



FIN 312

Investments



Challenge: How to invest optimally?

Stocks and other risky assets: IBM, MSFT, GE, INTC, GS, GOOG

Short-term Treasury securities (risk-free asset)

Long-term bonds



Outline

- The interest rate
- Return
- Variance risk
- Investors preference
- Risk-return trade off

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Real and Nominal Rates of Interest

- Interest rate
- US Treasury use as the risk-free rate
- Nominal interest rate
 - Growth rate of your money
- Real interest rate
 - Growth rate of your purchasing power
- If R is the nominal rate and r the real rate and i is the inflation rate:

$$R+1=(1+r)(1+i)$$

$$R \sim r+i$$

Factors Influencing Interest rates

- Supply
 - Households
- Demand
 - Businesses
- Government's Net Supply and/or Demand
 - Federal Reserve Actions

Figure 5.1 Determination of the Equilibrium Real Rate of Interest

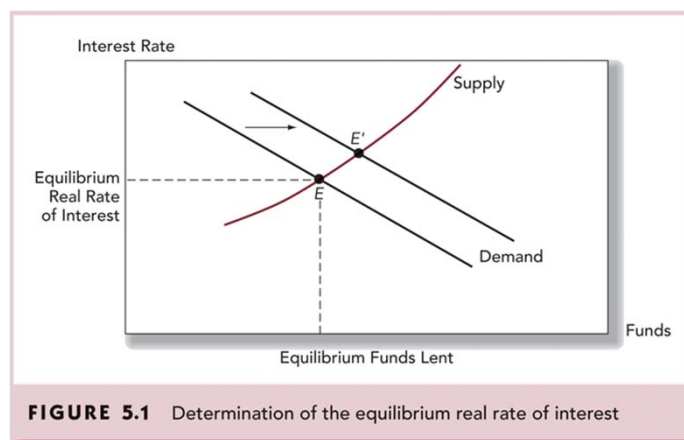


Table 5.2 History of T-bill Rates, Inflation and Real Rates for Generations, 1926-2005

Portfolio	Statistic	1926-2005	1966-2005	1981-2005	1971-1995	1961-1985	1951-1975	1941-1965	1931-1955	1926-1950
U.S. T-bills	Average	3.75	5.98	5.73	7.04	6.55	3.66	1.62	0.63	1.02
	Standard Deviation	3.15	2.84	3.15	2.87	3.15	1.97	1.16	0.57	1.33
	Serial Correlation	0.91	0.81	0.87	0.76	0.82	0.82	0.85	0.85	0.88
U.S. CPI inflation	Average	3.13	4.70	3.36	5.60	5.39	3.28	3.39	2.21	1.51
	Standard Deviation	4.29	3.02	1.62	3.38	3.63	2.99	4.28	5.75	6.02
	Serial Correlation	0.64	0.73	0.32	0.68	0.74	0.69	0.35	0.46	0.52
U.S. real rate	Average	0.72	1.25	2.28	1.41	1.14	0.40	-1.54	-1.25	-0.13
	Standard Deviation	3.97	2.35	2.18	2.76	2.60	1.61	4.40	5.68	6.35
	Serial Correlation	0.66	0.71	0.72	0.71	0.76	0.49	0.53	0.51	0.64

TABLE 5.2

History of T-bill rates, inflation, and real rates for generations, 1926-2005

Sources: T-bills: Salomon Smith Barney 3-month U.S. T-bill index; inflation data: Bureau of Labor Statistics.

excel

Please visit us at
www.mhhe.com/bkm

Figure 5.2 Interest Rates and Inflation, 1926-2005

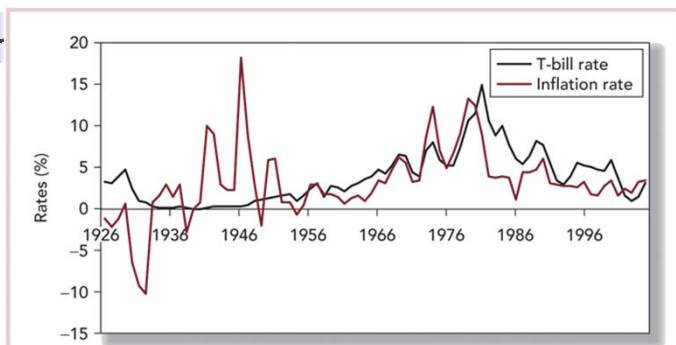


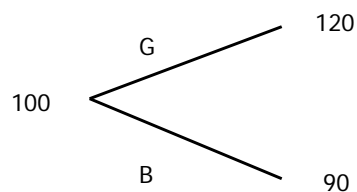
FIGURE 5.2 Interest and inflation rates, 1926-2005

Bills and Inflation, 1926-2005

- Entire post-1926 history of annual rates:
 - www.mhhe.com/bkm
- Average real rate of return on T-bills for the entire period was 0.72 percent
- Real rates are larger in late periods

Risky Prospects

The stock price of ABC is currently \$100. Assume the stock price is either going to increase to \$120 or decrease to \$90 over the next year. In both cases the firm will pay a dividend of \$5.





