EE320 (1/2013)

INTRODUCTORY MATHEMATICAL ECONOMICS

STATIC and COMPARATIVE STATIC EQUILIBRIUM ANALYSIS

Topics

- Meaning of Equilibrium
- Partial Market Equilibrium: Linear Models
  - Linear models in economics
  - Break-even analysis
  - Individual and market demand
  - Individual and market supply
  - Partial market equilibrium
- Partial Market Equilibrium: Nonlinear Models
- General Market Equilibrium
- Applications
  - Excise tax and market equilibrium
  - Macroeconomic models

Meaning of Equilibrium

- An equilibrium is *"a constellation of selected interrelated variables so adjusted to one another that no inherent tendency to change prevails in the model which they constitute."*

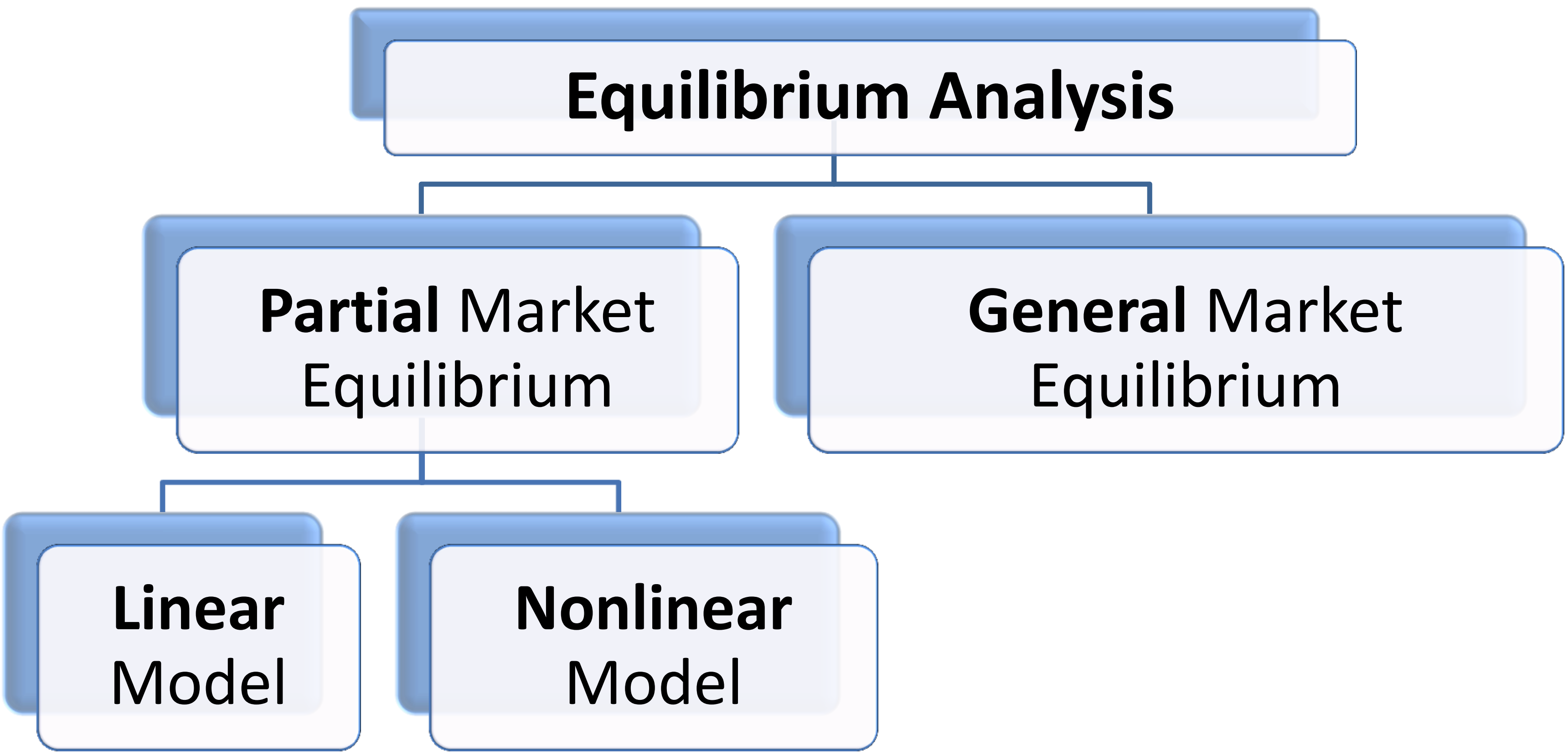
  - Machlup, F. (1958).

- In essence, an equilibrium for a specified model is a situation characterized by a lack of tendency to change.
- Due to the lack of tendency to change, the analysis of equilibrium is referred to as statics.

