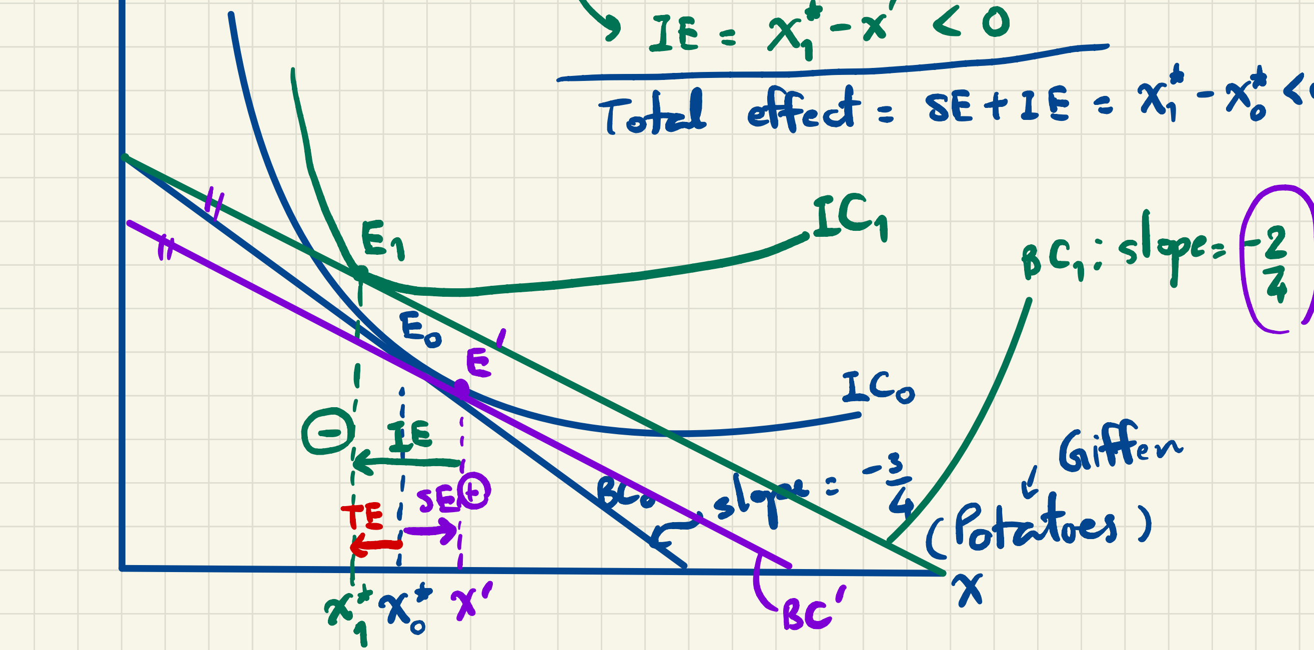
Let P_X=3, P_Y=4, and B = 120. If P_X decreases to 2, X* will decrease.

Total effect on x : $x_1^* - x_0^* < 0$

$\hookrightarrow SE = x' - x_0^* > 0$

$\hookrightarrow IE = x_1^* - x' < 0$

Total effect = SE + IE = $x_1^* - x_0^* < 0$



∴ As $P_x \downarrow$, x^* decreases because the negative IE outweighs the positive SE. (For Giffen good).

