BACHELOR of ECONOMICS
International Program

Thammasat University | Thailand

FN 312 Investments

Term Structure of Interest Rates

Outline

- The term structure of interest rates
- Forward rates
- Theories of the term structure of interest rates

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Spot Rate

	0	0.5	1.0	1.5	2.0	
		$1	$1	$1	$1	Cash Flow
	0	0.5	1.0	1.5	2.0	Years (t)
d(0.5)						
d(1.0)						
d(1.5)						
d(2.0)						

- The 'market' discount factors from zeros

Mat.	Price	t	d(t) *
0.5	97.087	0.5	0.97087
1	93.897	1	0.93897
1.5	90.909	1.5	0.90909
2	87.336	2	0.87336

* d(t) = Price/Face Value

Spot rate (semi annual)

- The spot rate, y_t, is the yield that is equivalent to the discount rate at time t.

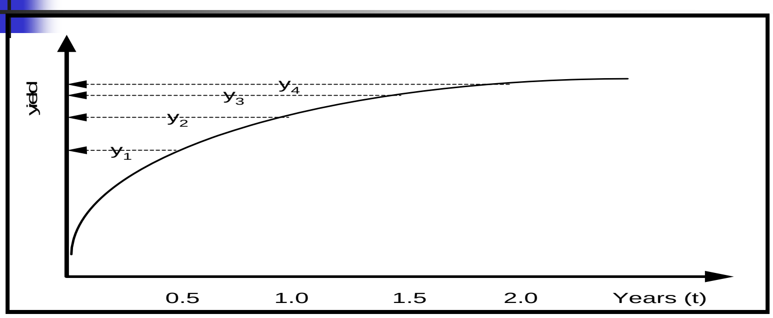
$$y_t = 2 * \left((1/d(t))^{1/year*2} - 1 \right)$$

- The relation between the semi-annual spot rate and the discount rate

$$y_t = 2 * \left(\left(\frac{F}{B_t} \right)^{1/year*2} - 1 \right)$$

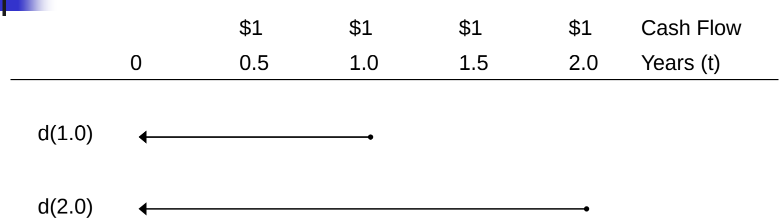
- Getting spot rates directly from zero-coupon bonds

Semi annual spot rates



t	d(t)	$y_t = 2 * \left((1/d(t))^{1/t*2} - 1 \right)$
0.5	0.97087	6.0%
1.0	0.93897	6.4%
1.5	0.90909	6.46%
2.0	0.87336	6.88%

Spot Rate (annual rates)



- The ‘market’ discount factors from zeros

Mat.	Price	t	d(t) *
1	93.897	1	0.93897
2	87.336	2	0.87336

* d(t) = Price/Face Value

Yield of zero coupon bonds (annual rates)

- From price of zeros, we can calculate the yield-to-maturity of zero coupon bonds as

$$P = \sum_{t=1}^{n} \frac{C_t}{(1+y)^t} = \frac{C_n}{(1+y)^n}$$

- The yield to maturity of zero coupon bond maturing in n years = discount rate for a cash flow at year n

$$\frac{P_n}{C_n} = \frac{1}{(1+y_n)^n} \quad \rightarrow \quad y_n = \left(\frac{C_n}{P_n}\right)^{1/n} - 1$$