Equilibrium Analysis

- However, an equilibrium does *not* necessarily constitute a desirable or an ideal state of affairs; i.e., there are both good and bad equilibria.
- "Desirable" equilibrium is referred to as "goal" equilibrium (to be discussed in optimization problems).
- "Non-goal" equilibrium, which is discussed in this topic, is a result from a process of interaction and adjustment of economic forces.
  - Equilibrium in a market under given demand and supply
  - Equilibrium of national income given conditions of consumption and investment

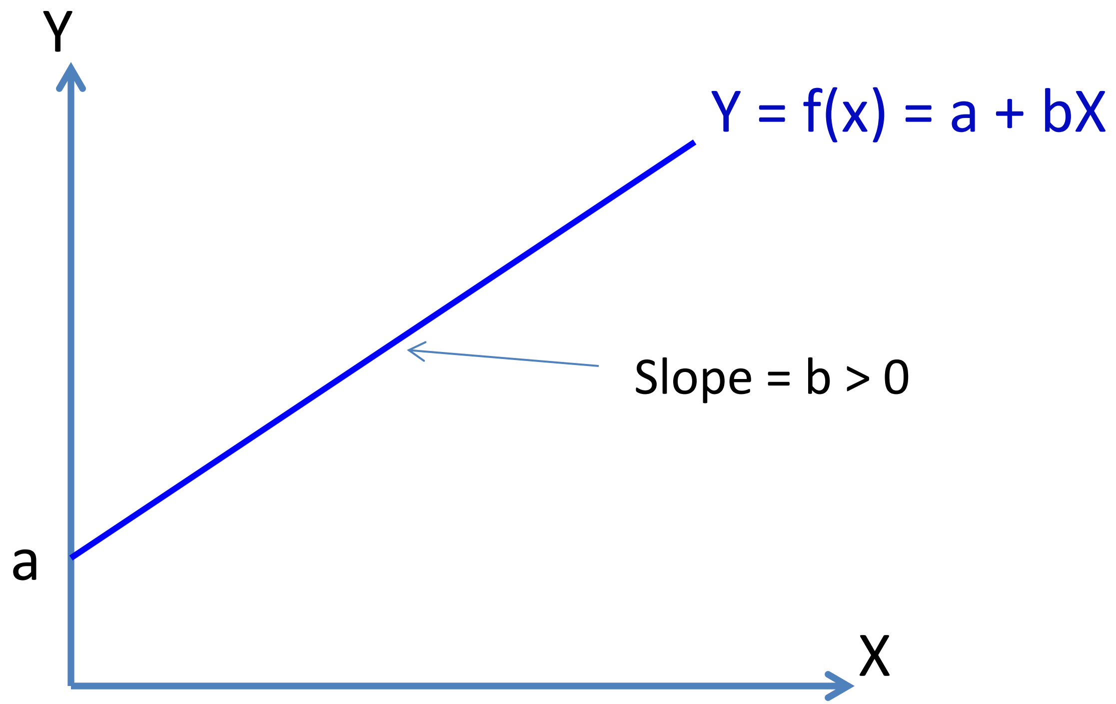
Types of Market Equilibrium Analysis



PARTIAL MARKET EQUILIBRIUM:
LINEAR MODELS

Linear Models in Economics

- General form of a linear model: Y = a + bX



Example: Cost Function (1)

- Theory: Cost of a firm consists of fixed costs and variable costs.
- In a mathematical model:

$$TC = TFC + TVC$$

$$TC(Q) = TFC + TVC(Q) \Rightarrow TC = f(Q)$$

- Assumption: Total cost function has a linear relationship.

$$TC(Q) = f(Q) = TFC + cQ,$$

where c is a constant.

- With this theoretical model and given data, we can approximate the cost function.
  - Example: Suppose TC = \$5,000 when Q = 50 units, and TC = \$9,000 when Q =100 units. What is the cost function? What is the total cost if Q =120 units?

Example: Cost Function (2)

- Answer: 1) TC = 1000 + 80Q.

  2) When Q = 120, TC = $10,600

Breakeven Analysis

- The breakeven level for a product is a level at which total revenue (TR) equals total costs (TC); i.e., normal profit (π) is 0.
- Theoretical model:

$$\pi \equiv TR - TC = 0$$

$$TR = PQ$$

$$TC = a + cQ$$

- Empirical model:

Suppose the 2nd-year BE students want to raise a fund by selling T-shirts. The price of a T-shirt purchased from a factory is 100 baht, and the cost of travelling to the factory and other fixed cost is 2000 baht. The students can sell T-shirts at a price 150 baht per piece. How many T-shirts should these students sell in order to breakeven?

Illustration of a Breakeven Analysis

Individual and Market Demand

- Theory - Demand for a good is determined by many factors:

  $Q^x_d = f(P^x, P^y, I, \text{taste, etc.})$

- Individual demand for a good as a function of its price:

  $Q_d = f(P)$, ceteris paribus

  $Q_d = a - bP$, where $b > 0$.

- Market demand:

  $Q_{Md} = Q_{d1} + Q_{d2} + \ldots + Q_{dn}$

  (Suppose that there are n buyers.)

- Example:

  Suppose $Q_{d1} = 500 – 4P$ and $Q_{d2} = 1000 – 4P$. What does a market demand function look like?

Illustration of a Market Demand Function

Individual and Market Supply

- Individual supply of a good as a function of its price:

$$Q_s = f(P), \quad \text{ceteris paribus}$$

$$Q_s = c + dP, \quad \text{where } d > 0.$$

- Market supply:

$$Q_{Ms} = Q_{s1} + Q_{s2} + \ldots + Q_{sq}$$

(Suppose that there are q sellers.)

- Example:

Suppose that there are 10 sellers, and each seller *i* has the same supply function: $Q_{si} = 23 + P$. What does the market supply function look like?

Illustration of a Market Supply Function

Some Notations:

System of Equations /Simultaneous Equations

- Systems of equations are two or more equations containing common multiple variables.

Example: $Q_d = a - bP$, where $b > 0$. - (1)

$Q_s = c + dP$, where $d > 0$. - (2)

➢ Note that any value of P makes this system correct.

- Simultaneous equations are two or more equations in which the common variables are jointly determined.

