# EE320 - Topic 3 (Additional Materials).

## Tax Incidence and Price Elasticity

Recall the concepts of price elasticities.

① Price elasticity of demand : $\eta = \dfrac{-\Delta Qd / Qd}{\Delta P / P} = -\dfrac{\Delta Qd}{\Delta P} \cdot \dfrac{P}{Qd}$

② Price elasticity of supply : $\varepsilon = \dfrac{\Delta Qs / Qs}{\Delta P / P} = \dfrac{\Delta Qs}{\Delta P} \cdot \dfrac{P}{Qs}$

Note ; When $\Delta$ is very small, we write "d" instead of $\Delta$.
Ex $\eta = -\dfrac{dQd}{dP} \cdot \dfrac{P}{Qd}$. But for now, let's use $\Delta$.

Objective : Determine how much tax affects the change in the "equilibrium" price?

① Impact of specific tax on consumer's price.

Recall ; The after-tax equilibrium price for consumer is :

$$P^d = \frac{a + c + dT}{b + d}.$$

Question : What is $\dfrac{\Delta P^d}{\Delta T}$ ?

Suppose tax increases from $T_0$ to $T_1$, so $\Delta T = T_1 - T_0$.

$$\Rightarrow \underbrace{P_1^d - P_0^d}_{= \Delta P^d} = \left(\frac{a + c + dT_1}{b + d}\right) - \left(\frac{a + c + dT_0}{b + d}\right) = \frac{d\overbrace{(T_1 - T_0)}^{\Delta T}}{b + d}$$

$$\Rightarrow \boxed{\frac{\Delta P^d}{\Delta T} = \frac{d}{b + d}} \quad — ①$$

We want to express $\dfrac{\Delta P^d}{\Delta T}$ in terms of $\eta$ and $\varepsilon$.

Recall, $Q^d = a - bP$ and $Q^s = -c + dP$

$\Rightarrow \dfrac{\Delta Q^d}{\Delta P} = -b$ and $\dfrac{\Delta Q^s}{\Delta P} = d$.

Substitute $\dfrac{\Delta Q^d}{\Delta P}$ and $\dfrac{\Delta Q^s}{\Delta P}$ in ①:

$$\Rightarrow \frac{\Delta P^d}{\Delta T} = \frac{d}{b+d} = \frac{\Delta Q_s / \Delta P}{-\frac{\Delta Q_d}{\Delta P} + \frac{\Delta Q_s}{\Delta P}}$$

Multiply the right hand side by $\frac{P}{Q}$ (both top and bottom):

$$\frac{\Delta P^d}{\Delta T} = \frac{\overbrace{\frac{\Delta Q_s}{\Delta P} \times \frac{P}{Q}}^{=\varepsilon}}{\underbrace{\left(-\frac{\Delta Q_d}{\Delta P}\right) \times \frac{P}{Q}}_{=\eta} + \underbrace{\left(\frac{\Delta Q_s}{\Delta P}\right) \times \frac{P}{Q}}_{=\varepsilon}}$$

At equilibrium $Q_s = Q_d = Q$.

$$\Rightarrow \boxed{\frac{\Delta P^d}{\Delta T} = \frac{\varepsilon}{\eta + \varepsilon}} \quad — ②$$

$\therefore$ If $\varepsilon = 0$, $\dfrac{\Delta P^d}{\Delta T} = 0$. (ie. If the supply is perfectly price inelastic, there will be no change in consumer's price)

If $\eta = 0$, $\dfrac{\Delta P^d}{\Delta T} = 1$. (ie. If the demand is perfectly price inelastic, the consumer's price will increase by the amount of tax).