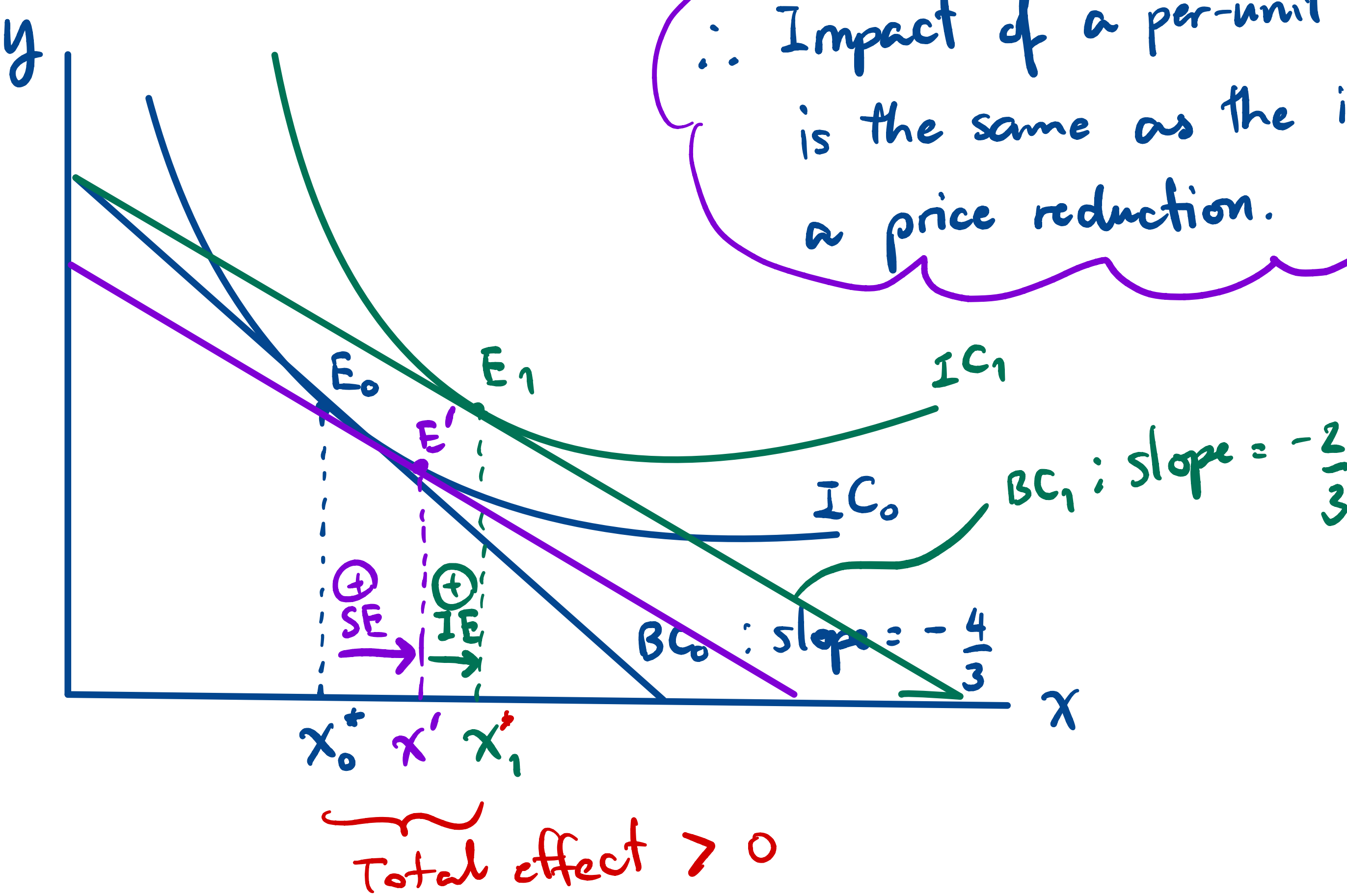
Application 2: Per-Unit Subsidy

$s = \$2/\text{unit}$

Ex: Suppose the gov't gives a $2 per-unit subsidy for good X.

Let $P_x = 4$, $P_y = 3$, $B = 120$ → $P_x' = P_x - 2 = 2$

∴ Impact of a per-unit subsidy is the same as the impact of a price reduction.

