Phoomphiphat Khachornsart
6104640849

1. In Table 1. X_i is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

Student	Y_i	X_i	$x_i Y_i$
1	2.8	63	176.4
2	3.4	72	244.8
3	3.0	78	234
4	3.5	81	283.5
5	3.6	87	313.2
6	3.0	75	225
7	2.7	75	202.5
8	3.7	90	333

$\Sigma X_i Y_i = 2012.4$

$\Sigma X_i = 621$

$\Sigma X_i^2 = (63)^2 + (72)^2 + (78)^2 + (81)^2 + (87)^2 + (75)^2 + (75)^2 + (90)^2$

$= 48717$

$\Sigma Y_i = 25.7$

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

1.2 Find $\hat{Y}_i$ and $\hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$

1.1) Find $\bar{X} = \dfrac{63+72+78+81+87+75+75+90}{8} = 77.625$

$\bar{Y} = \dfrac{2.8+3.4+3+3.5+3.6+3+2.7+3.7}{8} = 3.2125$

Find $\hat{\beta}_1$

$\hat{\beta}_1 = \bar{Y} - \beta_2 \bar{X}$

$= 3.2125 - \beta_2 (77.625)$

$\hat{\beta}_1 = 3.2125 - 77.625(\beta_2)$ — ①

Find $\hat{\beta}_2$

$$\hat{\beta}_2 = \frac{n \Sigma x_i Y_i - \Sigma x_i \Sigma Y_i}{n \Sigma x_i^2 - (\Sigma x_i)^2}$$

$$= \frac{8(2012.4) - (621)(25.7)}{8(48717) - (621)^2}$$

$$= \frac{16099.2 - 15959.7}{389736 - 385641} = 0.0341$$

substitute ② in ①

$\hat{\beta}_1 = 3.2125 - 77.625(0.0341)$

$= 0.5655$

$\therefore \hat{\beta}_1 = 0.5655$

$\hat{\beta}_2 = 0.0341$

Use 4 decimal places for numerical answers

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From $\hat{\beta}_1 = 0.5655$

$\hat{\beta}_2 = 0.0341$

1.2) $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + u_i$

$\hat{Y}_i = 0.44905 + 0.0356\, x_i + u_i$

From $\hat{Y}_i = \beta_1 + \beta_2 X_i$

Find Y_i :

$i=1$, $0.5655 + 0.0341(63) = 2.7138$

$i=2$, $0.5655 + 0.0341(72) = 3.0207$

$i=3$, $0.5655 + 0.0341(78) = 3.2253$

$i=4$, $0.5655 + 0.0341(81) = 3.3276$

$i=5$, $0.5655 + 0.0341(87) = 3.5322$

$i=6$, $0.5655 + 0.0341(75) = 3.123$

$i=7$, $0.5655 + 0.0341(75) = 3.123$

$i=8$, $0.5655 + 0.0341(90) = 3.6345$

Find $\hat{u}_i$, From $Y_i = \hat{u}_i + \hat{Y}_i \rightarrow \hat{u}_i = Y_i - \hat{Y}_i$

$i=1$, $2.8 - 2.7138 = 0.0862$

$i=2$, $3.4 - 3.0207 = 0.3793$

$i=3$, $3.0 - 3.2253 = -0.2253$

$i=4$, $3.5 - 3.3276 = 0.1724$

$i=5$, $3.6 - 3.5322 = 0.0678$

$i=6$, $3.0 - 3.123 = -0.123$

$i=7$, $2.7 - 3.123 = -0.423$

$i=8$, $3.7 - 3.6345 = 0.0655$

$$\sum_{i=0}^{n} \hat{u}_1 = 0.0862 + 0.3793 + (-0.2253) + (0.1724) + (0.0678) + (-0.123) + (-0.423) + 0.0655 = 0$$

Use 4 decimal places for numerical answers

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1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$

1.3) $$var(\hat{u}_i) = \hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{0.4347}{8-2} = 0.0725$$

$$var(\hat{\beta}_1) = \frac{\sum X_i^2}{n \sum x_i^2} \hat{\sigma}^2 = \frac{621 \cdot (0.0725)}{511.875} = 0.0880$$

$$var(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\sum x_i^2} = \frac{(0.0725)}{511.875} = 0.0001416$$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$
Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i), var(\hat{\beta}_1), var(\hat{\beta}_2)$

2.1) $\bar{X} = (10+12+14+16+18+22+24+26+28+30) \div 10 = 20$

$\bar{Y} = (0+2+5+6+7+10+10+15+16+20) \div 10 = 9.1$

$\sum X_i^2 = (10)^2 + 12^2 + 14^2 + \dots + 30^2 = 4440$

$\sum X_i Y_i = 10(0) + 12(2) + 14(5) + 16(6) + 18(7) + 22(10) + 24(10) + 26(15) + 28(16) + 30(20) = 2214$

$$\hat{\beta}_2 = \frac{\sum X_i Y_i - n\bar{X}\bar{Y}}{\sum X_i^2 - n\bar{X}^2} = \frac{2214 - 10(20)(9.1)}{4440 - 4000} = 0.8954$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$$
$$= 9.1 - 0.895(20) = -8.808$$