Rates of return

Rates of Return: Single Period

$$HPR = \frac{P_1 - P_0 + D_1}{P_0}$$

HPR = Holding Period Return

P_0 = Beginning price

P_1 = Ending price

D_1 = Dividend during period one



Rates of Return

- The rate of return in the G state is:

$$r_G = \frac{120 + 5 - 100}{100} = 25\%$$

- The rate of return in the B state is:

$$r_B = \frac{90 + 5 - 100}{100} = -5\%$$



Variance of Rate of Return

- The variance is defined as the expectation of the squared deviations from the expectation. In our case:

$$\text{Var}(\tilde{r}) = \sigma_r^2 = 0.4(0.25 - 0.07)^2 + 0.6(-0.05 - 0.07)^2 = 0.0216$$

- In general, the variance is given by:

$$\text{Var}(\tilde{r}) = \sigma_r^2 = E(\tilde{r} - E(\tilde{r}))^2 = \sum_{j=1}^n p_j (r_j - \bar{r})^2$$

- The standard deviation is defined as the squared root of the variance. In our example: $\sigma_r = \sqrt{0.0216} = 14.7\%$.

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Variance – An Alternative Method

- The variance can also be calculated as follows:

$$\text{Var}(\tilde{r}) = \sigma_r^2 = E(\tilde{r}^2) - E(\tilde{r})^2 = \sum_{j=1}^n p_j r_j^2 - \left(\sum_{j=1}^n p_j r_j \right)^2$$

- In our example:

$$\text{Var}(\tilde{r}) = 0.4 \cdot 0.25^2 + 0.6 \cdot (-0.05)^2 - 0.07^2 = 0.0216$$

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Expected Return

Expected returns

$$E(r) = \sum_s p(s)r(s)$$

$p(s)$ = probability of a state

$r(s)$ = return if a state occurs

s = state

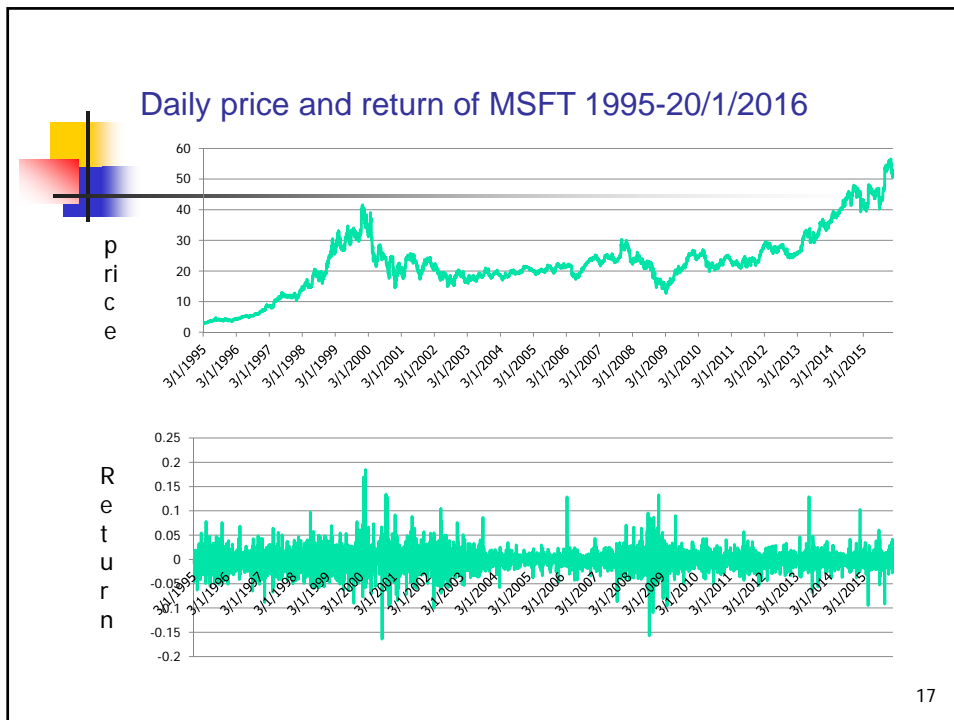


Scenario Returns: Example

<u>State</u>	<u>Prob. of State</u>	<u>r in State</u>
1	.1	-.05
2	.2	.05
3	.4	.15
4	.2	.25
5	.1	.35

$$E(r) = (.1)(-.05) + (.2)(.05) \dots + (.1)(.35)$$

$$E(r) = .15$$



Time Series Analysis of Past Rates of Return

- When using historical data to estimate expected return we assume that each data point has equal probability, therefore the probability = $1/n$
- Expected Returns and the Arithmetic Average

$$E(r) = \sum_{s=1}^n p(s)r(s) = \frac{1}{n} \sum_{s=1}^n r(s)$$

Variance and Standard Deviation

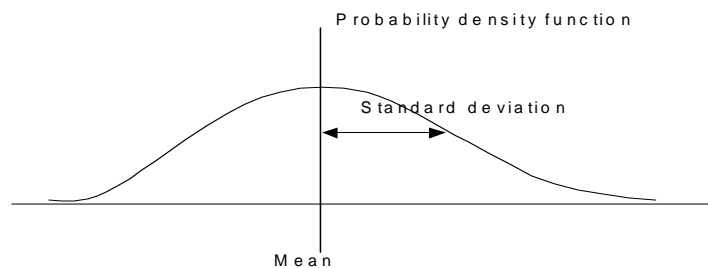
- Variance = expected value of squared deviations

$$\sigma^2 = \frac{1}{n} \sum_{s=1}^n [r(s) - \bar{r}]^2$$

- For sample data (not population data) when eliminating the bias because we use estimated expected return, Standard Deviation becomes:

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{j=1}^n [r(s) - \bar{r}]^2}$$