- y is often called the spot rate

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Present value of $1

Mat.	Price	t	d(t) *
1	93.897	1	0.93897
2	87.336	2	0.87336

•d(t) = Price/Face Value

T annual rates

1. 0.065
2. 0.007

$$y_n = \left(\frac{C_n}{P_n}\right)^{1/n} - 1$$

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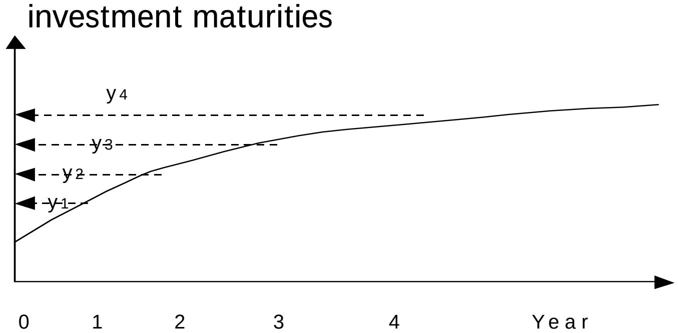
Term structure of interest rates (TIR)

- The term structure of interest rate (yield curve) presents yield of zero-coupon bonds for various investment maturities



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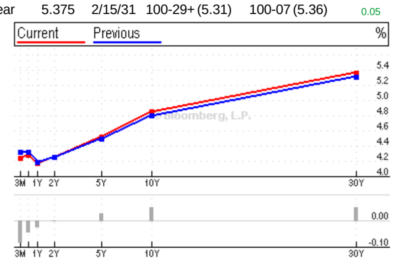
TIR

Bills		Mat Date	Previous Price/Yield	Current Price/Yield	Yld Chg	Prc Chg
3month		6/21/01	4.23 (4.33)	4.15 (4.25)	-0.08	-8
6month		9/20/01	4.17 (4.32)	4.14 (4.28)	-0.03	-3
1year		2/28/02	4.02 (4.20)	4.01 (4.18)	-0.01	-1

Notes/ Bonds	Coupon	Mat Date	Previous Price/Yield	Current Price/Yield	Yld Chg	Prc Chg
2year	4.625	2/28/03	100-21+ (4.26)	100-21 (4.26)	0.01	--0-01
5year	5.750	11/15/05	105-05+ (4.50)	105-01 (4.53)	0.03	--0-05
10year	5.000	2/15/11	101-14+ (4.81)	101-03+ (4.86)	0.04	--0-11
30year	5.375	2/15/31	100-29+ (5.31)	100-07 (5.36)	0.05	--0-23



Inflation Indexed Treasury	Coupon	Mat Date	Previous Price/Yield	Current Price/Yield	Yld Chg	Prc Chg
5year	3.625	7/15/02	101-27+ (2.17)	101-26 (2.20)	0.03	-0-01+
10year	3.500	1/15/11	102-01 (3.26)	101-24+ (3.29)	0.03	-0-08+
30year	3.875	4/15/29	107-27 (3.44)	107-15 (3.46)	0.02	-0-12

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Concept check

You have a project with the US government. You will received $100,000 from this project in 2 years. What is the present value of this cash flow?

TIR :	Maturity	yield
	1	5%
	2	6%

G) 100K/(1.05)(1.06)

Y) 100K/(1.06)^2

R) 100K/(1.06)

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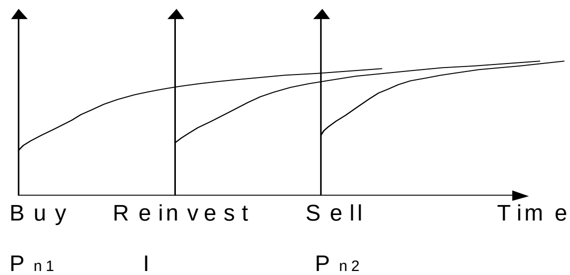
Bond return (realized yield)



$$\text{Holding period return} = \frac{P_{n2} - P_{n1} + \text{coupon} + I}{P_{n1}}$$

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Bond return

What is the return for holding a 10% 3-year coupon for 2 years?

TIR (today):

T	yield
1	4.6%
2	4.7%
3	4.8%

$$P_{n1} = \frac{100}{(1.046)} + \frac{100}{(1.047)^2} + \frac{1100}{(1.048)^3} = 1142.5$$

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Bond return

What is the return for holding a 10% 3-year coupon for 2 year?

