

Question 1: Static and Comparative Analysis (Demand and Supply)

A study has shown that there are three groups of iPhone users, namely, crazy, love-it, and just-live-with-it. Demand for iPhone of each group can be given by:

crazy: $Q_c = 100 - P$;

Love-it: $P = 50 - Q_L$;

Just-live-with-it: $Q_j = 20 - P$;

where Q_c is the quantity demanded by crazy group, Q_L is quantity demanded by love-it group, and Q_j is the quantity demanded by just-live-with-it group.

- Find the domain set of prices that justifies the demand equation for each group of iPhone user. And, rewrite each demand function in a more appropriate way.
- At what domain set of prices, do all the three types of iPhone users stay active in the market?
- Find the function for market demand for iPhone. Be precise about what is needed to make your equation justified.

a) Crazy : $P < 100$
 Love it : $P < 50$
 Just live with it : $P < 20$

b) $P < 20$

c) $Q = 0$; $P \geq 100$
 $Q = 100 - P$; $50 \leq P < 100$
 $Q = (100 - P) + (50 - P) = 150 - 2P$; $20 \leq P < 50$
 $Q = (150 - 2P) + (20 - P) = 170 - 3P$; $0 \leq P < 20$

Now, suppose that market supply equation is given by $p=4+3w+\frac{3}{8}Q$.

d) Find the equilibrium when $w = 1/3$ where w is wage rate for each unit of labour hired.

e) How much does each type of consumer consume in the equilibrium?

f) What is the likely effect on market equilibrium when wage drops? State your prediction and develop intuition for your result. (Note: Answer to this question could be made in qualitative sense. You don't need to get into algebraic solution with numbers solved.)

d) Market supply :
$$p = 4 + 3\left(\frac{1}{3}\right) + \frac{3}{8}Q$$

$$= 5 + \frac{3}{8}Q$$

The producer would supply iPhones at $P > 5 \rightarrow P = 5 + \frac{3}{8}(0) = 5$

$$P = 5 + \frac{3}{8}Q$$

$$(P-5) \times \frac{8}{3} = Q$$

$$\frac{8P}{3} - \frac{40}{3} = Q$$

Equilibrium :
$$\frac{8P}{3} - \frac{40}{3} = 170 - 2P$$

$$\frac{12P}{3} = \frac{550}{3}$$

$$P = 32.35$$

At $P = 32.35$, the 'just live with it' group would be unwilling to buy iPhones, hence market demand equation is $150 - 2P = Q$

$$150 - 2P = \frac{8P}{3} - \frac{40}{3}$$

$$\frac{490}{3} = \frac{14P}{3}$$

$$35 = P$$

$$Q = 150 - 2(35) = 80$$

e) Crazy : $Q_C = 100 - 35 = 65$
 Love it : $Q_L = 50 - 35 = 15$
 Just live with it : $Q_J = 0$

f) A drop in wage would increase the producer's supply. Thus it is likely that more output would be produced.

Question 2: Price Control and Welfare

- a) Consider the market for apartment rentals in Chicago. The price of rent is determined by the following systems of equations:

