Wednesday, February 26, 2014
3:27 PM

PRODUCTION AND THE COST OF PRODUCTION (CH. 6 & 7 IN PINDYCK)

INPUTS → FIRM: BLACK BOX → OUTPUTS

FIRM: AN ORGANIZATION THAT CONVERTS INPUTS SUCH AS LABOR, MATERIALS, ENERGY, AND CAPITAL INTO OUTPUTS, THE GOODS & SERVICES THAT IT SELLS

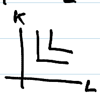
THE PRODUCTION FUNCTION TELLS US THE MAXIMUM OUTPUT THAT CAN BE PRODUCED BY THE FIRM FOR ANY GIVEN AMOUNT OF INPUTS.

$$Q = F(K, L)$$

- Q: OUTPUT (# UNITS / TIME PERIOD)
- K: CAPITAL (# UNITS / TIME PERIOD)
- L: LABOR (# WORKERS / TIME PERIOD) OR # WORKING HRS / TIME PERIOD)

EX:
1. $Q = 3L + 2K$ → L & K ARE SUBSTITUTES

2. $Q = \sqrt{LK}$
3. $Q = \min(10L, K)$ → WHEN L, K ARE COMPLEMENTS



4. $Q = AL^{\alpha}K^{\beta}$ WHERE $\alpha + \beta = 1$ → COBB-DOUGLAS PRODUCTION FUNCTION

PRODUCTION W/ ONE VARIABLE INPUT

$TP_L = F(\bar{K}, L)$ → SHORT-RUN PROD^n FUNCTION

$AP_L = \dfrac{TP_L}{L} = \dfrac{Q}{L}$ [AVERAGE PRODUCT OF LABOR]

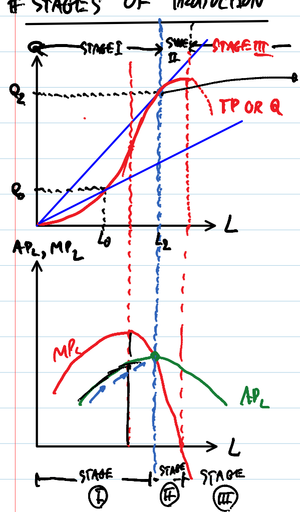
$MP_L = \dfrac{dTP}{dL} = \dfrac{dQ}{dL}$ [MARGINAL PRODUCT OF LABOR]

EX: CONSIDER $Q = \sqrt{LK}$. FIND AP_L AND MP_L.

$$AP_L = \frac{Q}{L} = \frac{L^{\frac{1}{2}}K^{\frac{1}{2}}}{L} = \frac{K^{\frac{1}{2}}}{L^{\frac{1}{2}}} = \sqrt{\frac{K}{L}}.$$

$$MP_L = \frac{dQ}{dL} = \frac{d(\sqrt{LK})}{dL} = \frac{d(L^{\frac{1}{2}}\cdot K^{\frac{1}{2}})}{dL} = \frac{1}{2}L^{-\frac{1}{2}}\cdot K^{\frac{1}{2}} = \frac{1}{2}\frac{\sqrt{K}}{\sqrt{L}} = \frac{1}{2}\sqrt{\frac{K}{L}}.$$

STAGES OF PRODUCTION



(SINCE) (THEN)

STAGE I: $MP_L > AP_L$, SO $AP_L \uparrow$

STAGE II: $MP_L < AP_L$, SO $AP_L \downarrow$

STAGE III: TP REACHES ITS MAXIMUM, MP BECOMES NEGATIVE, TP BEGINS TO FALL.

RELATIONSHIP BET. AP_L AND MP_L:

BEGIN W/ $Q = L \cdot AP_L$ $\left[\because AP_L = \dfrac{Q}{L}\right]$

$$\frac{dQ}{dL} = AP_L \cdot \frac{dL}{dL} + L \cdot \frac{d(AP_L)}{dL},$$

$$MP_L = AP_L + L \cdot (\text{SLOPE OF } AP_L)$$

$$\boxed{MP_L - AP_L = L \cdot \text{SLOPE OF } AP_L}$$

↓ (+)

$\oplus$

(I) when $MP_L - AP_L > 0$, slope of $AP_L > 0$

$\equiv$ when $MP_L > AP_L$, AP is rising.

(II) when $MP_L - AP_L < 0$, slope of $AP_L < 0$

$\equiv$ when $MP_L < AP_L$, AP is falling.

(III) when $MP_L - AP_L = 0$, slope of $AP_L = 0$

$\equiv$ when $MP_L = AP_L$, AP_L reaches its peak.