TIR (in 1 year):

t	yield
1	4.6%
2	4.7%
3	4.8%

- I= 100*(0.046)=4.6

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Bond return

TIR (2 years):

t	yield
1	4.65%
2	4.7%

- $$P_{n2} = \frac{1100}{(1.0465)} = 1051.12$$

- Total return for two years

$$= \frac{1051.12 - 1142.5 + 4.6 + 200}{1142.5} = 9.91\%$$

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The Forward Rate (for the n^{th} period)

Is the interest rate in the n^{th} period which equates the n-period holding return of

(a) An n-period zero-coupon bond and

(b) The strategy of holding an (n-1) period bond (zero for n-1 periods and rolling it over through the n^{th} period

y_n are interest rates for that particular period

$$(1+f_n)(1+y_{n-1})^{n-1} = (1+y_n)^n$$

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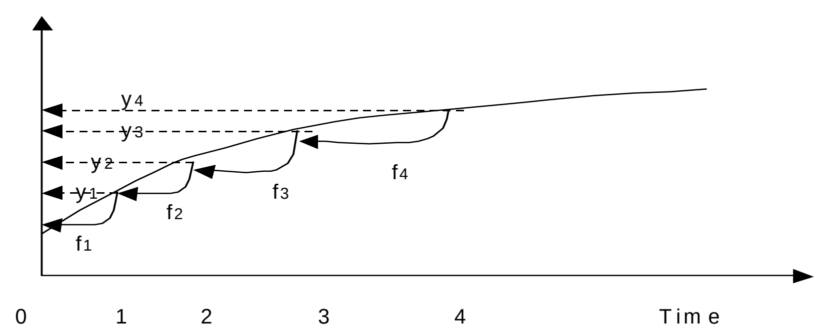
The Forward Rate (for the n^{th} period)

$$(1+f_n)(1+y_{n-1})^{n-1} = (1+y_n)^n$$



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The Forward Rate (for the n^{th} period)

- Equals $$f_n = \frac{(1+y_n)^n}{(1+y_{n-1})^{n-1}} - 1$$

- Equals the short rate when interest rates are certain. But in general forward rates do not equal future short rates

- Why use forward rate?

  Useful for understanding the theories of the TIR

  Use in a forward contract. A forward contract is a contract you enter into today to borrow/lend in the future at an interest rate specify today.

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Forward rates

- What is the price of a 3-year 10% coupon bond?
- TIR:

T	Price	yield	forward rate
1	934.58	7%	7%
2	857.34	8%	9%
3	782.91	8.5%	9.5%

Calculating forward rates

$$f_2 = \left[\frac{(1+.08)^2}{(1+.07)^1} - 1\right] = 9\%$$

Usage of forward rates

Calculate a 10% annual coupon bond maturing in 3 years:

Using yields

$$P = \frac{100}{1+0.07} + \frac{100}{(1+0.08)^2} + \frac{1100}{(1+0.085)^3} = 1040.4$$

Using forward rates

$$P = \frac{100}{1.07} + \frac{100}{(1.07)(1.09)} + \frac{1100}{(1.07)(1.09)(1.095)} = 1040.5$$

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The forward rate as future borrowing or lending rate

company will work on a short project that starts one year from today and ends 2 years from today. It will need a loan at the start of the project and will get all the revenues to pay back the loan at the end of the project. The analysis of the project is based on current interest rates so you want to fix the future borrowing rate **today**. How would you use positions in the 1-year and 2-year bonds to fix your borrowing rate between year 1 and 2 (disregard your default risk for now)?

G) Long 1-year and short 2-year zeros

Y) Short 1-year and long 2-year zeros

R) Long 1-year and long 2-year zeros

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The forward rate as future borrowing rate

You want to fix your borrowing rate today for borrowing $1000 between years 1 and 2.0. What to do?