Demand: $p = -2q + 160$

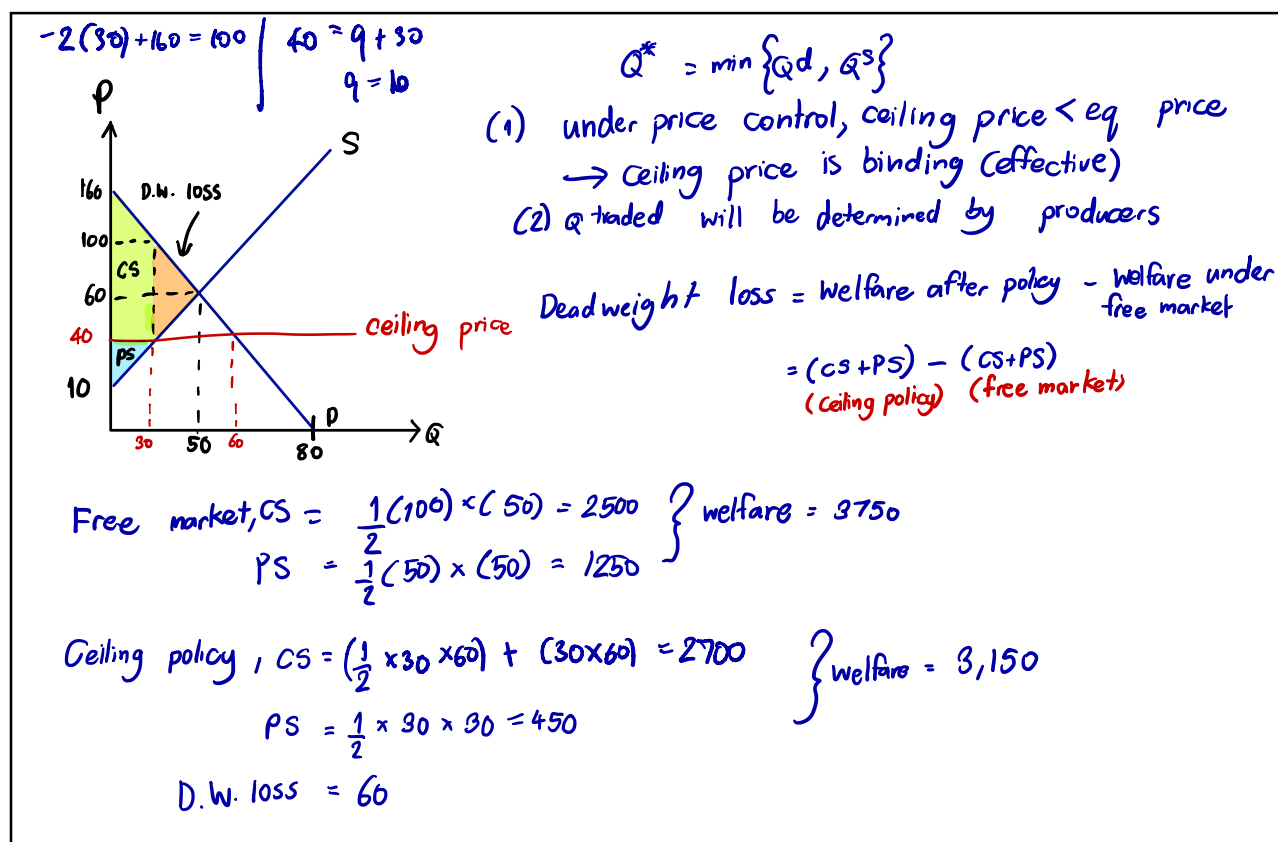
Supply: $p = q + 10$

$$Q_d = Q_s \rightarrow -2Q + 160 = Q + 10$$

$$Q^* = 50 \text{ units}$$

$$P^* = \$60/\text{unit}$$

- b) Suppose the government tries to control the rent prices through a price ceiling of \$40. Discuss the implication of this policy. Is there any deadweight loss?



Question 3: Inverse Matrix (Demand and Supply)

Given the following supply and demand functions: $Q_D = 100 - 3P$, $Q_S = 80 + 2P$

a) Write the equilibrium condition for this market and translate the system of equations into matrix notation.

$$\begin{aligned}
 Q_D &= 100 - 3P \rightarrow Q_D + 3P = 100 \\
 Q_S &= 80 + 2P \rightarrow Q_S - 2P = 80 \\
 Q_D - Q_S &= 0
 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} Q_D \\ Q_S \\ P \end{bmatrix} = \begin{bmatrix} 100 \\ 80 \\ 0 \end{bmatrix}$$

b) Use matrix inversion to solve for the equilibrium quantity and equilibrium price.

$$A^{-1} = \frac{\text{adj}(A)}{\det(A)}$$

$$\det(A) = \begin{vmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 1 & -1 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 1 & -1 & 0 \end{vmatrix} = -(3+2) = -5$$

$$\text{adj}(A) = C^T$$

$$C_{ij} = (-1)^{i+j} m_{ij}$$

$$m_{11} = \begin{bmatrix} 1 & -2 \\ -1 & 0 \end{bmatrix} = 0 - (-2) \rightarrow C_{11} = -2$$

$$m_{12} = \begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix} = 0 - (-2) = 2 \rightarrow C_{12} = -2$$

$$m_{13} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} = 0 - 1 = -1 \rightarrow C_{13} = -1$$