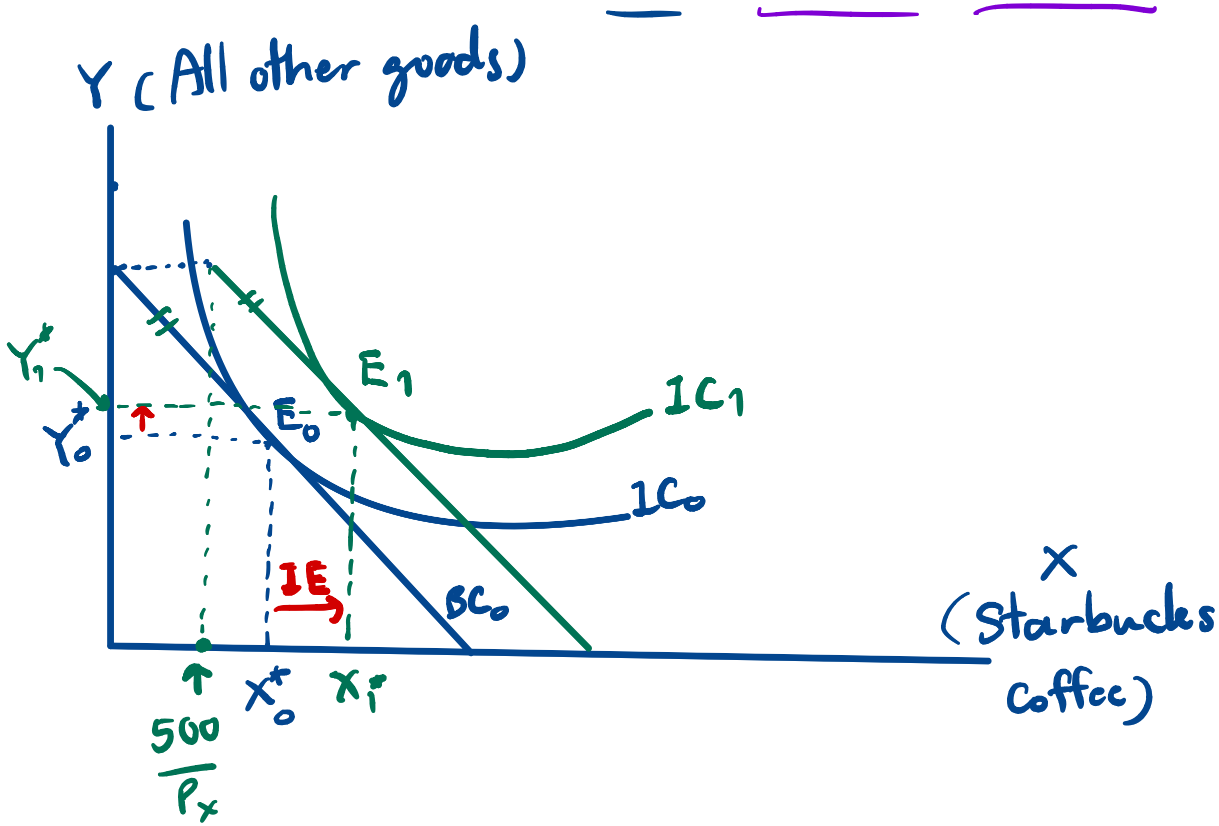


Application 3: Voucher

can be spent on x only

relative price does not change.

Ex: Suppose you receive a ฿500 starbucks coupon.



same as "food stamp"

Application 4: Work-Leisure Analysis

- Suppose consumer's utility depends on consumption of all goods (c) and leisure (ℓ). Constraint is time, say 24 hours.
- We can write utility function as U(c, ℓ).