Distribution of returns



$$\text{Mean} = E[\tilde{r}_i] = 1/N \sum \tilde{r}_i$$

$$\text{Variance of returns} = \sigma^2[\tilde{r}_i] = 1/(N-1) \sum (\tilde{r}_i - E[\tilde{r}_i])^2$$

Standard deviation = square root of variance

The Normal Distribution

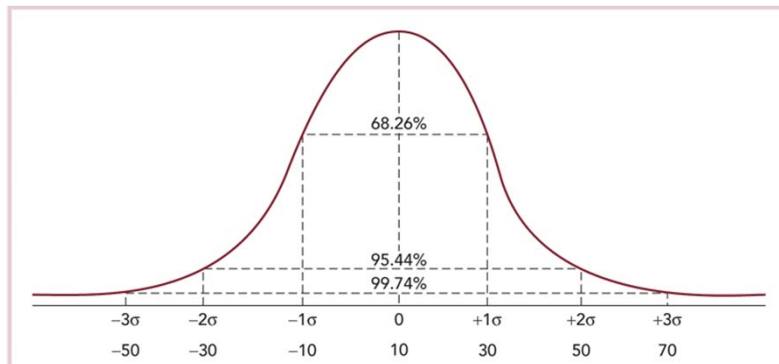


FIGURE 5.4 The normal distribution

Normal and Skewed Distributions (mean = 6% SD = 17%)

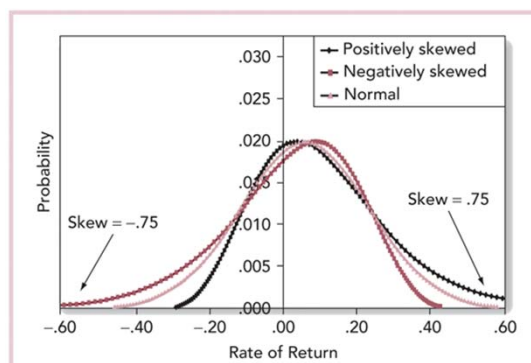


FIGURE 5.5A Normal and skewed distributions
(mean = 6%, SD = 17%)

Normal and Fat-Tailed Distributions (mean = .1, SD = .2)

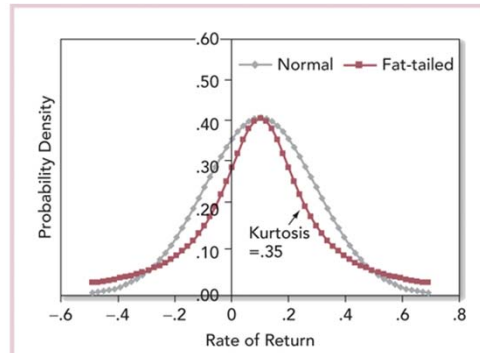


FIGURE 5.5B Normal and fat-tailed distributions (mean = .1, SD = .2)

