$\boxed{\text{Reading in williamson}}$ (2017 edition)

Chapter 7 (skip Multhusian Model)

Chapter 8

K Existing Capital stock → (y) production → Consumption ⇒ $(1-s)\cdot Y$

\+ Labor

a fixed proportion of investment out of income/output generated

saving ⇒ $s\cdot Y$.

↓

investment → add new Capital stock to the existing ones

after used "depreciate" "δ"

Evolution of K'

⇓

the level of ~~new~~ existing Capital stock K' for future.

→ future y' production output. → e.t.c.

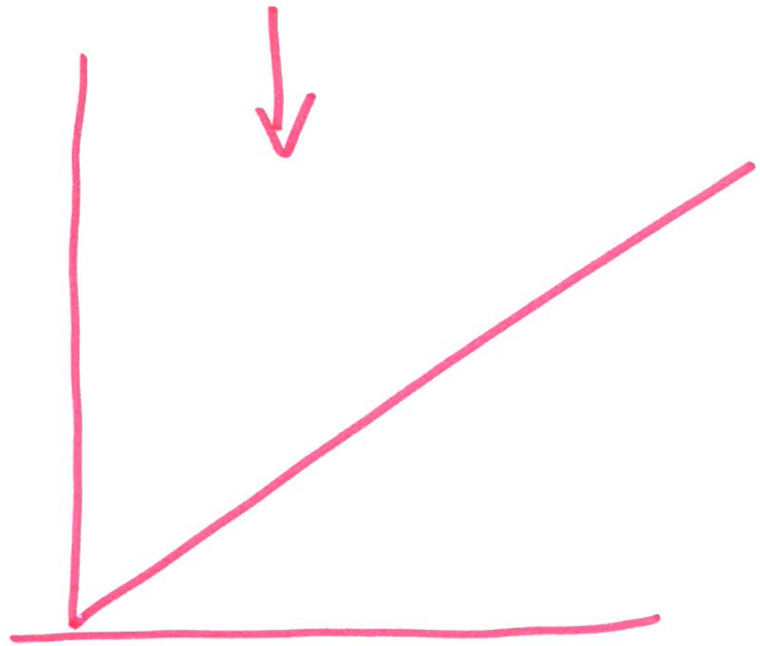
Capital accumulation helps increasing. Production frontier → generates growth

Once Expressed interm" of "per worker": $k = \frac{K}{N}$; c, $y = \frac{Y}{N}$

$$k' = \frac{s \cdot z \cdot f(k)}{(1+n)} + \frac{(1-d) \cdot k}{1+n}$$

current k $\longrightarrow$ output consumption $\longrightarrow$ saving = investment

$\longrightarrow$ future k' for tomorrow $\longrightarrow$ production capacity in future