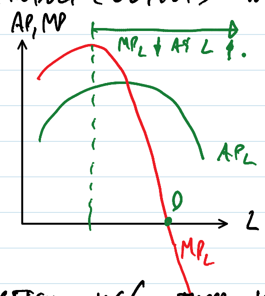
Q: Why do we get this result?

A: Law of diminishing marginal returns (operated in the short-run production)

↳ when more labor is added, marginal product (output) will eventually fall. (why?)

↓

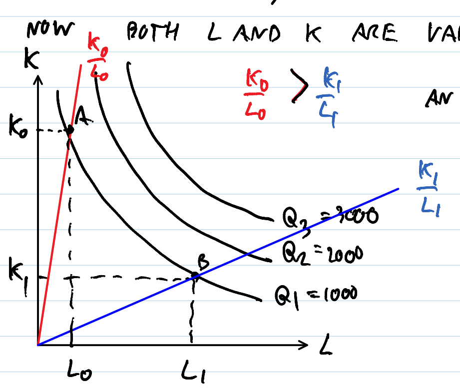
imbalance bet. fixed input & variable input



Production w/ two variable inputs

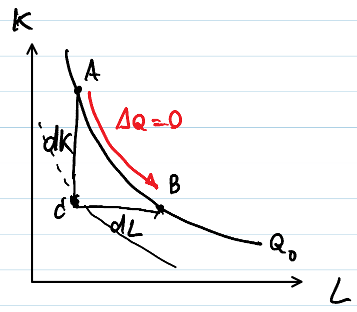
$$Q = F(L, K)$$

now both L and K are variable inputs



An isoquant: set of input mix that gives the same amount of output.

Slope of isoquant = MRTS = marginal rate of technical substitution = $\frac{\Delta K}{\Delta L}$



MRTS = amount by which the quantity of one input can be reduced when one extra unit of another input is used, so that output remains constant

Movement from $A \to C$: loss of output $= dK \cdot MP_K$ $(-5 \cdot 10 = -50)$

Movement from $C \to B$: gain of output $= dL \cdot MP_L$ $(+10 \cdot 5 = +50)$

$$\underset{A \to C}{dK \cdot MP_K} + \underset{C \to B}{dL \cdot MP_L} = \underset{A \to B}{\Delta Q} = 0$$

$$\boxed{MRTS = \frac{dK}{dL} = \frac{-MP_L}{MP_K}}$$

• **FIXED - PROPORTION PRODUCTION FUNCTION**

$$Q = \text{MIN}\{aK, bL\}$$

TO PRODUCE A CAR, WE NEED STRUCTURE (S) + WHEELS (W)

THIS PRODUCTION CAN BE DESCRIBED BY

$$Q = \min\left\{S, \frac{W}{4}\right\}$$

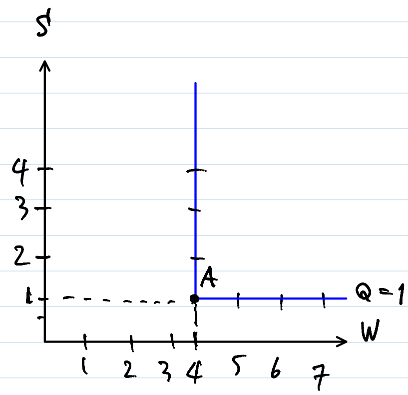
EX: $Q = \min\left\{S, \frac{W}{4}\right\}$: $S = 1$, $W = 4$