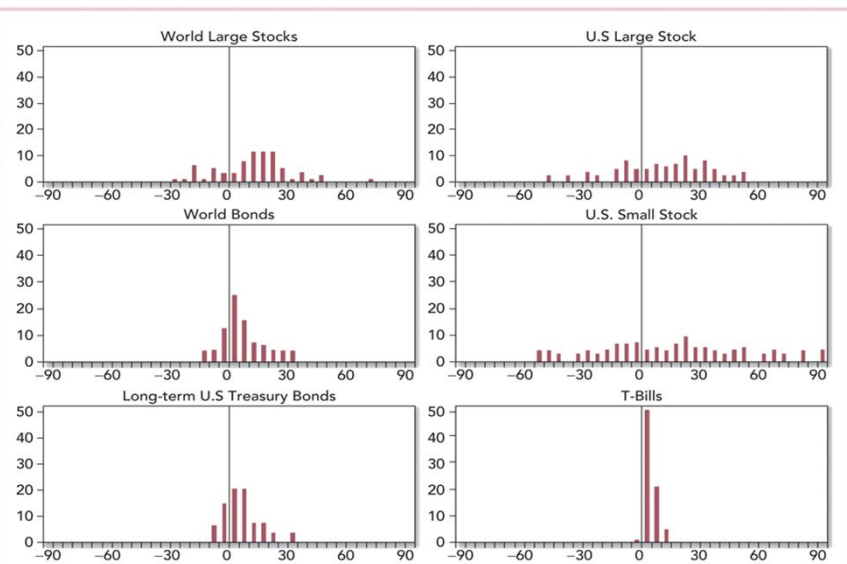


FIGURE 5.6 Frequency distributions of rates of return for 1926–2005

Portfolio	Statistic	1926–2005	1966–2005	1981–2005	1971–1995	1961–1985	1951–1975	1941–1965	1931–1955	1926–1950
World large stocks	Arithmetic avg.	11.46	12.12	13.45	14.20	11.17	12.28	13.01	10.80	7.70
	SD	18.57	17.72	17.84	17.59	16.10	17.64	14.18	21.42	21.61
	Geometric avg.	9.85	10.67	12.03	12.79	9.99	10.89	12.15	8.77	5.59
U.S. large stocks	Arithmetic avg.	12.15	11.64	13.65	13.51	10.92	11.90	15.70	13.16	10.34
	SD	20.26	16.97	16.02	16.62	16.74	19.28	17.17	25.40	25.98
	Geometric avg.	10.17	10.31	12.50	12.26	9.63	10.27	14.47	10.04	7.05
U.S. small stocks	Arithmetic avg.	17.95	14.98	12.27	16.01	18.37	14.64	23.09	28.41	23.40
	SD	38.71	29.58	20.24	27.21	33.65	35.68	33.00	51.80	55.46
	Geometric avg.	12.01	11.27	10.44	12.61	13.62	9.59	19.15	19.03	11.85
World bonds	Arithmetic avg.	6.14	9.40	11.22	11.48	7.10	3.92	1.69	2.23	2.74
	SD	9.09	9.56	10.89	9.96	8.39	4.58	5.16	8.76	8.89
	Geometric avg.	5.77	9.00	10.71	11.07	6.80	3.82	1.56	1.88	2.38
Long-term U.S. treasury bonds	Arithmetic avg.	5.68	8.17	10.28	9.94	5.52	2.75	2.31	3.34	3.94
	SD	8.09	9.97	10.80	10.20	8.59	6.37	4.45	3.96	3.90
	Geometric avg.	5.38	7.73	9.78	9.50	5.20	2.56	2.22	3.27	3.87
U.S. T-bills	Arithmetic avg.	3.75	5.98	5.73	7.04	6.55	3.66	1.62	0.63	1.02
	SD	3.15	2.84	3.15	2.87	3.15	1.97	1.16	0.57	1.33
	Geometric avg.	3.70	5.95	5.68	7.00	6.50	3.64	1.62	0.62	1.01

TABLE 5.3

History of rates of return of asset classes for generations, 1926–2005

Sources: World portfolio: Datastream (16 countries index returns weighted by market capitalization).

U.S. stock returns for 1926–1995: Center for Research in Security Prices (CRSP).

U.S. stock returns since 1996: Returns on appropriate index portfolios: Large stocks, S&P 500; Small stocks, Russell 2000.

World bonds: Elroy Dimson, Paul Marsh, and Mike Staunton (16 countries weighted by GDP).

Long-term Government bonds: Lehman Bros. LT Treasury index.

T-bills: Salomon Smith Barney 3-month U.S. T-bill index.

Portfolio	Statistic	1926–2005	1966–2005	1981–2005	1971–1995	1961–1985	1951–1975	1941–1965	1931–1955	1926–1950
World large stocks	Average excess return	7.71	6.14	7.73	7.16	4.63	8.62	11.39	10.18	6.68
	SD of excess return	18.90	18.21	18.33	18.33	16.67	18.87	14.30	21.38	21.66
	Sharpe ratio	0.41	0.34	0.42	0.39	0.28	0.46	0.80	0.48	0.31
	Skew	-0.61	-0.62	-0.53	-0.93	-0.78	-0.65	-0.12	-0.70	-0.57
	Kurtosis	0.98	-0.38	-0.57	0.48	0.32	0.38	0.55	3.05	1.88
	Serial correlation	0.14	0.05	0.13	-0.01	-0.16	0.03	0.04	0.03	0.23
U.S. large stocks	Average excess return	8.39	5.66	7.92	6.47	4.38	8.24	14.08	12.54	9.32
	SD of excess return	20.54	17.10	16.12	16.97	17.22	20.47	17.43	25.39	26.01
	Sharpe ratio	0.41	0.33	0.49	0.38	0.25	0.40	0.81	0.49	0.36
	Skew	-0.80	-0.70	-0.65	-1.00	-0.79	-0.39	-0.14	-1.15	-0.91
	Kurtosis	1.03	-0.20	-0.46	0.91	-0.02	0.03	-0.67	1.62	0.62
	Serial correlation	0.08	0.02	0.07	-0.15	-0.18	-0.08	-0.20	-0.05	0.16
U.S. small stocks	Average excess return	14.20	9.00	6.54	8.97	11.82	10.98	21.47	27.78	22.38
	SD of excess return	39.31	29.89	20.70	27.50	34.06	36.38	33.40	51.92	55.86
	Sharpe ratio	0.36	0.30	0.32	0.33	0.35	0.30	0.64	0.54	0.40
	Skew	-0.22	-0.30	-0.42	-0.86	-0.41	-0.07	0.45	-0.28	-0.31
	Kurtosis	0.86	0.20	-0.20	0.41	-0.03	-0.26	-0.84	0.75	-0.17
	Serial correlation	0.16	0.07	-0.26	0.12	0.12	0.01	0.11	0.05	0.26
World bonds	Average excess return	2.39	3.42	5.49	4.44	0.55	0.26	0.07	1.61	1.72
	SD of excess return	8.97	10.36	11.58	11.20	9.02	4.71	4.92	8.78	8.81
	Sharpe ratio	0.27	0.33	0.47	0.40	0.06	0.05	0.01	0.18	0.20
	Skew	0.48	0.23	-0.06	0.02	0.36	0.14	-1.13	0.69	0.65
	Kurtosis	0.70	-0.42	-0.63	-0.36	1.29	0.09	2.26	2.50	2.39
	Serial correlation	0.13	0.11	-0.14	0.16	0.07	0.16	0.13	0.19	0.16
Long-term U.S. Treasury bonds	Average excess return	1.93	2.18	4.55	2.90	-1.02	-0.91	0.69	2.72	2.92
	SD of excess return	7.91	10.18	11.01	10.70	8.44	6.36	4.60	4.21	4.19
	Sharpe ratio	0.24	0.21	0.41	0.27	-0.12	-0.14	0.15	0.64	0.70
	Skew	0.23	0.23	-0.04	0.45	0.87	0.17	0.08	-0.21	-0.38
	Kurtosis	0.28	-0.57	-0.78	-0.49	1.60	-0.01	-0.51	-0.20	0.09
	Serial correlation	-0.07	-0.05	-0.31	0.01	0.23	0.02	-0.19	-0.14	-0.25

TABLE 5.4

History of excess rates of return of asset classes for generations, 1926–2005

* Skew, kurtosis, and serial correlation are estimated from continuously compounded excess rates of return.