<u>Ex</u>

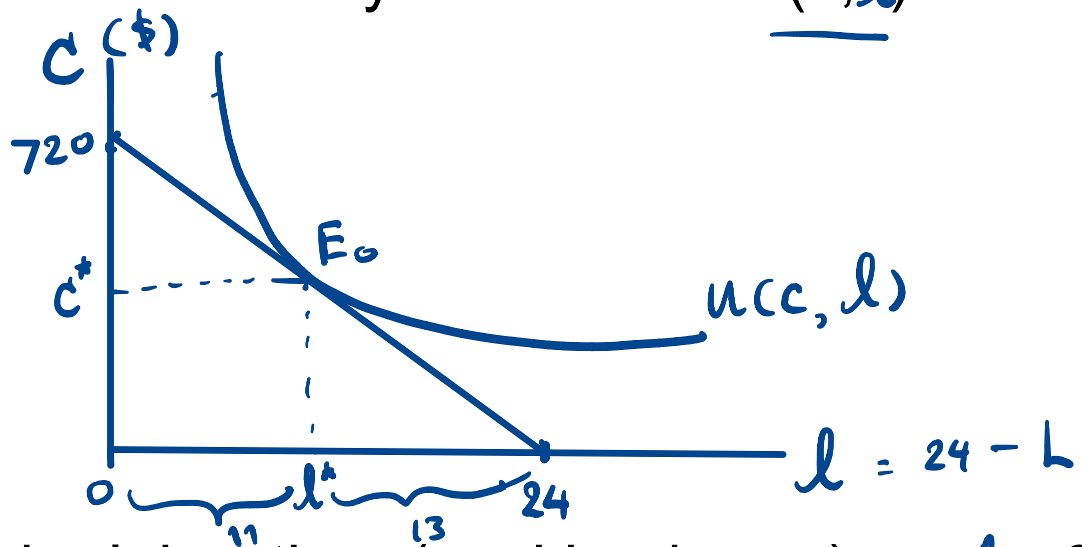
$W_0 = \$30/\text{hr.}$

$W_0 \times 24 = \$30 \times 24$

$= \$720$

suppose $\ell^* = 11$

$\Rightarrow L^* = 13$.



labor

- Let L be labor time (working hours), so $\ell = 24 - L$. Suppose the wage rate per hour is w. The constraint can be written as: (Assume $p_c = 1$)

$$\cancel{p_c}^{1} \cdot C = w \cdot L = w(24 - \ell) \Rightarrow C = w \times 24 - w\ell$$

① IE < SE

SE on $\ell = \ell' - \ell_0^* < 0$ (big).

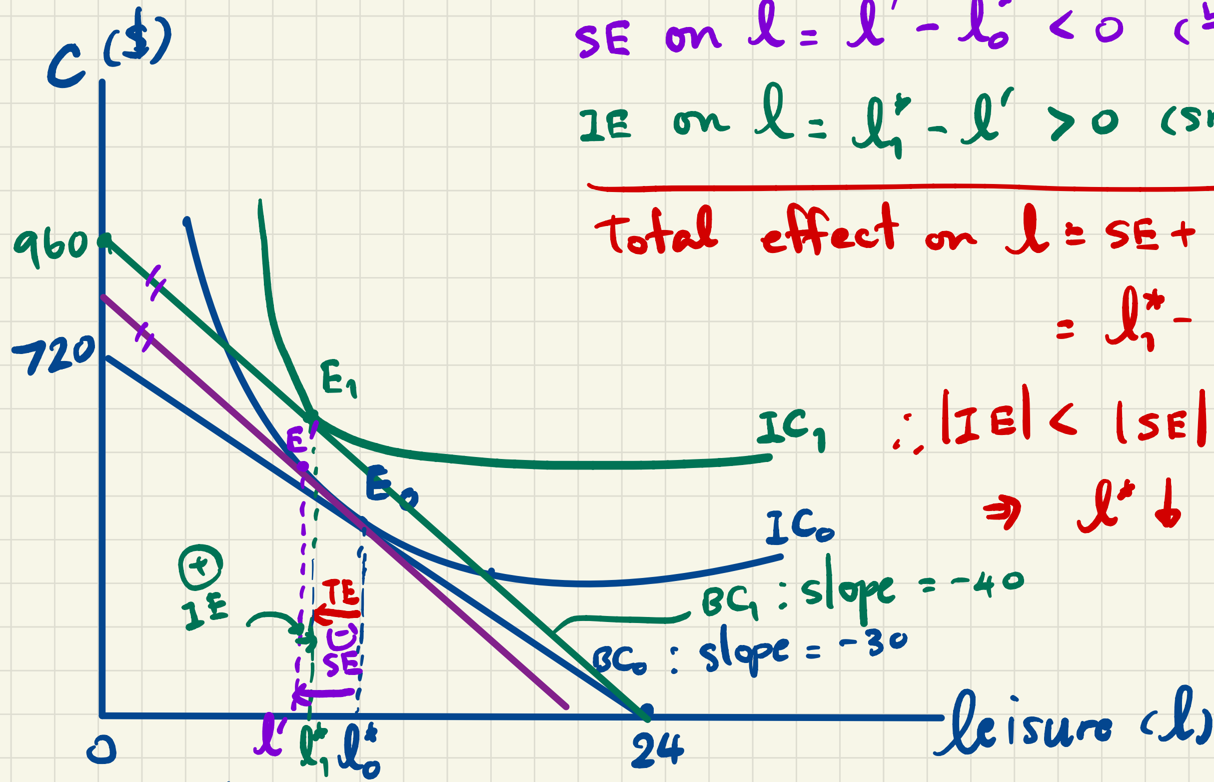
IE on $\ell = \ell_1^* - \ell' > 0$ (small)