$$k' = \frac{s \cdot z \cdot f(k)}{1+n} + \frac{(1-d)}{1+n} \cdot k$$

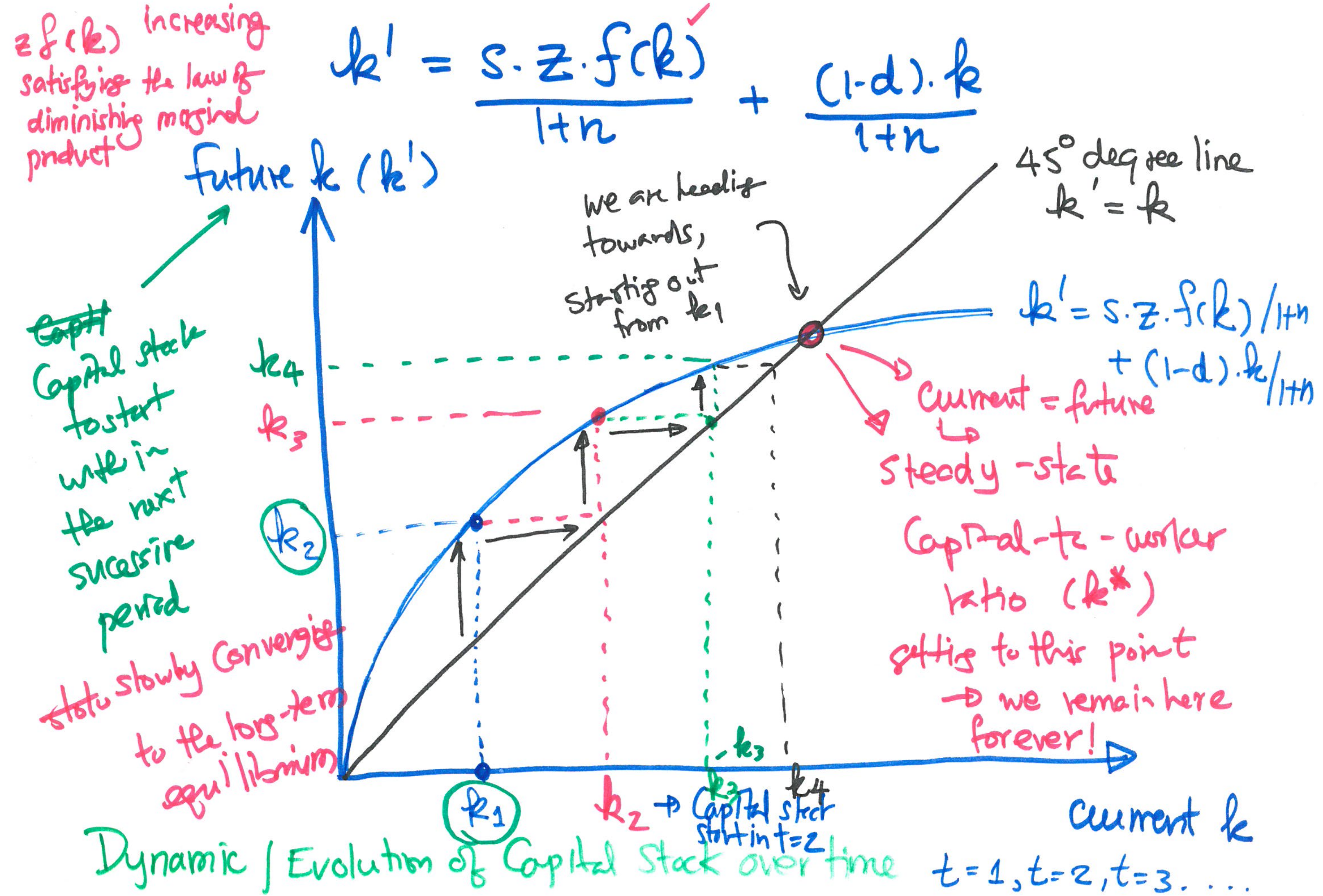
1. k can grow forever

   $k\uparrow \rightarrow k'\uparrow \rightarrow$ future capacity

2. $\rightarrow$ True that $k' \rightarrow \boxed{\text{Limitation}}$

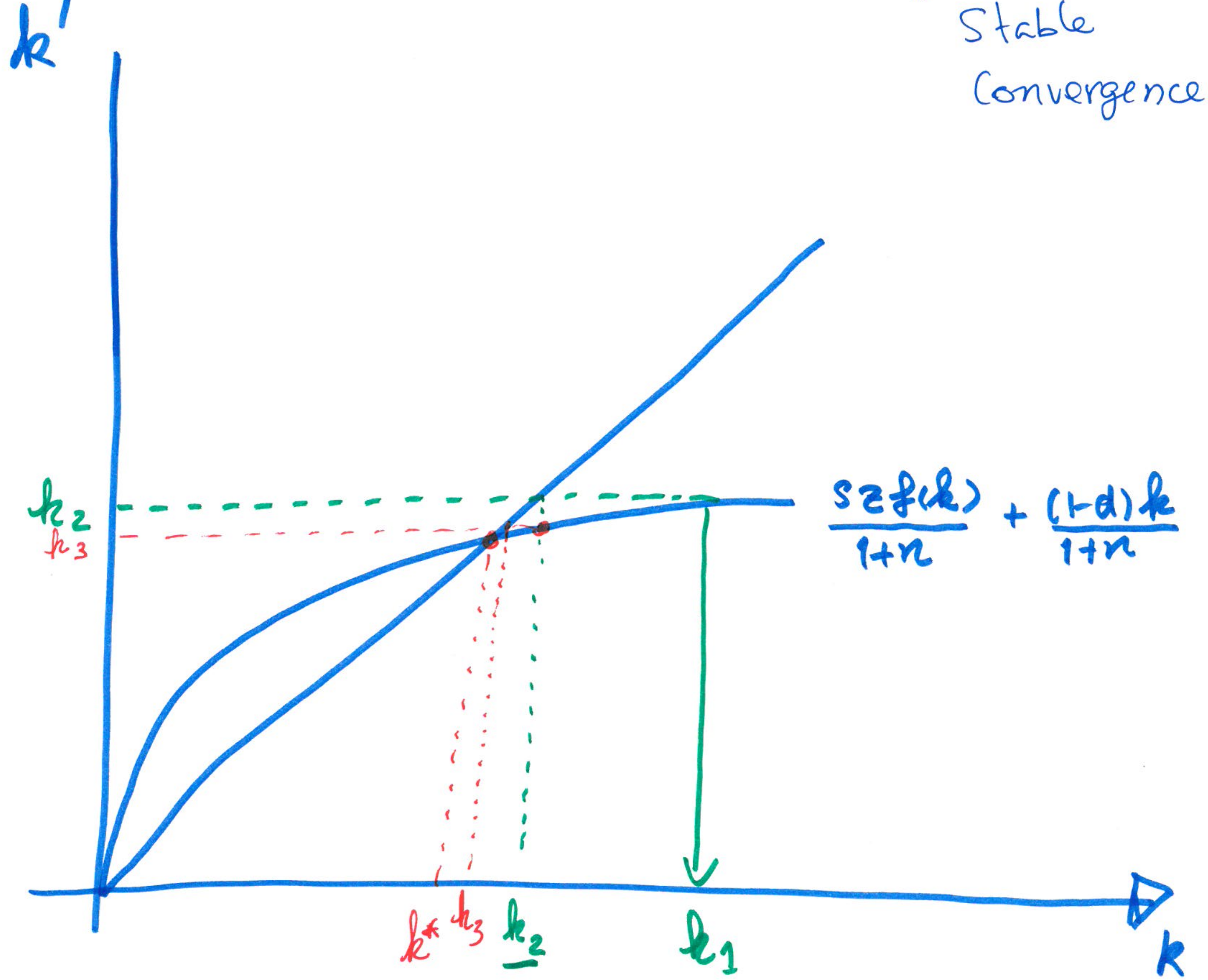
Time path of Capital over time into the future.