Sources: World portfolio: Datastream (16 countries index returns weighted by market capitalization).

U.S. stock returns for 1926–1995: Center for Research in Security Prices (CRSP).

U.S. stock returns since 1996: Returns on appropriate index portfolios: Large stocks, S&P 500; Small stocks, Russell 2000.

World bonds: Elroy Dimson, Paul Marsh, and Mike Staunton (16 countries weighted by GDP).

Long-term Government bonds: Lehman Bros. LT Treasury index.

T-bills: Salomon Smith Barney 3-month U.S. T-bill index.

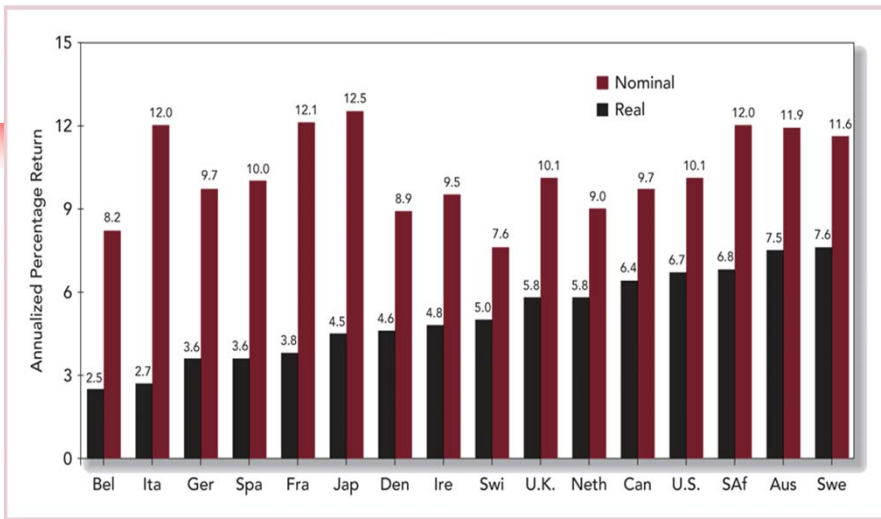


FIGURE 5.7 Nominal and real equity returns around the world, 1900–2000

Source: Elroy Dimson, Paul Marsh, and Mike Staunton, *Triumph of the Optimists: 101 Years of Global Investment Returns* (Princeton University Press, 2002), p. 50. Reprinted by permission of the Princeton University Press.

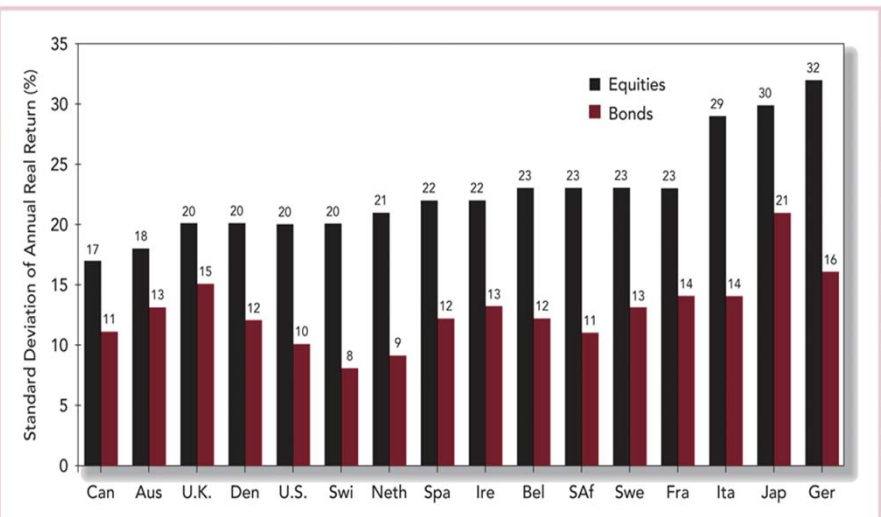


FIGURE 5.8 Standard deviations of real equity and bond returns around the world, 1900–2000

Source: Elroy Dimson, Paul Marsh, and Mike Staunton, *Triumph of the Optimists: 101 Years of Global Investment Returns* (Princeton University Press, 2002), p. 61. Reprinted by permission of the Princeton University Press.