Example: $Q_d = a - bP$, where $b > 0$. - (1)

$Q_s = c + dP$, where $d > 0$. - (2)

$Q_d = Q_s$ - (3)

➢ This system contains simultaneous equations because there is a unique value of P* that must be true in all equations.

➢ Equation (3) specifies the equilibrium condition in the market.

Partial Market Equilibrium

- Partial market equilibrium analysis deals with price determination in an isolated market. That is, only one commodity is being considered.
- All relevant factors except the price in question (e.g. prices of all substitutes and complements and income levels of consumers) are constant.
- This analysis can be used to determine *changes in other exogenous parameters*, e.g. the impact of a tax, without worrying about side-effects from other markets.
  - The analysis of a change in the equilibrium condition as a result of a *change in the parameter* in the model is called "comparative static analysis."

Partial Market Equilibrium: A Linear Model

- Equilibrium occurs in the market *if and only* if the excess demand is zero (i.e. if and only if the market is clear).
- 1st step: Construct the model.
  - One conditional equation: $Q_d = Q_s$.
  - Two behavioral equations:

$$Q_d = a - bP, \quad (a, b > 0)$$
$$Q_s = -c + dP, \quad (c, d > 0)$$

  - *a, b, c,* and *d* are parameters, whereas Q_d, Q_s, and P are endogenous variables.
- 2nd step: Solve the model for the 3 endogenous variables (Q^*_d, Q^*_s, and P^*). These values are the equilibrium values of the model.

Solution by Elimination of Variables

- Show that $P^* = \dfrac{a+c}{b+d}$ and $Q^* = \dfrac{ad-bc}{b+d}$.

PARTIAL MARKET EQUILIBRIUM:
NONLINEAR MODELS

Example: A Nonlinear Model

- Suppose that a market demand function is nonlinear. The model can be rewritten as:

$$Q_d = a - bP^2, \quad (a, b > 0)$$
$$Q_s = -c + dP, \quad (c, d > 0)$$
$$Q_d = Q_s.$$

$\rightarrow$ $-bP^2 + dP - a - c = 0$

- Numerical example: $Q_d = 4 - P^2$ and $Q_s = 4P - 1$. What are the equilibrium price and quantity in this model?

Graphic Solution of a Nonlinear Model

- $D = \{(P, Q) | Q = 4 – P^2 \}$
- $S = \{(P,Q) | Q = 4P - 1\}$

Quadratic Formula

- Given a quadratic equation of the form

$$aX^2 + bX + c = 0,$$

there are 2 roots, which can be obtained from the quadratic formula:

$$\boxed{X_1^*, X_2^* = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

If $b^2 - 4ac > 0$, $X^*_1 \neq X^*_2$.

If $b^2 - 4ac = 0$, $X^*_1 = X^*_2$.

If $b^2 - 4ac > 0$, no real value root exists.

- Own-practice: Use the process called "completing the square" to show the 2 roots of a quadratic equation.
  - Hint: Start by dividing both sides by a.

Quadratic Formula

GENERAL MARKET EQUILIBRIUM

EXCISE TAX AND MARKET EQUILIBRIUM

General Market Equilibrium

- General market equilibrium deals with simultaneous equilibria in a group of related markets, e.g. input and output markets.
  - Ex: A change in the price of an input good may affect the price of the output good, with a possible subsequent effect on the price of the input good.
- The equilibrium condition of an *n*-commodity market model involves *n* equations:

$$E_i \equiv Q_{di} - Q_{si} = 0 \qquad (i = 1, 2, \dots, n)$$

$$Q_{di} = Q_{di}(P_1, P_2, \dots, P_n)$$

$$Q_{si} = Q_{si}(P_1, P_2, \dots, P_n)$$

  - ➢ If a solution exists, there will be a set of price P^*_i and quantity Q^*_i such that all *n* equations in the equilibrium conditions are simultaneously satisfied.

2-Commodity Market Model

- Market for good 1:

$$Q_{d1} - Q_{s1} = 0$$
$$Q_{d1} = a_0 + a_1 P_1 + a_2 P_2$$
$$Q_{s1} = b_0 + b_1 P_1 + b_2 P_2$$

- Market for good 2:

$$Q_{d2} - Q_{s2} = 0$$
$$Q_{d2} = \alpha_0 + \alpha_1 P_1 + \alpha_2 P_2$$
$$Q_{d2} = \beta_0 + \beta_1 P_1 + \beta_2 P_2$$

- Let $c_i \equiv a_i - b_i$ and $\gamma_i \equiv \alpha_i - \beta_i$

➢ Show that $\boxed{P_1^* = \dfrac{c_2\gamma_0 - c_0\gamma_2}{c_1\gamma_2 - c_2\gamma_1}}$ and $\boxed{P_2^* = \dfrac{c_0\gamma_1 - c_1\gamma_0}{c_1\gamma_2 - c_2\gamma_1}}$

2-Commodity Market Model (cont'd)

Numerical Example (Own Practice!)

- Suppose that the demand and supply functions of a two-commodity market model are as follows:

$$Q_{d1} = 8 - 3P_1 + P_2 \quad \text{and} \quad Q_{s1} = 2 + 2P_1$$
$$Q_{d2} = 12 + 5P_1 - 2P_2 \quad \text{and} \quad Q_{s2} = 2 + 3P_2$$

Find P^*_i and Q^*_i $(i = 1, 2)$.