$$k' = \frac{s \cdot z \cdot f(k)}{1+n} + \frac{(1-d) \cdot k}{1+n}$$



Dynamic | Evolution of Capital Stock over time

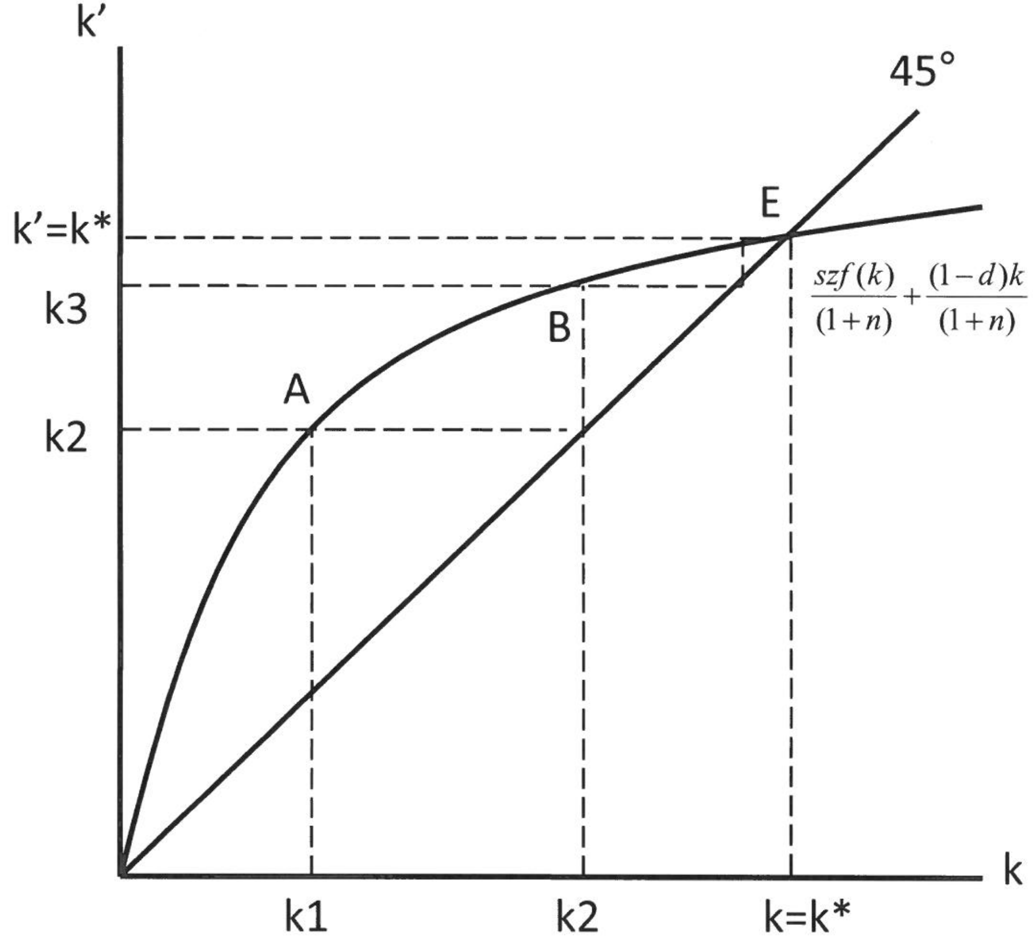
"Stable Convergence"



↳ steady-state k^* (Capital-to-Worker ratio)

The steady-state capital per worker

- At A, k2 > k1; k is growing.
- At B, k3 > k2; k is growing.
- k = k*; steady-state capital per worker.