Growth rate of investment

$$TV_n = (1+r_1)(1+r_2)x \dots x = (1+r_n)$$

TV = Terminal Value of the Investment of one dollar

g = geometric average rate of return

$$g = TV^{1/n} - 1$$

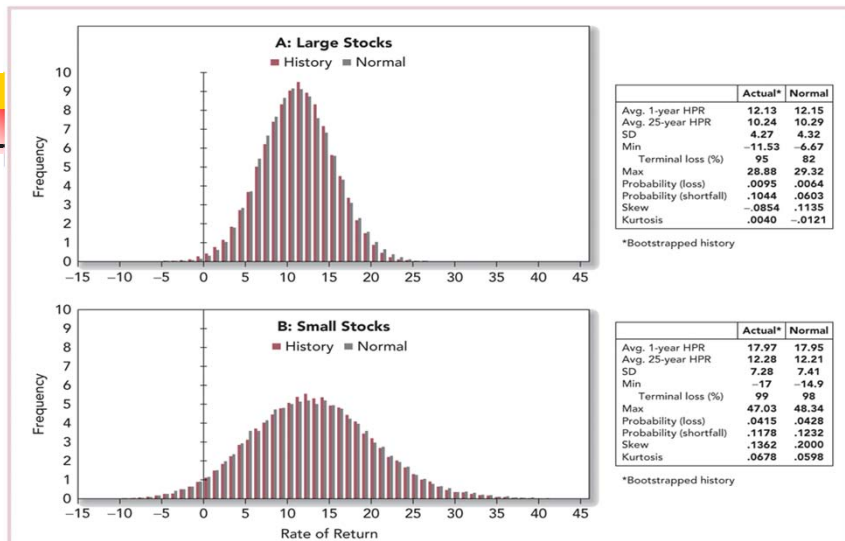


FIGURE 5.10 Annually compounded, 25-year HPRs from bootstrapped history and a normal distribution (50,000 observations)

Table 5.5 Risk Measures for Non-Normal Distributions

	Large U.S. Stocks		Small U.S. Stocks	
	History	Normal	History	Normal
Value at Risk				
VaR 1%	0.02%	0.18%	-0.63%	-0.64%
VaR 5%	1.16	1.27	0.17	0.13
VaR 10%	2.17	2.26	1.13	1.04
VaR 50%	10.58	10.29	16.41	15.99
Conditional Tail Expectation				
CTE 1%	-0.28%	-0.14%	-0.77%	-0.76%
CTE 5%	0.46	0.62	-0.33	-0.35
CTE 10%	1.07	1.20	0.16	0.12
CTE 50%	5.07	4.99	5.80	5.49
Lower Partial Standard Deviation				
LPSD of 25-year HPR	4.34%	4.23%	7.09%	7.14%
LPSD of 1-year HPR	21.71	21.16	35.45	35.72
Average 1-year HPR	12.13	12.15	17.97	17.95

TABLE 5.5

Risk measures for non-normal distributions

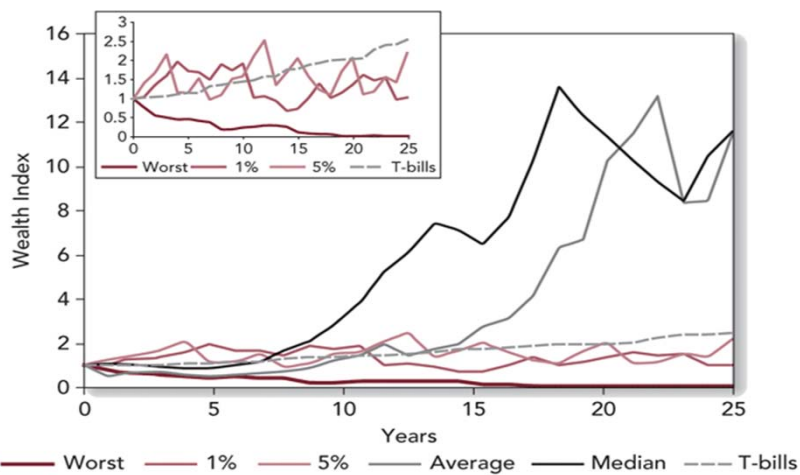


FIGURE 5.12 Wealth indexes of selected outcomes of large stock portfolios and the average T-bill portfolio. Inset: Focus on worst, 1%, 5% outcome versus bills.



Covariance

- The covariance of two random rates of return is

$$\text{Cov}(\tilde{r}_1, \tilde{r}_2) = \sigma_{12} = E((\tilde{r}_1 - E(r_1))(\tilde{r}_2 - E(r_2))) = \sum_{j=1}^n p_j (r_{1j} - \bar{r}_1)(r_{2j} - \bar{r}_2)$$

- An alternative formula

$$\text{Cov}(\tilde{r}_1, \tilde{r}_2) = \sigma_{12} = E(\tilde{r}_1 \tilde{r}_2) - E(\tilde{r}_1)E(\tilde{r}_2)$$

- Question: What is $\text{Cov}(\tilde{r}, \tilde{r})$?

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Correlation Coefficient

- The correlation coefficient is

$$\rho_{12} = \frac{\text{Cov}(r_1, r_2)}{\sigma_1 \sigma_2}$$

- It satisfies

$$-1 \leq \rho_{12} \leq 1$$

- What is the meaning of $\rho = 0, 1, -1$?