<u>Answer</u>:

$$(Q_1^*, P_1^*) = (6, 2)$$
$$(Q_2^*, P_2^*) = (14, 4)$$

APPLICATIONS OF MARKET EQUILIBRIUM

Excise Tax and Market Equilibrium

- Excise tax (a.k.a. excise duty) is a type of tax charged on goods produced within the country, as opposed to custom duties which are charged on goods from outside the country.
- Two types of excise tax:

1. Specific tax – tax imposed based on physical units (e.g. pieces, gallons, or individual items) of the taxed item good.
   - $P^t = P + T$, where P is price without tax.
2. Ad-valorem tax – tax imposed based on the monetary value of the taxed item. Suppose that a *t*% ad-valorem tax is imposed.
   - $P^t = (1+t)P$, where P is price without tax.

Specific Tax (imposed on consumers)

- Suppose the government imposes a *T* baht specific tax on a good. Find the P*, Q*, P^t (market price), and tax revenue.

➢ Without tax: $Q_D = a - bP$ and $Q_S = -c + dP$

➢ With tax: $P_D' = P + T = P^t$ and $P_S = P$,

At the new equilibrium, $Q_D' = Q_S$.

$$a - b(P + T) = -c + dP$$

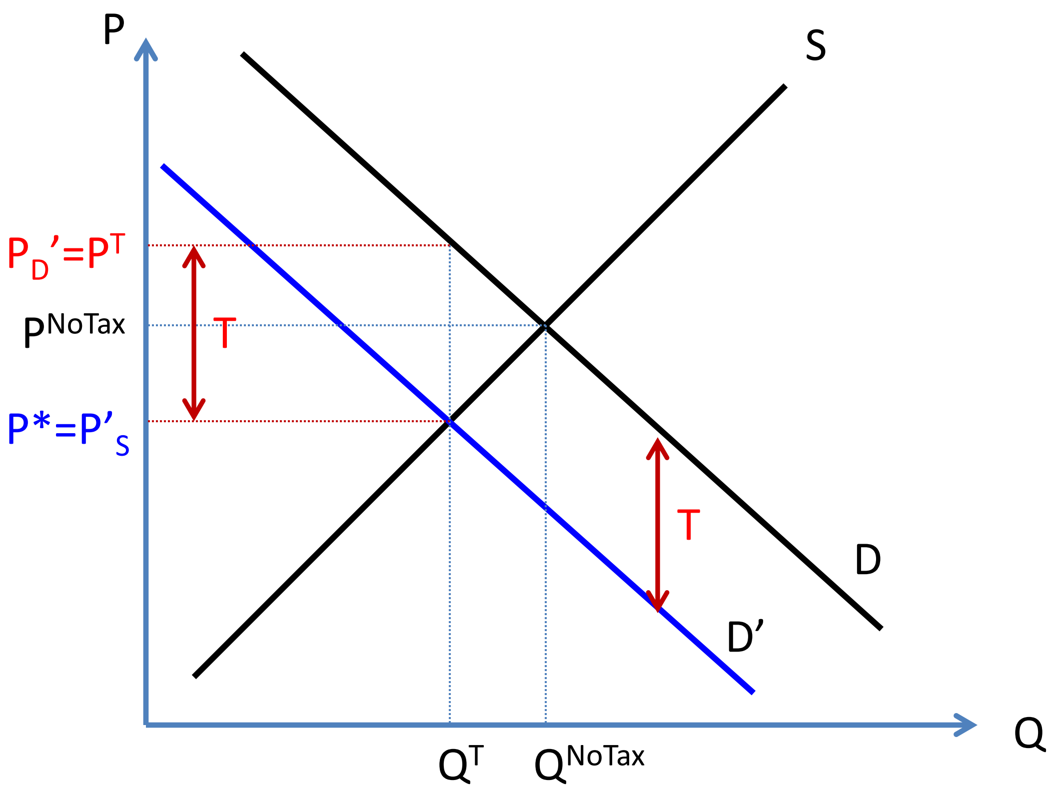
➔ $\boxed{P^* = \dfrac{a+c-bT}{b+d} = P_S'}$: Price received by producer

➔ $\boxed{P^t = P + T = \dfrac{a+c+dT}{b+d} = P_D'}$: Price (tax included) paid by consumer

Specific Tax (imposed on consumers) – Cont'd

- Consumer tax burden: $P^{t} - P^{NoTax} = \dfrac{dT}{b+d}$
- Producer tax burden: $P^{NoTax} - P_{S}^{'} = -\dfrac{bT}{b+d}$
- Equilibrium quantity after tax: $Q^{*} = \dfrac{ad - bc - bdT}{b+d}$
- Tax revenue: $Tax = TQ^{*} = \dfrac{(ad - bc)T - bdT^{2}}{b+d}$

Graph: Specific Tax (imposed on consumers)



Specific Tax (imposed on producers)

- Suppose the government imposes a T baht specific tax on a good. Find the P*, Q*, P^t (price excluding tax received by producer), and tax revenue.

➢ Without tax: $Q_D = a - bP$ and $Q_S = -c + dP$

➢ With tax: $P_S' = P - T = P^t$ and $P_D = P$,

At the new equilibrium, $Q_D = Q_S'$.

$$a - bP = -c + d(P - T)$$

➔ $\boxed{P^* = \dfrac{a + c + dT}{b + d} = P_D'}$: Price (tax included) paid by consumer