Diminishing returns on k

$MP_k \downarrow$ as $k \uparrow$

- At E, $k = k' = k^*$ so that k^* is steady.
  - To the left of k^*, $k' > k$ so that k is increasing.
  - To the right of k^*, $k' < k$ so that k is decreasing.
- As k is increasing, MP_k is falling so that **y is increasing at a decreasing rate**.
- Finally, **investment** (or new capital) is just sufficient to keep up with **population growth** and **depreciation**, so that k (and y) is stagnant.

Per head = $\frac{\text{Aggregate}\uparrow}{\#\text{worker}}$ → "n"

Steady-state aggregates

fixed fixed fixed

- With k^* at the steady state, y^*, c^* and $szf(k^*)$ are all at **the steady-state**.
  - No further improvement in output per worker (y).
- Given population growth (n), total factor productivity (z) and the savings rate (s), the steady-state growth rate is 'n' for aggregate quantities:
  - Capital stock (K) and output (Y);
  - Consumption (C), savings (S) and investment (I).

K Y C I → "Balanced growth Path"

k^* $k' = \frac{s \cdot z \cdot f(k^*)}{1+n} + \frac{(1-d)k^*}{1+n}$

Analysis of the steady-state

Steady State Can be solved for

$k' = k = k^*$

$$k^* = \frac{szf(k^*)}{(1+n)} + \frac{(1-d)k^*}{(1+n)}$$

Multiplying k* by (1+n)

$$\boxed{\underbrace{szf(k^*)}_{\text{investment}} = (n+d)k^*}$$

⟹ Ensuring Condition for the steady state Capital Stock.

- Or steady-state savings = steady-state investment.