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Return and standard deviation of a portfolio

Given two uncertain stock returns, X and Y:

$$E[\tilde{r}_X] \quad \sigma[\tilde{r}_X] \quad E[\tilde{r}_Y] \quad \sigma[\tilde{r}_Y] \quad \rho_{X,Y}$$

A portfolio, P, that invests W_1 in X and W_2 in Y:

Mean

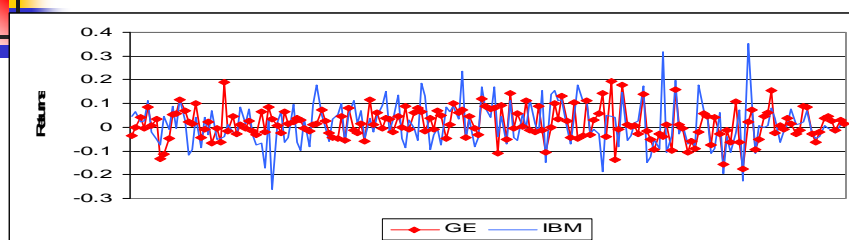
$$E[\tilde{r}_P] = w_1 E[\tilde{r}_X] + w_2 E[\tilde{r}_Y]$$

Standard deviation (STD)

$$\sigma[\tilde{r}_P] = \sigma_P = \sqrt{w_1^2 \sigma_X^2 + w_2^2 \sigma_Y^2 + 2w_1 w_2 \rho_{X,Y} \sigma_X \sigma_Y}$$

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Invest 50% in each stock; correlation= 0.34



GE: mean = 0.0146, standard deviation = 0.0657

IBM: mean = 0.01345 , standard deviation = 0.09233

Portfolio mean = $0.5 \times 0.0146 + 0.5 \times 0.01345 = 0.014$ (monthly)

Portfolio STD = $\sqrt{w_1^2 \sigma_X^2 + w_2^2 \sigma_Y^2 + 2w_1 w_2 \rho_{X,Y} \sigma_X \sigma_Y} = 0.06512$

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Some Useful Formulas

$$E(\tilde{r}_1 + \tilde{r}_2) = E(\tilde{r}_1) + E(\tilde{r}_2)$$

$$E(\alpha\tilde{r}) = \alpha E(\tilde{r})$$

$$\text{Var}(\alpha\tilde{r}) = \alpha^2 \text{Var}(\tilde{r})$$

$$\text{Var}(\tilde{r}_1 + \tilde{r}_2) = \text{Var}(\tilde{r}_1) + \text{Var}(\tilde{r}_2) + 2\text{Cov}(\tilde{r}_1, \tilde{r}_2) = \text{Var}(\tilde{r}_1) + \text{Var}(\tilde{r}_2) + 2\sigma_1\sigma_2\rho_{12}$$

$$\text{Cov}(\tilde{r}_1 + \tilde{r}_2, \tilde{r}_3) = \text{Cov}(\tilde{r}_1, \tilde{r}_3) + \text{Cov}(\tilde{r}_2, \tilde{r}_3)$$

$$\text{Cov}(\alpha\tilde{r}_1, \tilde{r}_2) = \alpha\text{Cov}(\tilde{r}_1, \tilde{r}_2)$$

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More Useful Stuff

If α is a constant then:

$$\text{Var}(\alpha) = 0$$

$$\text{Cov}(\tilde{r}, \alpha) = 0$$

$$E(\alpha) = \alpha$$

Also:

$$\text{Cov}(\tilde{r}_1, \tilde{r}_2) = \text{Cov}(\tilde{r}_2, \tilde{r}_1)$$

$$\rho_{12} = \rho_{21}$$

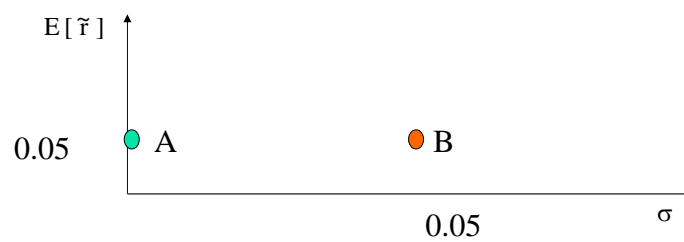
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Investor's preference

Choose between

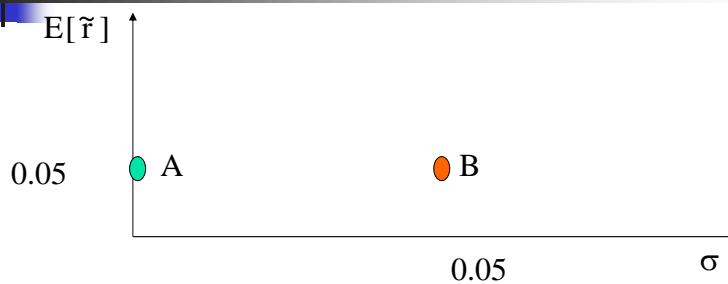
Security A: certain investment that returns 5%

Security B: uncertain investment with expected return of 5% and STD = 5%



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Investor's preference



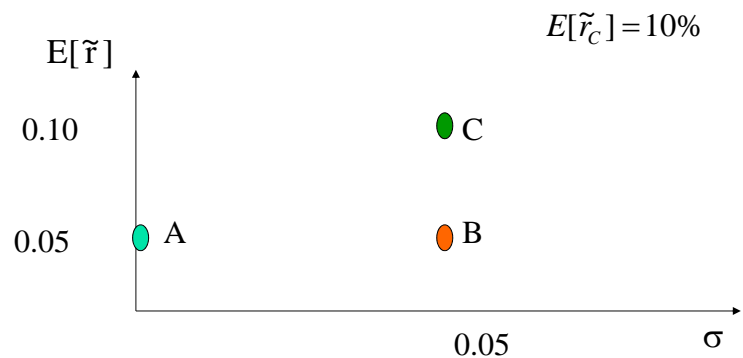
Risk averse investors prefer A; A is the dominant security

- (1) $E[\tilde{r}_A] \geq E[\tilde{r}_B]$
- (2) $\sigma_A \leq \sigma_B$, and strict inequality in at least one case