$$m_{21} = \begin{bmatrix} 0 & 3 \\ -1 & 0 \end{bmatrix} = 0 - (-3) = 3 \rightarrow C_{21} = -3$$

$$m_{22} = \begin{bmatrix} 1 & 3 \\ 1 & 0 \end{bmatrix} = 0 - 3 = -3 \rightarrow C_{22} = -3$$

$$m_{23} = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} = -1 \rightarrow C_{23} = 1$$

$$m_{31} = \begin{bmatrix} 0 & 3 \\ 1 & -2 \end{bmatrix} = 0 - 3 = -3 \rightarrow C_{31} = -3$$

$$m_{32} = \begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix} = -2 \rightarrow C_{32} = 2$$

$$m_{33} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1 \rightarrow C_{33} = 1$$

$$C = \begin{bmatrix} -2 & -2 & -1 \\ -3 & -3 & 1 \\ -3 & 2 & 1 \end{bmatrix}$$

$$C^T = \begin{bmatrix} -2 & -3 & -3 \\ -2 & -3 & 2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj}(A)}{\det(A)}$$

$$= \frac{1}{-5} \begin{bmatrix} -2 & -3 & -3 \\ -2 & -3 & 2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2/5 & 3/5 & 3/5 \\ 2/5 & 3/5 & -2/5 \\ 1/5 & -1/5 & -1/5 \end{bmatrix}$$

$$AX = B$$

$$X = BA^{-1}$$

$$X = \begin{bmatrix} 2/5 & 3/5 & 3/5 \\ 2/5 & 3/5 & -2/5 \\ 1/5 & -1/5 & -1/5 \end{bmatrix} \begin{bmatrix} 100 \\ 80 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} Q_D \\ Q_S \\ P \end{bmatrix} = \begin{bmatrix} 2/5(100) + 3/5(80) + 0 \\ 2/5(100) + 3/5(80) + 0 \\ 1/5(100) - 1/5(80) \end{bmatrix} = \begin{bmatrix} 88 \\ 88 \\ 4 \end{bmatrix}$$

Question 4: Matrix and Cramer's Rule (IS-LM Model)

Consider the macroeconomic model. $C = a + bY_d$; $0 < b < 1$

$$Y_d = Y - T + R$$

$$I = I_a + iY$$
; $0 < i < 1$

$$G = G_0$$

$$T = T_0 + tY$$
; $0 < t < 1$

$$R = R_0$$

intercept

$$Y = a + bX$$

$a = 3$, $b = 5$

where R is the government transfer and G is the government purchase. All the remaining are defined as usual.

- a) Determine all the endogenous and exogenous variables in the model.

Endogenous : C, Y_d, Y, T, I
 Exogenous : G_0, T_0, R_0

- b) State the condition that characterizes the equilibrium of this model.

$$Y = C + I + G$$

- c) Simplify the model into a 3-variable system of equations that only includes on Y, C, I .

1) $Y = C + I + G_0 \rightarrow Y - C - I = G_0$
 2) $C = a + b(Y - T_0 - tY + R_0)$
 $C = a + bY - bT_0 - btY + bR_0 \rightarrow C - Y[(1-t)b] = a - bT_0 + bR_0$
 $C = a - bT_0 + Y[(1-t)b] + bR_0$
 3) $I - iY = I_a$

- d) Rewrite the system of equations in 2.3 in the form of matrix.

$$\begin{bmatrix} 1 & -1 & -1 \\ -(1-t)b & 1 & 0 \\ -i & 0 & 1 \end{bmatrix} \begin{bmatrix} Y \\ C \\ I \end{bmatrix} = \begin{bmatrix} G_0 \\ a - bT_0 + bR_0 \\ I_a \end{bmatrix}$$

e) Solve for the solution of Y, C and I. Use the Cramer's rule method.