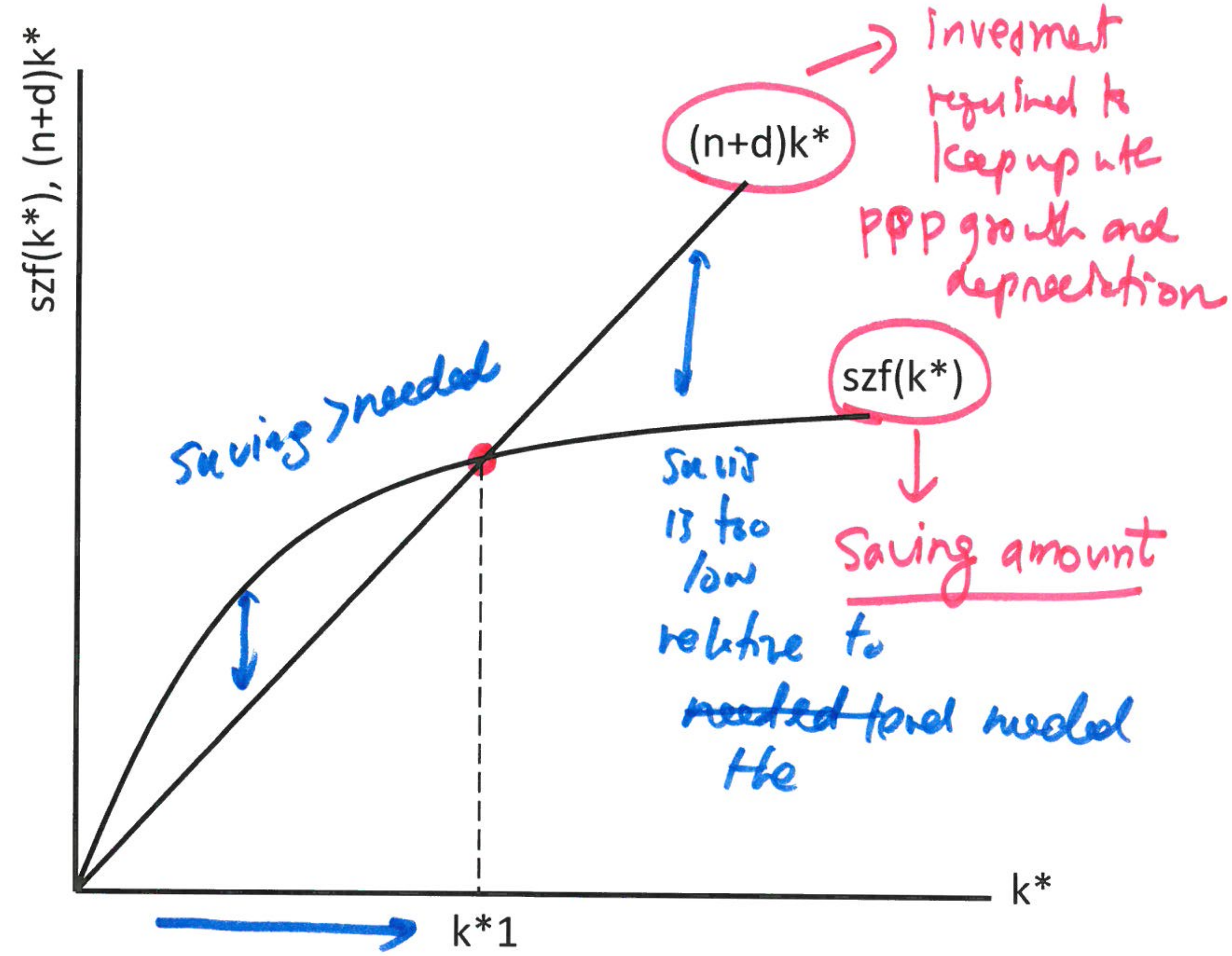
$$szf(k^*) = (n+d)k^*$$

- szf(k*) = savings per worker;
- (n+d)k* = investment per worker needed to keep up with population growth and depreciation.
- At k*, the capital stock is still growing, but just sufficient to equip each worker with the same k and depreciation (so k* is steady).
  - ‘**Capital widening**’: growing K just to keep the steady k and y.

$s \cdot z \cdot f(k^*) = (n+d) \cdot k^*$

Determination of steady-state k*

- szf(k*) is concave due to zf(k*).
- (n+d)k* has the slope = (n+d).