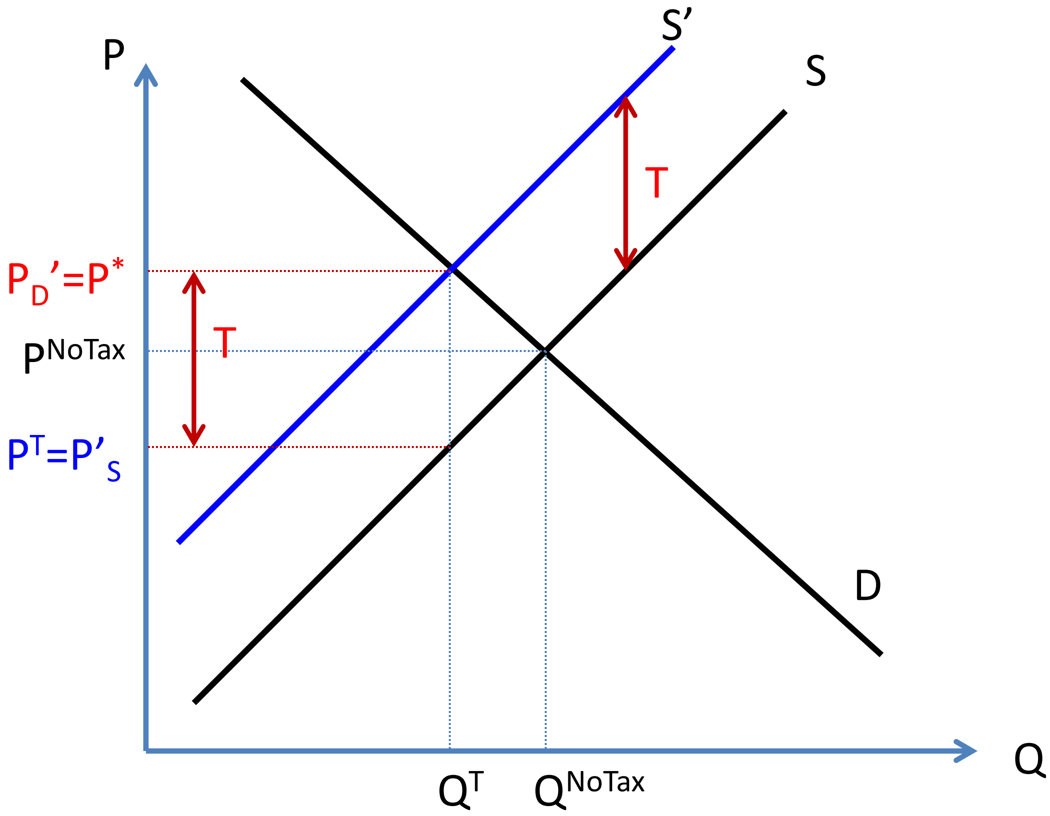
➔ $\boxed{P^T = P_D' - T = \dfrac{a + c - bT}{b + d} = P_S'}$: Price received by producer

Specific Tax (imposed on producers) – Cont'd

- Consumer tax burden: $P_D^{'} - P^{NoTax} = \dfrac{dT}{b+d}$
- Producer tax burden: $P^{NoTax} - P_S^{'} = -\dfrac{bT}{b+d}$
- Equilibrium quantity after tax: $Q^{*} = \dfrac{ad - bc - bdT}{b+d}$
- Tax revenue: $Tax = TQ^{*} = \dfrac{(ad-bc)T - bdT^{2}}{b+d}$

➢The results (PD', PS', Q*, Tax) are exactly the same as regardless of whom the government imposes the specific tax on.

Graph: Specific Tax (imposed on producers)



Ad-Valorem Tax (imposed on consumers)

- Suppose the government imposes a *t%* ad-valorem tax on a good. Find the P*, Q*, and P^t (market price).

➢ Without tax: $Q_D = a - bP$ and $Q_S = -c + dP$

➢ With tax: $P_D' = P + tP = (1+t)P = P^t$ and $P_S = P$,

At the new equilibrium, $Q_D' = Q_S$.

$$a - b[(1+t)P] = -c + dP$$

➔ $\boxed{P^* = \dfrac{a+c}{b(1+t)+d} = P_S'}$: Price received by producer

➔ $\boxed{P^t = P_S' + T = \dfrac{(1+t)(a+c)}{b(1+t)+d} = P_D'}$: Price (tax included) paid by consumer

➔ Equilibrium quantity after tax: $Q^* = \dfrac{ad - (1+t)bc}{b(1+t)+d}$

Ad-Valorem Tax (imposed on producers)

- Suppose the government imposes a *t%* ad-valorem tax on a good. Find the P*, Q*, and P^t (price excluding tax received by producer).

➢ Without tax: $Q_D = a - bP$ and $Q_S = -c + dP$

➢ With tax: $P_D = P$ and $P_S' = P - tP = (1-t)P$,

At the new equilibrium, $Q_D = Q_S'$.

$$a - bP = -c + d[(1-t)P]$$

➔ $\boxed{P^* = \dfrac{(a+c)}{b+d(1-t)} = P_D'}$: Price paid by consumer

➔ $\boxed{P^t = P_D' - T = \dfrac{(1-t)(a+c)}{b+d(1-t)} = P_S'}$: Price received by producer

➔ Equilibrium quantity after tax: $Q^* = \dfrac{ad(1-t) - bc}{b+d(1-t)}$

Elasticity Concept

- Recall the concept of price elasticity:

$$E_p = \frac{\%\Delta Q}{\%\Delta P} = \frac{\Delta Q / Q}{\Delta P / P} = \frac{\Delta Q}{\Delta P} . \frac{P}{Q}$$

- When Δ is very small, we use notation *d* instead of Δ: $E_p = \dfrac{dQ}{dP} . \dfrac{P}{Q}$

- Price elasticity of demand:

$$\eta = -\frac{dQ_d}{dP} . \frac{P}{Q_d}$$

- Price elasticity of supply:

$$\varepsilon = \frac{dQ_s}{dP} . \frac{P}{Q_s}$$

Tax Incidence and Price Elasticity (1)

- Case 1 - Specific tax imposed on consumers.
- Consumer's price: $P^t = \dfrac{a+c+dT}{b+d}$. Find dP^t/dT = ?