Cash flow		↓	↑	
	0	1000	?	
	0	1	2.0	years

Buy a 1 year zero

Sell a 2.0 year zero

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The forward rate as future borrowing rate

t	price	yield	forward
1	952.38	5%	5%
2	898.45	5.5%	6.0
3	839.62	6%	7.12

Cash flow (year)	0	1	2
Buy 1 one-year zero	-952.38	1000	
Sell 1.06 (952.38/898.45) of 2.0 year zero	952.38	0	-1060
Net	0	1000	-1060

Interest rate = (1060-1000)/1000 = 6%

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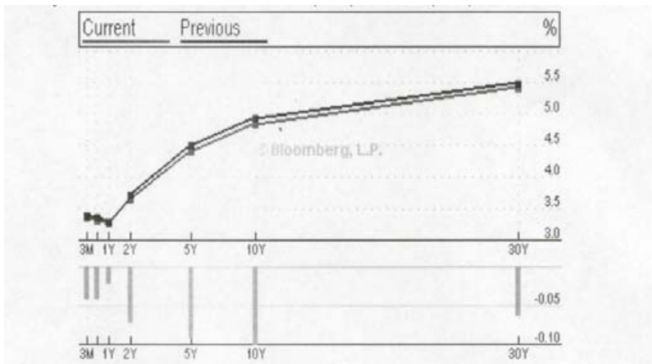
Theories of the term structure of interest rates

What can we deduce from the shape of the TIR?



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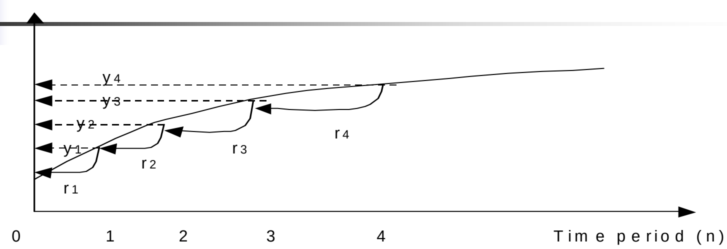
When future short rates are certain



- The future 'short' rate is the interest rate that will prevail in a future period. In general it is not known for certain today.
- When interest rates are certain, future short rate equals forward rate

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Theory of the term structure

When the future is uncertain what should be our best estimate of the future short rate?

- Market Expectations Theory
  - Term structure determined solely by expectations of future interest rates

$$E[\tilde{r}_n] = f_n$$

- Liquidity Preference Theory
  - Term structure determined also by different levels of risk associated with different maturities

$$f_n = E[\tilde{r}_n] + premium$$

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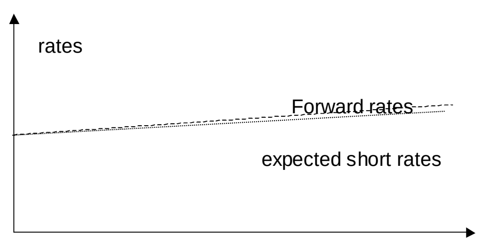
Expectations theory



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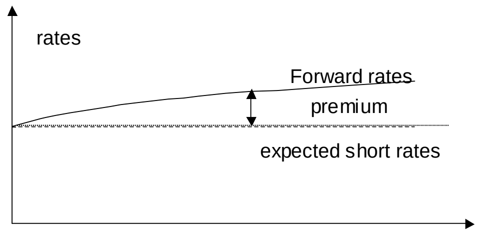
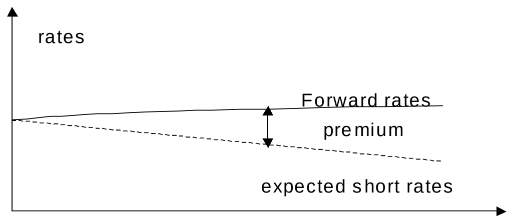
If most investors are short-term investors





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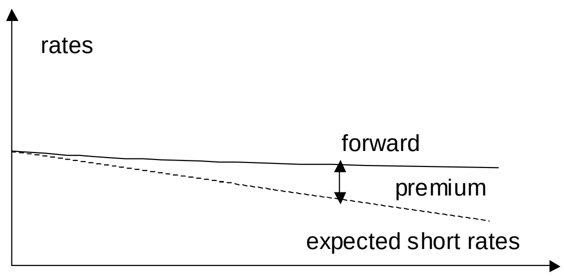
If most investors are short-term investors

What can we deduce when the TIR is downward sloping?



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Summary

- The term structure of interest rates
- Forward rates
- Theories of the term structure of interest rates

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