Total effect on ℓ = SE + IE

$= \ell_1^* - \ell_0 < 0$

$\therefore |IE| < |SE|$

$\Rightarrow \ell^* \downarrow \Rightarrow L^* \uparrow$

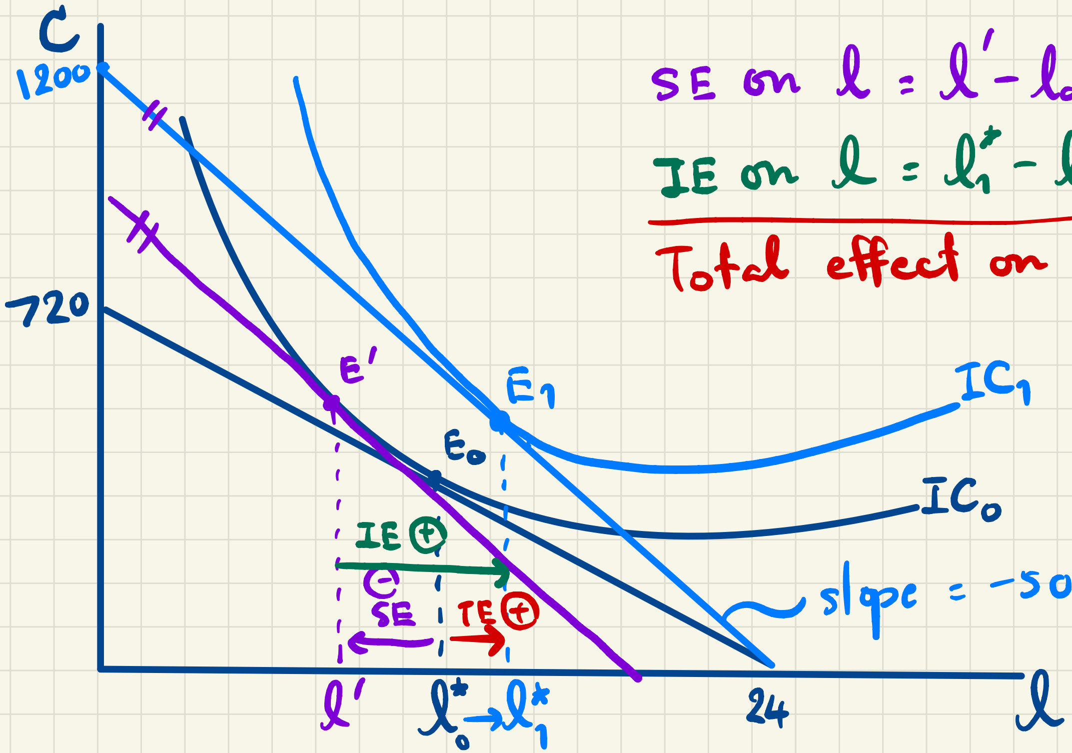


Let $w_0 = \$30/hr$.

w_0 increases to $w_1 = \$40$. $\Rightarrow$ Max $I = w_1 L_{max} = 960$ ($L_{max} = 24$)

② SE < IE

(Suppose $w_0 = \$30/hr$.
$w_1 = \$50/hr$).