Suppose $\Delta T = T_1 - T_0$. We want to know ΔP^t = ?

- $P^{t0} = \dfrac{a+c+dT_0}{b+d}$ and $P^{t1} = \dfrac{a+c+dT_1}{b+d}$
- $\Delta P^t = P^{t1} - P^{t0} = \dfrac{a+c+dT_1}{b+d} - \dfrac{a+c+dT_0}{b+d}$
- $\Delta P^t = \dfrac{d}{b+d}(T_1 - T_0) = \dfrac{d}{b+d}\Delta T$

➔ The impact of tax change on equilibrium price when tax is imposed on consumer is: $\boxed{\dfrac{\Delta P^t}{\Delta T} = \dfrac{d}{b+d}}$ -- (1)

Specific tax imposed on consumers (cont'd)

- Rewrite the impact of tax change on equilibrium price, when tax is imposed on producers:
  - Recall $Q_D = a - bP$ and $Q_S = -c + dP$.
  - Thus, $\Delta Q_D / \Delta P = -b$ and $\Delta Q_S / \Delta P = d$.
- From (1), $\dfrac{\Delta P^t}{\Delta T} = \dfrac{d}{b+d} = \dfrac{\Delta Q_S / \Delta P}{-\Delta Q_D / \Delta P + \Delta Q_S / \Delta P}$ -- (2)
- Recall $\eta = \dfrac{\Delta Q_d}{\Delta P} . \dfrac{P}{Q_d}$ and $\varepsilon = \dfrac{\Delta Q_s}{\Delta P} . \dfrac{P}{Q_s}$
- Rearrange (2) and we obtain:

$$\boxed{\frac{dP^t}{dT} \approx \frac{\Delta P^t}{\Delta T} = \frac{\varepsilon}{\eta + \varepsilon}}$$

- If $\varepsilon = 0$, then $dP^t/dT = 0$.
- If $\eta = 0$, then $dP^t/dT = 1$.

Tax Incidence and Price Elasticity (2)

- Case 2 - Specific tax imposed on producers.
- Producer's price: $P^t = \dfrac{a+c-bT}{b+d}$. Find dP^t/dT = ?
- Suppose $\Delta T = T_1 - T_0$. We want to know ΔP^t = ?

➢ $P^{t0} = \dfrac{a+c-bT_0}{b+d}$ and $P^{t1} = \dfrac{a+c+dT_1}{b+d}$

➢ $\Delta P^t = \dfrac{-b}{b+d}(T_1 - T_0) = \dfrac{-b}{b+d}\Delta T$

➔ The impact of tax change on equilibrium price when tax is imposed on producer is: $\boxed{\dfrac{\Delta P^t}{\Delta T} = \dfrac{-b}{b+d}}$ -- (3)

Specific tax imposed on producers (cont'd)

- Rewrite the impact of tax change on equilibrium price, when tax is imposed on producers:

From (3) and following the same method, we get:

$$\frac{\Delta P^t}{\Delta T} = \frac{-b}{b+d} = \frac{\Delta Q_D / \Delta P}{-\Delta Q_D / \Delta P + \Delta Q_S / \Delta P}$$

$$\frac{\Delta P^t}{\Delta T} = \frac{\Delta Q_D / \Delta P}{-\Delta Q_D / \Delta P + \Delta Q_S / \Delta P} \cdot \frac{P/Q}{P/Q}$$

$$\rightarrow \boxed{\frac{dP^t}{dT} \approx \frac{\Delta P^t}{\Delta T} = \frac{-\eta}{\eta + \varepsilon}}$$

- If $\varepsilon = 0$, then $dP^t/dT = -1$.
- If $\eta = 0$, then $dP^t/dT = 0$.

MARKET EQUILIBRIUM

MARKET EQUILIBRIUM IN NATIONAL-INCOME ANALYSIS

Simplest Keynesian National-Income Model

- Consider a *closed* economy (i.e. no trade).
- The equilibrium is: $Y = C + I_0 + G_0 = AE$