Policy experiments

- Change in saving rate (s)
- Change in population growth (n)
- Change in the level of technology (z)

Effect of an increase in s

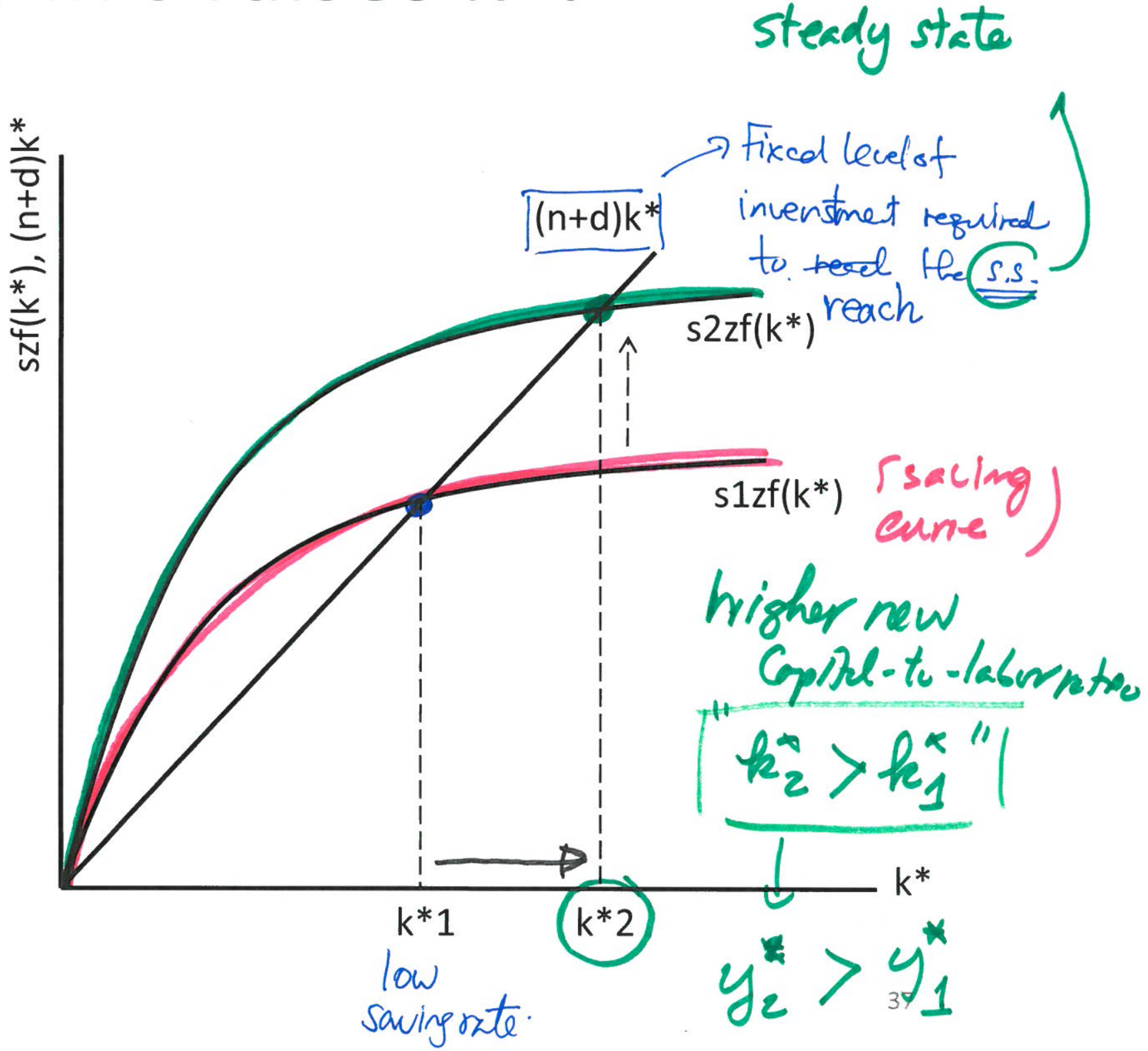
- Savings rate may increase due to changes in consumers' propensity or government policy.
- Assume a permanent increase in s:
  - $szf(k^*)$ rotates upwards.
  - Higher steady-state k^* and y^* (on a different 'growth path').
  - Higher growth of K and Y is transitional.
  - Convergence to the same steady-state growth rate of 'n'.