## ② Impact of specific tax on producer's price.

Recall: The after-tax equilibrium price for producer:

$$P^s = \frac{a+c-bT}{b+d}.$$

Question: $\dfrac{\Delta P^s}{\Delta T} = ?$

Again, Suppose tax increases from $T_0$ to $T_1$ $(\Delta T = T_1 - T_0)$.

$$\Rightarrow \underbrace{P^s_1 - P^s_0}_{\Delta P^s} = \frac{-b(\overbrace{T_1 - T_0}^{\Delta T})}{b+d} \Rightarrow \boxed{\frac{\Delta P^s}{\Delta T} = \frac{-b}{b+d}} \quad — ③$$

Recall: $\dfrac{\Delta Q^d}{\Delta P} = -b$ and $\dfrac{\Delta Q^s}{\Delta P} = d.$

$b = \dfrac{-\Delta Q^d}{\Delta P}$

$$\Rightarrow \frac{\Delta P^s}{\Delta T} = \frac{-b}{b+d} = -\left[\frac{-\Delta Q_d/\Delta P}{\dfrac{-\Delta Q_d}{\Delta P} + \dfrac{\Delta Q_s}{\Delta P}}\right]$$

Use the same method, we have:

$$\frac{\Delta P^s}{\Delta T} = -\frac{\boxed{-\left(\dfrac{\Delta Q_d}{\Delta P} \times \dfrac{P}{Q}\right)} \leftarrow = \eta}{\underbrace{\left(-\dfrac{\Delta Q_d}{\Delta P}\right)\left(\dfrac{P}{Q}\right)}_{= +\eta} + \underbrace{\left(\dfrac{\Delta Q_s}{\Delta P}\right)\left(\dfrac{P}{Q}\right)}_{= \varepsilon}}$$

$\eta = -\dfrac{\Delta Q_d}{\Delta P} \times \dfrac{P}{Q_d}.$

$$\Rightarrow \boxed{\frac{\Delta P^s}{\Delta T} = -\left(\frac{\eta}{\eta + \varepsilon}\right)} \quad — ④$$

$\therefore$ If $\varepsilon = 0$, $\dfrac{\Delta P^s}{\Delta T} = -1.$, ie. Producer's price is reduced by the amount of tax.

If $\eta = 0$, $\dfrac{\Delta P^s}{\Delta T} = 0.$, ie. no change in the producer's price.

## Numerical Example

Given $Q_d = 150 - \frac{1}{2}P$ and $Q_s = 20 + 2P$.

Suppose $T_0 = \$1$ per unit. We have shown that $P_0^d = 52.8$, $P_0^s = 51.8$ & $Q_T^* = 123.6$.

Suppose $T_1 = \$2$ per unit. What is $P_1^d$ & $P_1^s$ ?

① $\frac{\Delta P^d}{\Delta T} = ?$

At $P_0^d$, $\eta = -\frac{\Delta Q_d}{\Delta P^d} \cdot \frac{P^d}{Q_d} = -\left(-\frac{1}{2}\right)\left(\frac{52.8}{123.6}\right) \approx 0.21.$

$$\varepsilon = \frac{\Delta Q_s}{\Delta P^d} \cdot \frac{P^d}{Q_s} = (2)\left(\frac{52.8}{123.6}\right) \approx 0.85$$

We have shown that $\frac{\Delta P^d}{\Delta T} = \frac{\varepsilon}{\eta + \varepsilon}$

$\therefore \frac{\Delta P^d}{\Delta T} = \frac{0.85}{0.21 + 0.85} = 0.8 \Rightarrow \Delta P^d = (0.8) \cdot \Delta T.$

$\therefore$ If $\Delta T = \$2 - \$1$, then $\Delta P^d = (0.8)(1) = 0.8$.

$\Rightarrow P_1^d = 52.8 + 0.8 = \$53.6$

② $\frac{\Delta P^s}{\Delta T} = ?$

At $P_0^s$, $\eta = -\left(-\frac{1}{2}\right)\left(\frac{51.8}{123.6}\right) \approx 0.21$

$$\varepsilon = (2)\left(\frac{51.8}{123.6}\right) \approx 0.84$$

$\Rightarrow \frac{\Delta P^s}{\Delta T} = \frac{-\eta}{\eta + \varepsilon} = \frac{-0.21}{0.21 + 0.84} = -0.2$

$\Rightarrow \Delta P^s = (-0.2) \cdot \Delta T.$

$\therefore$ If $\Delta T = 1$, then $\Delta P^s = (-0.2)(1) = -0.2$.

$\Rightarrow P_1^s = 51.8 - 0.2 = \$51.6$

$\therefore$ Demand is more price inelastic, so the tax burden will be passed on to consumer more.