$Q = \min\left\{1, \frac{4}{4}\right\}$

$Q = 1$

$Q_1 = \min\left\{S, \frac{W}{4}\right\}$: $S = 2$, $W = 8$

$Q_1 = \min\left\{2, \frac{8}{4}\right\} = 2.$



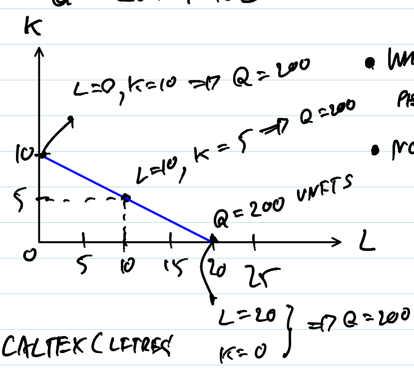
$\boxed{S = 3, \quad W, 15}$

$Q_2 = \min\left\{3, \frac{15}{4}\right\} = \min\{3, 3...\} = 3$

• **LINEAR PRODUCTION FUNCTION**

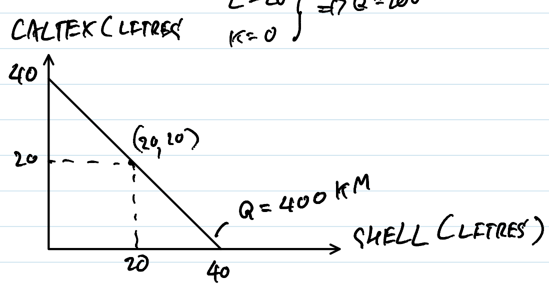
$$Q = aK + bL.$$

EX: $Q = 20K + 10L$



• WHEN THE TWO INPUTS ARE PERFECT SUBSTITUTES

• NOTE THAT <u>MRTS IS CONSTANT</u>.



OR
(A DEGREE)

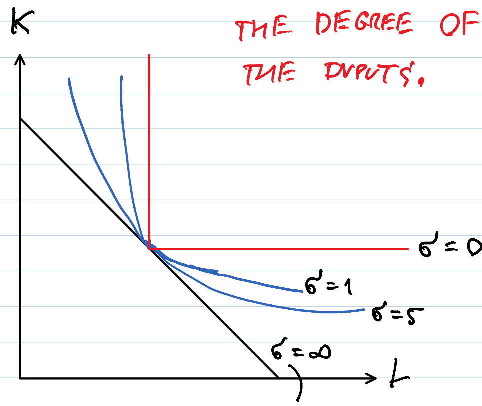
ELASTICITY OF SUBSTITUTION : TO MEASURE AN ABILITY OF HOW THE TWO INPUTS CAN BE SUBSTITUTED

$$\sigma = \frac{\text{PERCENTAGE CHANGE IN CAPITAL-LABOR RATIO}}{\text{PERCENTAGE CHANGE IN MRTS}}$$

$$= \frac{\% \Delta (K/L)}{\% \Delta \text{ MRTS}} = \frac{d \ln (K/L)}{d \ln \text{MRTS}}$$

NOTE ① $0 \leq \sigma \leq \infty$.

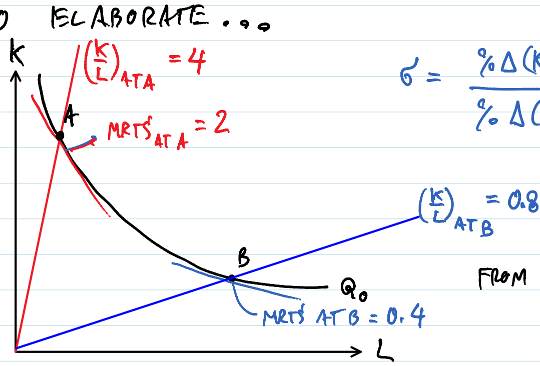
② THE SHAPE OF ISOQUANT INDICATES THE DEGREE OF SUBSTITUTABILITY OF THE INPUTS.



[FOR FIXED-PROPORTION PRODUCTION FUNCTION] CHARACTERISED BY L-SHAPED ISOQUANT.

[FOR LINEAR PRODUCTION FUNCTION CHARACTERISED BY LINEAR ISOQUANT]

TO ELABORATE...



$$\sigma = \frac{\% \Delta (K/L)}{\% \Delta (\text{MRTS})}$$

SLOPE OF ISOQUANT

FROM $A \rightarrow B$: $\% \Delta \text{MRTS} = \frac{0.4 - 2}{2} \times 100 = -80\%$

$$\% \Delta \left(\frac{K}{L}\right) = \frac{0.8 - 4}{4} \times 100 = -80\%$$

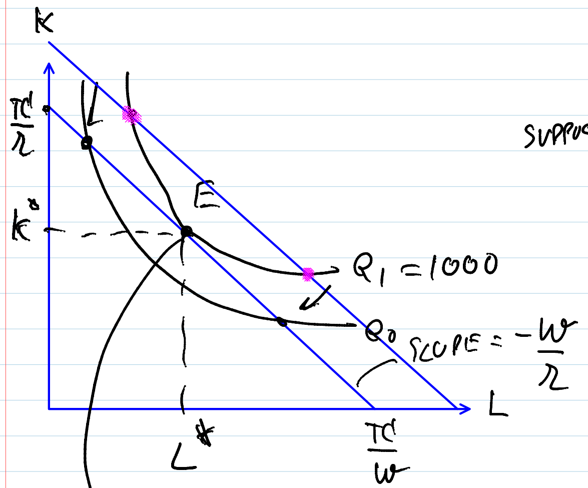
$$\sigma = \frac{-80\%}{-80\%} = 1 \Rightarrow \text{SUGGEST THAT}$$