$$Y = \frac{D_Y}{D}, \quad C = \frac{D_C}{D}, \quad I = \frac{D_I}{D}$$

$$D_Y = \begin{vmatrix} G_0 & -1 & -1 \\ a-bT_0+bR_0 & 1 & 0 \\ I_a & 0 & 1 \end{vmatrix} \begin{vmatrix} G_0 & -1 \\ a-bT_0+bR_0 & 1 \\ I_a & 0 \end{vmatrix}$$

$$D_Y = G_0 - (I_a - a + bT_0 - bR_0) = G_0 + I_a + a - bT_0 + bR_0$$

$$D_C = \begin{vmatrix} 1 & G_0 & -1 \\ -(1-t)b & a-bT_0+bR_0 & 0 \\ -i & I_a & 1 \end{vmatrix} \begin{vmatrix} 1 & G_0 \\ -(1-t)b & a-bT_0+bR_0 \\ -i & I_a \end{vmatrix}$$

$$= a - bT_0 + bR_0 + ((1-t)b)(I_a) - ((a - bT_0 + bR_0)i - (1-t)bG_0)$$

$$= a - bT_0 + bR_0(1-i) + ((1-t)b)(I_a + G_0)$$

$$D_I = \begin{vmatrix} 1 & -1 & G_0 \\ -(1-t)b & 1 & a-bT_0+bR_0 \\ -i & 0 & I_a \end{vmatrix} \begin{vmatrix} 1 & -1 \\ -(1-t)b & 1 \\ -i & 0 \end{vmatrix}$$

$$= (I_a + (a - bT_0 + bR_0)(i)) - (-iG_0 + (1-t)bI_a)$$

$$= i(a - bT_0 + bR_0 + G_0) + I_a(1 - (1-t)b)$$

$$= i(a - bT_0 + bR_0 + G_0) + I_a(1 - b + bt)$$

f) Compare the multipliers of G and R. Which one has a bigger impact? Why?

e) Solve for the solution of Y, C and I. Use the Cramer's rule method.

$$D = \begin{bmatrix} 1 & -1 & -1 \\ -(1-t)b & 1 & 0 \\ -i & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -(1-t)b & 1 \\ -i & 0 \end{bmatrix}$$

$$= (1) - (i + (1-t)b)$$

$$= 1 - i - (1-t)b$$

$$Y = \frac{G_0 + I_a + a - bT_0 + bR_0}{1 - i - (1-t)b}$$

$$C = \frac{a - bT_0 + bR_0(1-i) + ((1-t)b)(I_a + G_0)}{1 - i - (1-t)b}$$

$$I = \frac{i(a - bT_0 + bR_0 + G) + I_a(1 - b + bt)}{1 - i - (1-t)b}$$

f) Compare the multipliers of G and R. Which one has a bigger impact? Why?

$$\frac{\Delta Y}{\Delta G} = \frac{1}{1 - i - (1-t)b} \quad \therefore \frac{\Delta Y}{\Delta G} > \frac{\Delta Y}{\Delta R}$$

$$\frac{\Delta Y}{\Delta R} = \frac{b}{1 - i - (1-t)b}$$

Question 5: Optimization without constraints (Monopoly and Subsidy Program)

A monopolist firm faces the market demand equation given by $P = 150 - 0.5Q$ and operates under a technology with cost function given by $TC = 100 + 3Q + 7Q^2$. Consider the following questions

- a. By using the derivative method, find the level of profit-maximizing output and price. Verify that your answer is the correction solution that results in maximized profit.

$$\begin{aligned}
 MR &= 150 - Q \\
 MC &= 3 + 14Q \\
 \text{FOC: } MR &= MC \\
 150 - Q &= 3 + 14Q \\
 Q &= \frac{147}{15} \\
 \text{SOC:} \\
 \pi'' &= MR' - MC' < 0 \\
 MR' &= -1 \\
 MC' &= 14 \\
 \pi'' &= -15 < 0
 \end{aligned}$$

So, $Q = \frac{147}{15}$ is the level of profit-maximizing output.

Continue with all the information given above, but now add another assumption to the questions. That is, we now assume that government subsidizes the monopolist for \$3 for each unit of output.

- b. Write the cost function of the monopolist when subsidization is taken into account.

$$TC = 100 + 3Q + 7Q^2 - 3Q = 100 + 7Q^2.$$

c) Find the level of profit-maximizing output and price under the subsidy program.

program.

Redo the same as in (a), but now we use the new TC as obtained in (b).

FOC:

$$MR = MC$$

$$150 - Q = 14Q$$

$$Q = 10.$$

SOC. $\pi'' = -15 < 0$. Thus, $Q = 10$ is the level of profit-maximizing output.

Given this Q , price would be \$145. (This can be obtained by plugging Q into the demand equation.)

Additional Notes:

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