A rise in s raises k*.

- Higher savings rate results in a higher k* and y*.

s_1 : original level of saving rate

s_2 : new higher level of the saving rate! (save more)



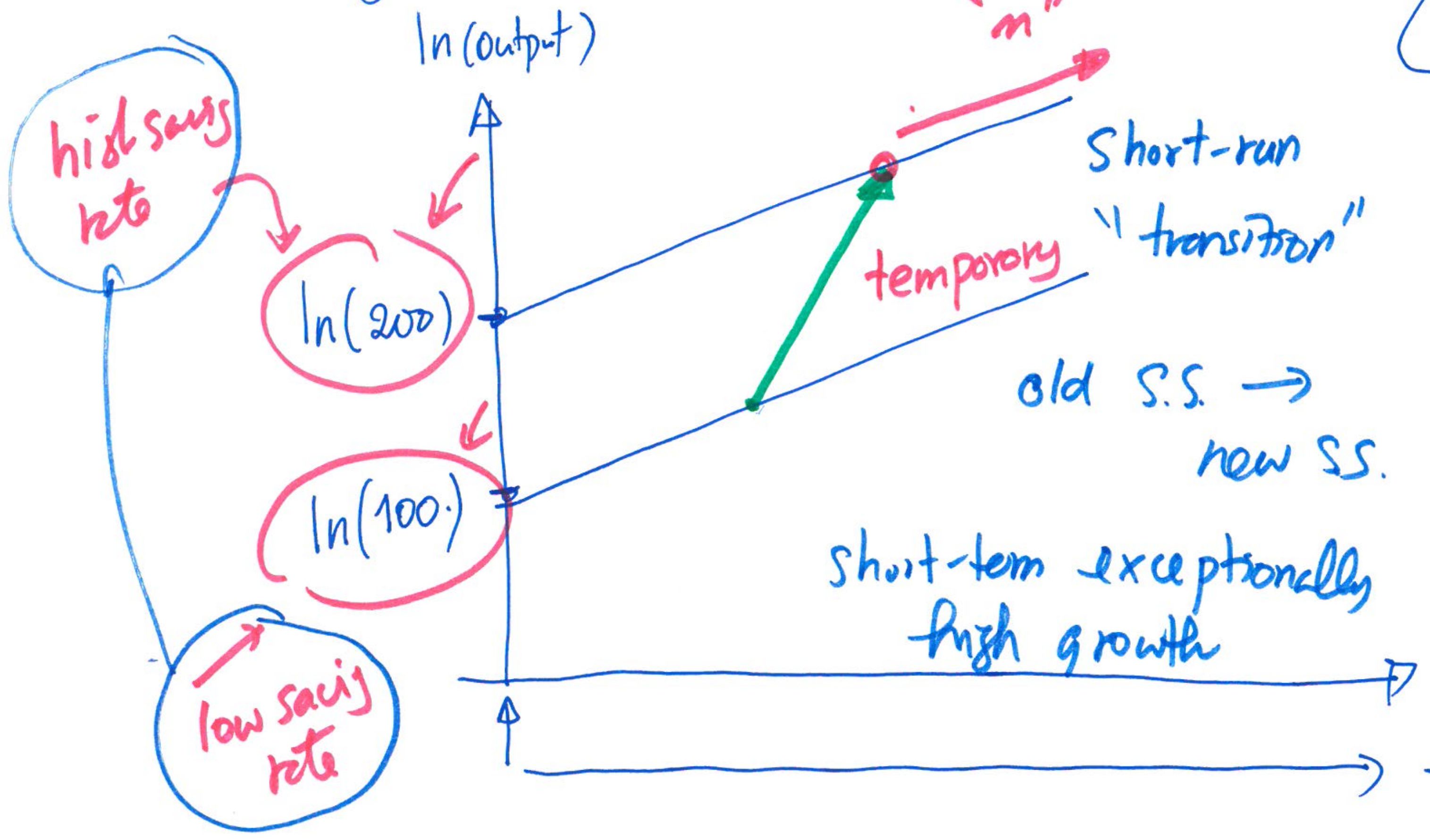
Pop $= 1$ ppl!