ISOQUANT ARE CURVES (NOT L SHAPED) (NOT LINEAR)

LEAST-COST COMBINATION

GOLDEN RULE : $\frac{MP_L}{MP_K} = \frac{P_L}{P_K}$ OR $\frac{MP_L}{MP_K} = \frac{w}{r}$

OR $\frac{MP_L}{w} = \frac{MP_K}{r}$



SUPPOSE $\boxed{TC = w \cdot L + r \cdot K}$ ISOCOST

$w =$ wage rate

$r =$ rental rate

$$K = \frac{TC - w \cdot L}{r}$$

$$\boxed{K = \frac{TC}{r} - \frac{w}{r} \cdot L}$$

SLOPE OF ISOQUANT = SLOPE OF ISOCOST

$\frac{TC}{r}$: INTERCEPT Y-AXIS

$\frac{w}{r}$: = SLOPE OF ISOCOST = RELATIVE PRICE OF INPUTS

$$MRTS = -\frac{w}{r}$$

$$-\frac{MP_L}{MP_K} = -\frac{w}{r}$$

$$\boxed{\frac{MP_L}{MP_K} = \frac{w}{r}} \quad \text{OR} \quad \frac{MP_L}{w} = \frac{MP_K}{r}$$

CONSUMER'S OPTIMIZATION

$$\frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$

TO MINIMIZE THE COST OF PRODUCING $Q_1 = 1000$, THE MANAGER MUST BUY (OR CHOOSE) INPUTS SUCH THAT

MARGINAL PRODUCT FROM THE LAST BAHT SPENT ON L

= MARGINAL PRODUCT FROM THE LAST BAHT SPENT ON K!

IF $\frac{MP_L}{w} > \frac{MP_K}{r}$, A REALLOCATION OF INPUTS IS NEEDED IF HE WANT TO MINIMIZE COSTS.

HE SHOULD USE MORE L AND LESS K TO SAVE MORE COSTS.

Q: IF HE DOUBLE BOTH L AND K, WOULD Q ALSO DOUBLE?

A: IT DEPENDS ON THE PRODUCTION FUNCTION.

IN OTHER WORDS, RETURNS TO SCALE...

RETURNS TO SCALE (RTS): THE RATE AT WHICH OUTPUT INCREASES

AS INPUTS INCREASED PROPORTIONATELY.

IRS : A SITUATION IN WHICH OUTPUT INCREASES MORE THAN DOUBLES WHEN ALL INPUTS ARE DOUBLED.

CRS : A SITUATION IN WHICH OUTPUT DOUBLES WHEN ALL INPUTS ARE DOUBLED,

DRS : A SITUATION IN WHICH OUTPUT LESS THAN DOUBLES WHEN ALL INPUTS ARE DOUBLED

SUPPOSE $\boxed{Q = F(K, L) = 2KL}$.

$$F(cK, cL) = 2(cK)(cL) = c^2\, 2KL$$

EX: IF $c = 2$ (DOUBLING OF EACH INPUT), YOU GET

$$F(2K, 2L) = 2(2K)(2L) = 4 \cdot 2KL = 2^2 \cdot 2KL$$

, A QUADRUPLING OF OUTPUT.

THIS PRODUCTION FUNCTION EXHIBITS IRS.

SUMMARY

INCREASING RETURNS TO SCALE : $F(cL, cK) > c^1 F(K, L)$

CONSTANT RETURNS TO SCALE $F(cL, cK) = c^1 F(K, L)$

DECREASE RETURNS TO SCALE $F(cL, cK) < c\, F(K, L)$.

EXERCISE:

$$\left.\begin{aligned} Q &= 2\sqrt{KL} \\ Q &= 4K + 2L \\ Q &= 4K^{\frac{1}{2}}L^{\frac{1}{2}} \\ Q &= K^{0.5} \cdot L^{0.6} \end{aligned}\right\}$$

CHECK IF CRS, IRS, OR DRS.

	MP_L	MP_K	$MRTS_{LK}$	DIMINISHING MP_L ?	DIMINISHING MP_K	DIMINISHING MRTS ?
• $Q = L + K$						
• $Q = \sqrt{LK}$						
• $Q = \sqrt{L} + \sqrt{K}$						
• $Q = L^3 K^3$						
• $Q = L^2 + K^2$						