2.2) $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$, Find $\hat{Y}_i$

when $i=1$, $-8.808 + 0.8954(10) = 0.1455$

$i=2$, $-8.808 + 0.8954(12) = 1.9364$

$i=3$, $-8.808 + 0.8954(14) = 3.7273$

$i=4$, $-8.808 + 0.8954(16) = 5.5182$

$i=5$, $-8.808 + 0.8954(18) = 7.3091$

$i=6$, $-8.808 + 0.8954(22) = 10.8909$

$i=7$, $-8.808 + 0.8954(24) = 12.6818$

$i=8$, $-8.808 + 0.8954(26) = 14.4727$

$i=9$, $-8.808 + 0.8954(28) = 16.2636$

$i=10$, $-8.808 + 0.8954(30) = 18.0549$

Find $\hat{u}_i$

when $i=1$, $\hat{u}_1 = -0.145$

$i=2$, $\hat{u}_2 = 0.064$

$i=3$, $\hat{u}_3 = 1.273$

$i=4$, $\hat{u}_4 = 0.482$

$i=5$, $\hat{u}_5 = -0.304$

$i=6$, $\hat{u}_6 = -0.891$

$i=7$, $\hat{u}_7 = -2.682$

$i=8$, $\hat{u}_8 = 0.527$

$i=9$, $\hat{u}_9 = -0.264$

$i=10$, $\hat{u}_{10} = 1.945$

$$\sum_{i=0}^{n} \hat{u}_i = (-0.145) + 0.064 + 1.273 + 0.482 + (-0.304) + (-2.682) + 0.527 - 0.264 + 1.945 = 0$$

3

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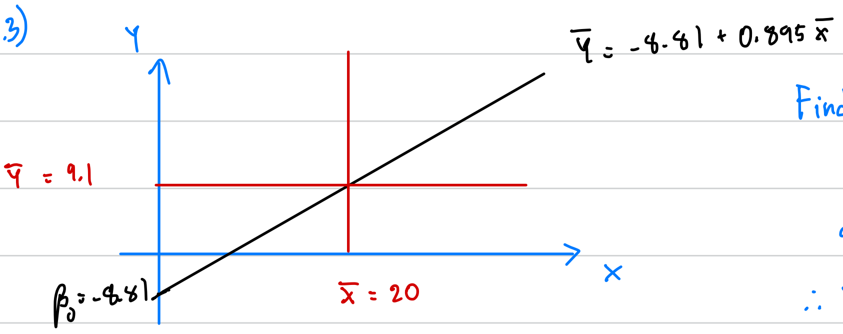
2.3)



Find out if the line pass $(\bar{X}, \bar{Y})$

$\bar{Y} = -8.81 + 0.895(20)$

$9.1 = 9.1$

$\therefore$ The line pass through $(\bar{X}, \bar{Y})$

2.4) $Y_i = \beta_0 + \beta_1 X_i$

$= -8.81 + 0.895(16)$

$= 5.51$

2.5) $Var(\hat{u}_i) = \hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{14.091}{8} = 1.7614$

$Var(\hat{\beta}_1) = \frac{\hat{\sigma}^2}{\sum(x_i - \bar{x})^2} = \frac{1.7614}{440} = 0.0040$

$Var(\hat{\beta}_2) = \frac{\sum x_i^2 (\hat{\sigma}^2)}{n\sum(x_i - \bar{x})^2} = \frac{440(1.7614)}{10(440)} = 1.7774$

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3. Consider the below regression function: consider the two-variable model

$$Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator.

Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

$\rightarrow$ $Y_i = \beta_1 + \beta_2 X_i + u_i$ are suit in CLRM assumption 1, linear regression model

Since $\hat{\beta}_1$ is unbiased estimator, $E(\hat{\beta}_1) = \beta_1$

From $\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$ —— ①

$\hat{\beta}_2 = \dfrac{\sum x_i Y_i}{\sum x_i^2}$ Given $k = \dfrac{x_i}{\sum x_i^2}$

Substitute k in ①

$\hat{\beta}_1 = \bar{Y} - \sum k Y_i \bar{X}$ —— ②

From Proof: $\sum_{n=1}^{i} k_i = 0$

$Y_i = \beta_1 + \beta_2 X_i + u_i$

From ② $\hat{\beta}_1 = \dfrac{\sum Y_i}{n} - \sum k Y_i \bar{X} = \sum \left(\dfrac{1}{n} - k\bar{X} \right) Y_i$

$= \sum \left(\dfrac{1}{n} - k\bar{X} \right)(\beta_1 + \beta_2 X_i + u_i)$

$= \sum \left(\dfrac{\beta_1}{n} - \beta_1 k \bar{X} + \dfrac{\beta_2 X_i}{n} - \beta_2 k \bar{X} + \dfrac{u_i}{n} - u_i k \bar{X} \right)$

$= \sum \left(\dfrac{\beta_1}{n} + \dfrac{\beta_2 X_i}{n} + \dfrac{u_i}{n} - \beta_1 k \bar{X} - \beta_2 k \bar{X} - u_i k \bar{X} \right)$

$= \sum \dfrac{\beta_1}{n} + \beta_2 \dfrac{\sum x_i}{n} + \dfrac{\sum u_i}{n} - \bar{X} \beta_1 \sum k_i - \beta_2 \sum k_i \bar{X} - k\bar{X} \sum u_i$

$\hat{\beta}_1 = \beta_1 \cancel{+ \beta_2 \bar{X}} - \dfrac{\sum u_i}{n} \cancel{- \beta_2 \bar{X}} - \bar{X} \sum k_i u_i$

$\hat{\beta}_1 = \beta_1 - \dfrac{\sum u_i}{n} - \bar{X} \sum k_i u_i$ —— ③

Add expected value to the equation ③

$E(\hat{\beta}_1) = E(\beta_1) + \bar{X} E(\sum k_i u_i)$

$\rightarrow$ From Assumption 3, Zero mean value of disturbance u_i treat x as given

$E(u_i | X_i)$

$E(\hat{\beta}_1) = E(\beta_1) + \bar{X} \sum k_i \underbrace{E(u_i)}_{=0}$

$E(\hat{\beta}_1) = \beta_1$

Therefore $\hat{\beta}_1$ is an unbiased estimator of β_1