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Risk and return trade off

- Consider another security C with $\sigma[\tilde{r}_C] = 5\%$



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Risk and return trade off

- The risk premium is how much an investor is being compensated beyond the riskfree rate
 $= E[\tilde{r}] - r_f$
- Risk premium of security C = $0.1 - 0.05 = 0.05$ (5%)
- The Sharpe ratio is how much an investor is being compensated beyond the riskfree rate per one standard deviation of uncertainty
- Sharpe ratio of C = $\frac{E[\tilde{r}] - r_f}{\sigma} = \frac{0.05}{0.05} = 1$

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Risk and return trade off

- How much risk premium does an investor require in order to take an investment depends on his/her preference
- To compare investment opportunities an investor can assign a welfare, or utility score to competing investments
- A utility number is a cardinal number
- What should be part of the utility function?

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Risk and return trade off

- What investors like?
- What investors dislike?
- Utility function?

$U =$

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Risk and return trade off

- What investors like? $E[\tilde{r}]$
- What investors dislike? σ^2
- Utility function?
$$U = E[\tilde{r}] - \frac{A}{2} \sigma^2$$

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Risk and return trade off

- One common utility measure used in economics is:

$$U = E[\tilde{r}] - 0.5 A \sigma^2$$

↑
Parameter measuring
sensitivity to risk

- What $A > 0$ means?

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Risk and return trade off

- To compare investment opportunities investors can assign a utility score to competing investments
- The higher utility measure, the preferred investment

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Concept check



What should be the value of A (the coefficient of risk aversion) if investors are risk neutral, i.e., do not care about risk? What sign should A have if investors are risk loving, i.e., like risk.

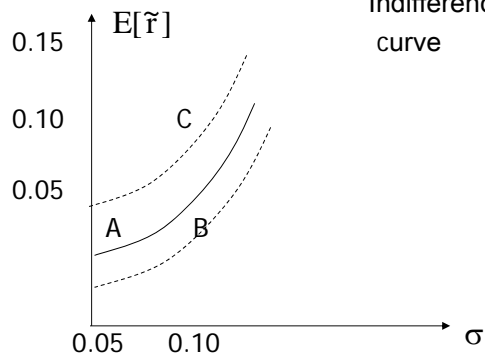
- G) Risk neutral $A=0$, risk loving $A<0$
- Y) Risk neutral $A=1$, risk loving $A<0$
- R) Risk Neutral $A= -1$, risk loving $A=0$

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Risk-return tradeoff

Let $A=10$



Indifference
curve

$E[\tilde{r}]$	σ	U
0.05	0	0.05
0.066	0.04	0.05
0.075	0.05	0.05
0.15	0.10	0.05

$$U = E[\tilde{r}] - 0.5A\sigma^2$$

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Risk-return tradeoff: example

A stock portfolio has an expected return of 20% and a standard deviation of 20%. T-bills yield 7% for sure. Which investment is preferred if $A = 4$?

$$U = E[r] - .5A\sigma^2$$

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Risk-return tradeoff: example

A stock portfolio has an expected return of 20% and a standard deviation of 20%. T-bills yield 7% for sure. Which investment is preferred if $A = 8$?

$$U = E[r] - .5A\sigma^2$$

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Risk-return tradeoff

- The level of risk aversion determines whether an investor chooses to invest in risky assets and how much he/she chooses to invest in risky assets.
- What determines the risk aversion level of an investor?

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Risk-return tradeoff

- The level of risk aversion determines whether an investor chooses to invest in risky assets and how much he/she chooses to invest in risky assets.
- What determines the risk aversion level of an investor?
 - Current wealth
 - Future income
 - Personality
 - Investment horizon

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Homework for next class

- The level of risk aversion determines whether an investor chooses to invest in risky assets and how much he/she chooses to invest in risky assets.
- What determines the risk aversion level of an investor?
 - Current wealth
 - Future income
 - Personality
 - Investment horizon

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Risk-return tradeoff

- How is risk aversion level determined in practice?

One way is to use survey methods to assess the relative level of risk aversion of an investor compared to that of an average investor which is approximately 3.0 (See how to compute this number in class notes for asset allocation).

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Historical Data of Common Stocks

	S&P 500 Stocks		One-month Bills		Excess Return	
	$E[r]$	σ	r_f	σ	$E[r] - r_f$	σ
1926-1942	7.2	29.6	1.0	1.7	6.2	29.9
1943-1960	17.4	18.0	1.4	1.0	16.0	18.3
1961-1979	8.6	17.0	5.2	2.0	3.4	17.7
1980-1999	18.6	13.1	7.0	3.0	11.6	13.8
1926-1999	13.1	20.2	3.8	3.3	9.3	20.6

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Historical facts (1929-1999)

- Rate of return of the S&P500 = 13.1 %.
- One month T-bill rate 3.8 %
- Risk premium of the S&P500 is 9.3% per year
- Volatility of the market = 20.6%
- Sharpe ratio = $9.3/20.6 = 0.45$

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Summary

- Returns and other statistics
 - Arithmetic return, geometric return
 - Historical data
- Risk and return trade off
 - Utility is a welfare score used to formalized preference
 - What is in an investment utility function
 - Investors with higher risk aversion levels choose investments with lower risk, all else equal

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Homework for next class

- Design one question such that you can ask two investors: A and B and you are about to determine which investor has higher risk aversion.
- The question much address risk aversion in terms of financial risk.