Low Saving rate $\Rightarrow$ $y^* = 100$

$100 \to (1+n)$

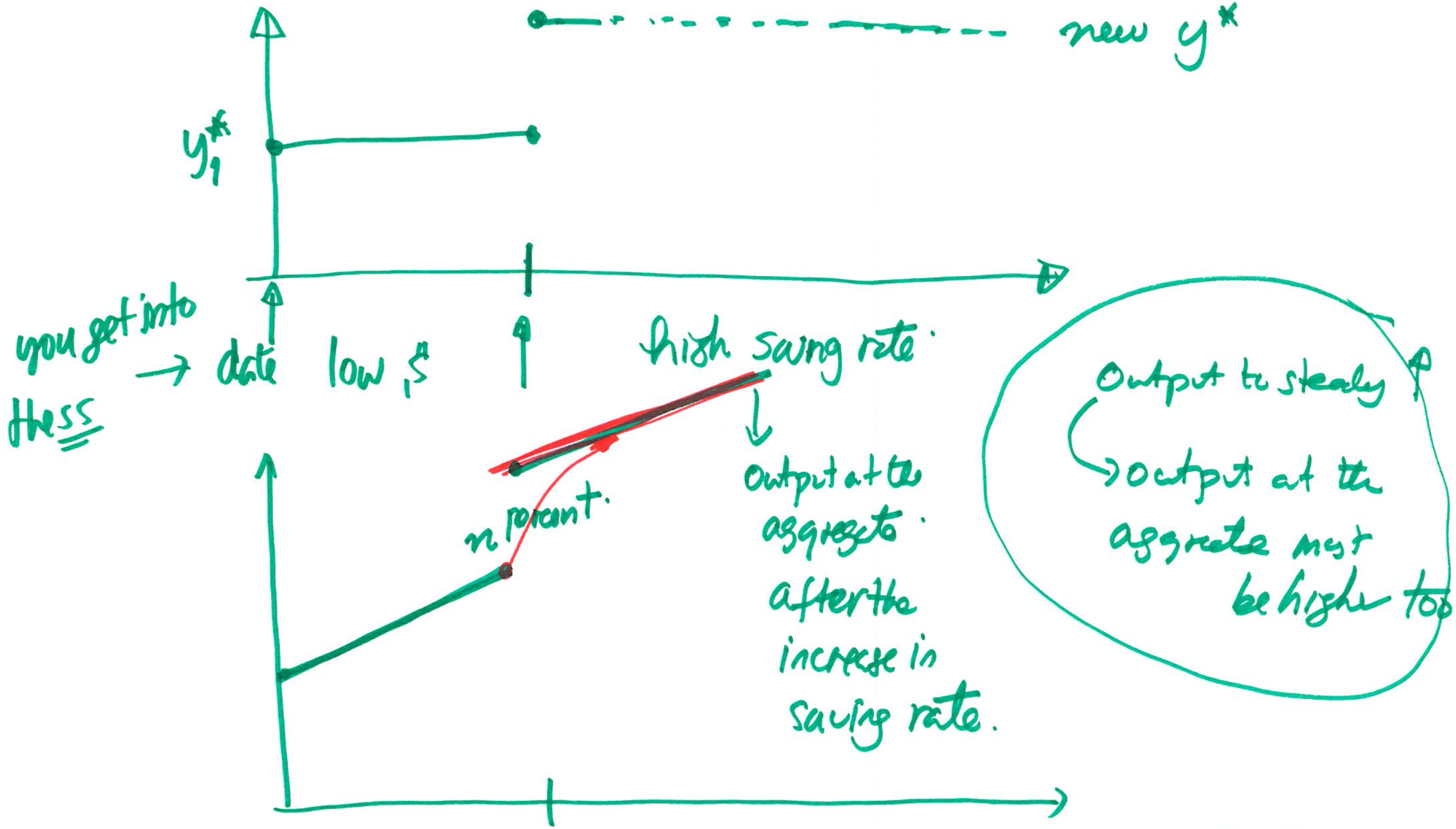
$200 \to (1+n)$

high Saving rate $\Rightarrow$ $y^* = 200$



low Saving Economy

↳ Encourage ppl to have the same rate of the ~~same~~ saving as the high-saving Economy



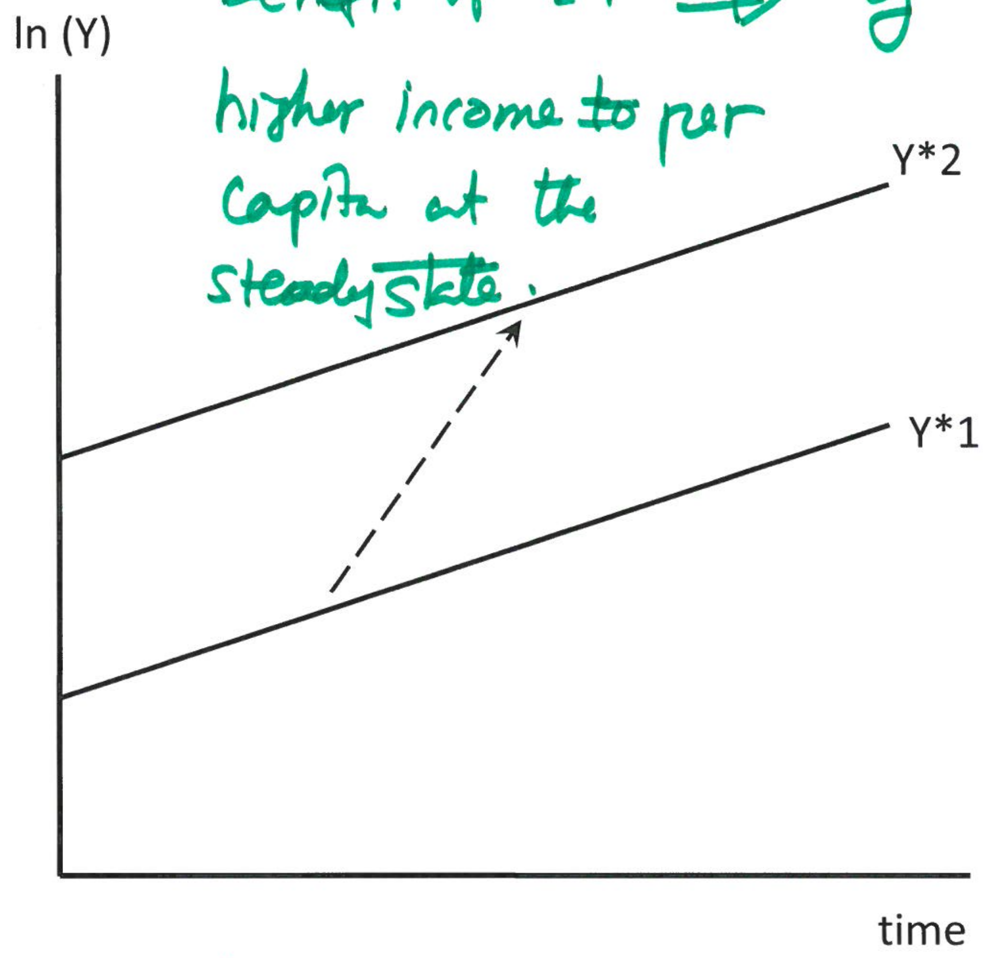
"growing Economy above the balanced growth path" > "n"

→ where you eventually grow at the rate of "n" in the long-term

Temporary gain in growth rate

Benefit of $s\uparrow \Rightarrow y^*\uparrow$

higher income ~~to~~ per capita at the steady state.



- K and Y move to new 'growth paths'.
- Higher growth rates of K and Y are transitional, converging to n.

↳ $K, Y \Rightarrow$ "n"