  where $C = a + bY$ $(a > 0, 0 < b < 1)$

- At equilibrium, $Y = a + bY + I_0 + G_0$

$$\Rightarrow \boxed{Y^* = \frac{a + I_0 + G_0}{1 - b}}$$

- Corresponding consumption: $C^* = a + bY^*$

$$\Rightarrow C^* = \frac{a + b(I_0 + G_0)}{1 - b}$$

Closed Economy with Proportional Income Tax

- Equilibrium condition: $Y = C + I_0 + G_0 = AE$

  where $C = a + bY_d \quad (a > 0, 0 < b < 1)$

  $Y_d = Y - T$

  $T = tY \quad (0 < t < 1)$

➢ At equilibrium, $Y = a + bY_d + I_0 + G_0$

$$Y = a + b(1-t)Y + I_0 + G_0$$

$$\rightarrow \quad \boxed{Y^* = \frac{a + I_0 + G_0}{1 - b(1-t)}}$$

$$\rightarrow \quad C^* = \frac{a + b(1-t)(I_0 + G_0)}{1 - b(1-t)}$$

Open Economy with Proportional Income Tax

- Equilibrium condition: $Y = C + I_0 + G_0 + X_0 - M = AE$

  where $C = a + bY_d \quad (a > 0, 0 < b < 1)$

  $T = tY \quad (0 < t < 1)$

  $M = mY$

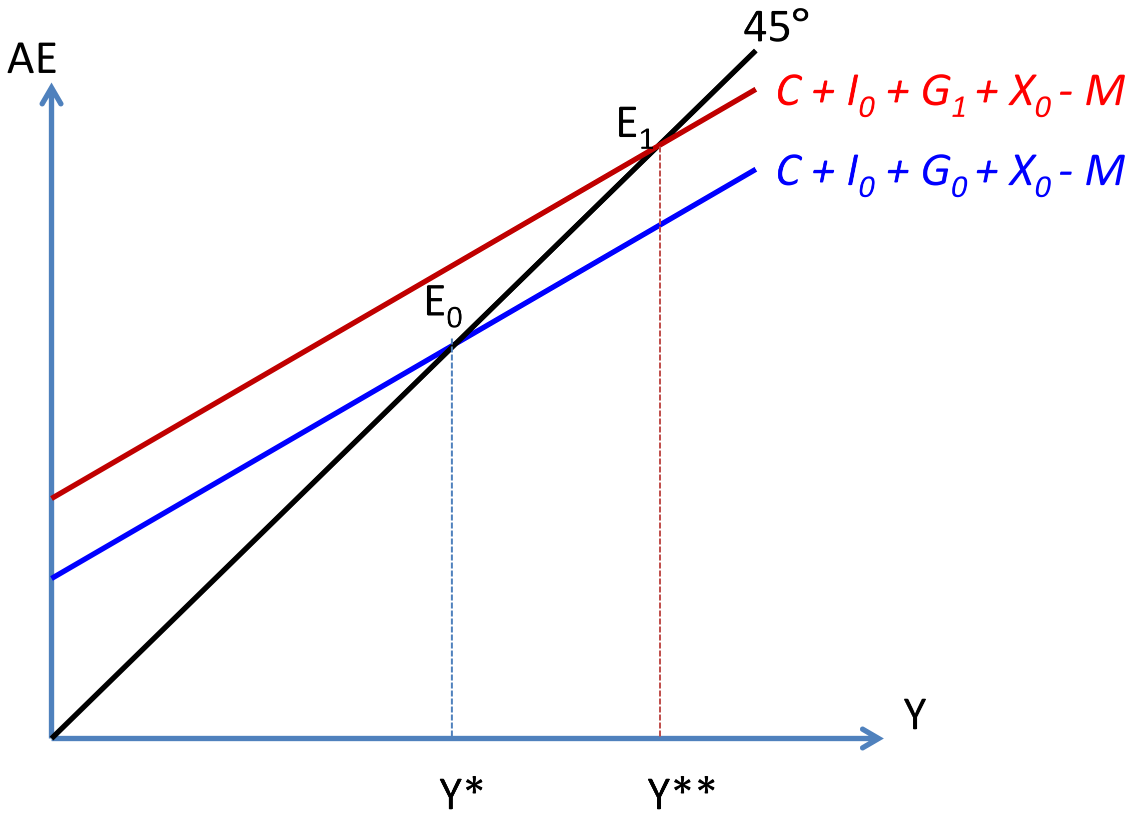
- At equilibrium, $Y = a + b(1-t)Y + I_0 + G_0 + X_0 - mY$

  $$\rightarrow \boxed{Y^* = \frac{a + I_0 + G_0 + X_0}{1 - b(1-t) + m}}$$

  $$\rightarrow C^* = \frac{a(1+m) + b(1-t)(I_0 + G_0 + X_0)}{1 - b(1-t) + m}$$

- If $M = mY_d$, $Y^* = ?$ (own practice!)

Graph: Keynesian National-Income Model



Suppose G increases. What is the new value of Y*?

Comparative Statics Analysis

- Suppose government increases its expenditure, what's the impact on the equilibrium national income (i.e. $\Delta Y^*/\Delta G=?$)
- Recall: $Y^* = \dfrac{a + I_0 + G_0 + X_0}{1 - b(1-t) + m}$, and suppose $\Delta G = G_1 - G_0$.

➢ $$Y^{*1} - Y^{*0} = \frac{a + I_0 + G_1 + X_0}{1 - b(1-t) + m} - \frac{a + I_0 + G_0 + X_0}{1 - b(1-t) + m}$$

➢ $$\Delta Y^* = Y^{*1} - Y^{*0} = \frac{\Delta G_1}{1 - b(1-t) + m}$$

➔ $$\boxed{\frac{\Delta Y^*}{\Delta G_1} = \frac{1}{1 - b(1-t) + m} > 0}$$

Thus, if *1 – b(1-t) + m < 1*, then *ΔY*/ΔG > 1*.