SE on ℓ : $\ell' - \ell_0^* < 0$ (small)

IE on ℓ : $\ell_1^* - \ell' > 0$ (Big)

Total effect on $\ell = \ell_1^* - \ell_0^* > 0$

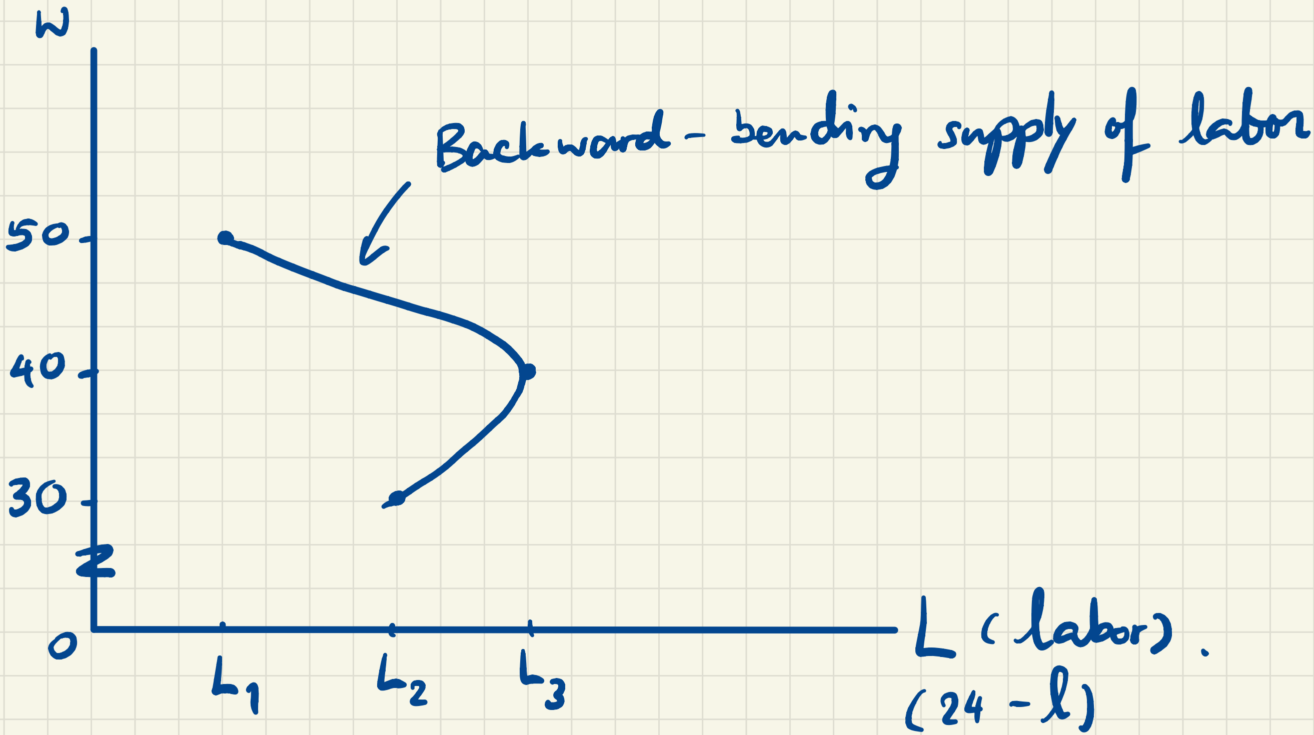
$\therefore$ As $w \uparrow$,

$\ell^* \uparrow$

$\Longleftrightarrow L^* \downarrow$

downward-slope supply L.

$|SE| < |IE|$ $\Longrightarrow$



w
Backward-bending supply of labor
50
40
30
0
L_1
L_2
L_3
L (labor)
(24 - l)