IS-LM Model

- The previous Keynesian model only deals with the equilibrium in the goods (or commodity) market. It seeks to determine the equilibrium level of national income.
- The IS-LM model deals with not only the commodity market but also the money market. It determines the equilibrium levels of national income and interest rate in the economy.
- The IS curve represents the equilibrium in the commodity market.
  - Planned "investment" = Planned "savings"
- The LM curve represents the equilibrium in the money market.
  - Liquidity preference (money demand) = Money supply

IS-LM Model: Closed Economy & No Tax

- Commodity Market:

  $Y = C + I + G_0$

  $C = a + bY$ $(a > 0,\ 0 < b < 1)$

  $I = I_0 - ir$ (I_0, $i > 0$, and r is interest rate)

  ➢ $Y = a + bY + I_0 - ir + G_0$ ➔ $\boxed{Y = \frac{(a + I_0 + G_0) - ir}{1-b}}$: IS

- Money Market:

  $M^S = M_0$

  $M^D = kY - hr$ $(k, h > 0)$

  ➢ At equilibrium, $M_0 = kY - hr$ ➔ $\boxed{Y = \frac{M_0}{k} + \frac{h}{k}r}$: LM

➢ From the two equilibrium conditions, we can solve for Y^* and r^*:

$\boxed{Y^* = \frac{(a + I_0 + G_0)h + iM_0}{ik + h(1-b)}}$ and $\boxed{r^* = \frac{(a + I_0 + G_0)k - (1-b)M_0}{ik + h(1-b)}}$

IS-LM Model: Closed Economy with Tax

- Commodity Market:

$Y = C + I + G_0$

$C = a + bY_d$ $(a > 0, 0 < b < 1)$

$I = I_0 - ir$ $(I_0, i > 0)$

$Y_d = Y - T$ where $T = tY$ $(0 < t < 1)$

$\rightarrow$ $Y = a + b(1-t)Y + I_0 - ir + G_0$ $\rightarrow$ $\boxed{Y = \dfrac{(a + I_0 + G_0) - ir}{1 - b(1-t)}}$: IS

- Money Market:

$M^S = M_0$

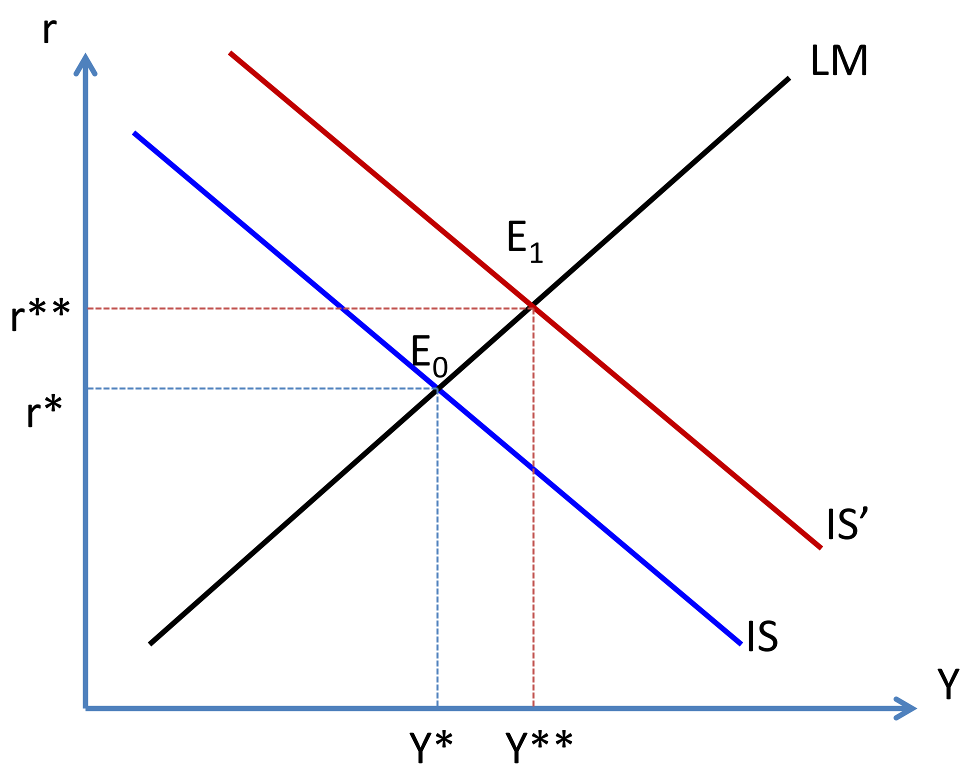
$M^D = kY - hr$ $(k, h > 0)$

$\rightarrow$ $M_0 = kY - hr$ $\rightarrow$ $\boxed{Y = \dfrac{M_0}{k} + \dfrac{h}{k}r}$: LM

- Equilibrium values for Y^* and r^*:

$$\boxed{Y^* = \frac{(a + I_0 + G_0)h + iM_0}{ik + h[1 - b(1-t)]}} \quad \text{and} \quad \boxed{r^* = \frac{(a + I_0 + G_0)k - [1 - b(1-t)M_0]}{ik + h[1 - b(1-t)]}}$$

Graph: Equilibrium in IS-LM Model



- When the government increases its expenditure, what's the impact on the equilibrium Y and r?

Comparative Statics Analysis: IS-LM Model

- In a closed economy with tax, suppose G increases (*ΔG= G₁ – G₀*), ΔY*/ΔG=? and Δr*/ΔG=?
- Recall: $Y^* = \frac{(a+I_0+G_0)h+iM_0}{ik+h[1-b(1-t)]}$ and $r^* = \frac{(a+I_0+G_0)k-[1-b(1-t)]M_0}{ik+h[1-b(1-t)]}$

$$\Delta Y^* = Y^{*1} - Y^{*0} = \frac{h}{ik+h[1-b(1-t)]}\Delta G \;\Rightarrow\; \boxed{\frac{\Delta Y^*}{\Delta G} = \frac{h}{ik+h[1-b(1-t)]} > 0}$$

$$\Delta r^* = r^{*1} - r^{*0} = \frac{k}{ik+h[1-b(1-t)]}\Delta G \;\Rightarrow\; \boxed{\frac{\Delta r^*}{\Delta G} = \frac{k}{ik+h[1-b(1-t)]} > 0}$$