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EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. X_i is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3.0	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

Normal distribution

1.2 Find $\hat{Y}_i \quad and\ \hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1), var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$\boldsymbol{Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)}$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

“Practice makes Perfect.”

Table 1

1.)

Student	Y_i	X_i	X_iY_i	X_i^2	$X_i - \bar{X} = x_i$	x_i^2
1	2.8	63	176.4	3969	-14.625	213.8906
2	3.4	72	244.8	5184	-5.625	31.6406
3	3.0	78	234	6084	0.375	0.1406
4	3.5	81	283.5	6561	3.375	11.3906
5	3.6	87	313.2	7569	9.375	87.8906
6	3.0	75	225	5625	-2.625	6.8906
7	2.7	75	202.5	5625	-2.625	6.8906
8	3.7	90	333	8100	12.375	153.1406
Sum	25.7	621	2012.4	48717		511.8748
Mean	3.2125	77.625	251.55	6089.625		

1.1.) $\hat{\beta}_2 = \dfrac{n\sum X_iY_i - \sum X_i \sum Y_i}{n\sum X_i^2 - (\sum X_i)^2}$

$= \dfrac{8(2012.4) - (621)(25.7)}{8(48717) - (621)^2}$

$= \dfrac{16099.2 - 15959.7}{389736 - 385641}$

$= \dfrac{139.5}{4095}$

$\hat{\beta}_2 = 0.0341$

$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$

$= 3.2125 - (0.0341)(77.625)$

$= 3.2125 - 2.6393$

$\hat{\beta}_1 = 0.5655$

$\therefore$ Regression equation is, $\hat{Y}_i = 0.5655 + 0.0341 X_i$.

If BE student get 0 in test exam their GPA likely to be 0.5655 and every 1 score increase on the test exam by average, GPA will increase by 0.0341.

1.2)

i	X_i	$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$	$Y_i - \hat{Y}_i = \hat{u}_i$
1	63	$0.5655 + 0.0341(63) = 2.7138$	$2.8 - 2.7138 = 0.0862$
2	72	$0.5655 + 0.0341(72) = 3.0207$	$3.4 - 3.0207 = 0.3793$
3	78	$0.5655 + 0.0341(78) = 3.2253$	$3.0 - 3.2253 = -0.2253$
4	81	$0.5655 + 0.0341(81) = 3.3276$	$3.5 - 3.3276 = 0.1724$
5	87	$0.5655 + 0.0341(87) = 3.5322$	$3.6 - 3.5322 = 0.0678$
6	75	$0.5655 + 0.0341(75) = 3.123$	$3.0 - 3.123 = -0.123$
7	75	$0.5655 + 0.0341(75) = 3.123$	$2.7 - 3.123 = -0.423$
8	90	$0.5655 + 0.0341(90) = 3.6345$	$3.7 - 3.6345 = 0.0655$

$\sum_{i=1}^{8} \hat{u}_i = 0.0862 + 0.3793 + (-0.2253) + 0.1724 + 0.0678 + (-0.123) + (-0.423) + 0.0655 = -0.0001$

$\simeq 0$

1.3.) Since $Var(u_i | X_1, X_2 \dots X_n) = \sigma^2$ but σ^2 is unknown, so we will find $\hat{\sigma}^2$ instead.

$$\hat{\sigma}^2 = \frac{\Sigma \hat{u}_i^2}{n-2} = \frac{(0.0862)^2 + (0.3799)^2 + (-0.2253)^2 + (0.1724)^2 + (0.0678)^2 + (-0.1237)^2 + (-0.4229)^2 + (0.0655)^2}{8-2}$$

$$= \frac{0.0074 + 0.1479 + 0.0508 + 0.0297 + 0.0046 + 0.0151 + 0.1789 + 0.0043}{6}$$

$$= \frac{0.4347}{6}$$

$$Var(\hat{u}_i) = 0.0725 = \hat{\sigma}^2$$

$$Var(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\Sigma x_i^2} = \frac{0.0725}{511.8748}$$

$$= 0.0001$$

$$Var(\hat{\beta}_1) = \frac{\Sigma X_i^2}{n \Sigma x^2} \cdot \sigma^2$$

$$= \frac{48717}{8(511.8748)} \cdot 0.0725$$

$$= \frac{3531.9825}{4094.9984}$$

$$= 0.8625$$

2.)

i	X_i	Y_i	$X_i Y_i$	X_i^2	$X_i - \bar{X} = x_i$	x_i^2
1	10	0	0	100	10 - 20 = -10	100
2	12	2	24	144	12 - 20 = -8	64
3	14	5	70	196	14 - 20 = -6	36
4	16	6	96	256	16 - 20 = -4	16
5	18	7	126	324	18 - 20 = -2	4
6	22	10	220	484	22 - 20 = 2	4
7	24	10	240	576	24 - 20 = 4	16
8	26	15	390	676	26 - 20 = 6	36
9	28	16	448	784	28 - 20 = 8	64
10	30	20	600	900	30 - 20 = 10	100
Sum	200	91	2214	4440	0	440
Mean	20	9.1	221.4	444	0	44

2.1) $\hat{\beta}_2 = \dfrac{n\sum X_iY_i - \sum X_i \sum Y_i}{n\sum X_i^2 - (\sum X_i)^2}$

$= \dfrac{10(2214) - (200)(91)}{10(4440) - (200)^2}$

$= \dfrac{22140 - 18200}{44400 - 40000}$

$= \dfrac{3940}{4400}$

$\hat{\beta}_2 = 0.8955$

$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2\bar{X}$

$= 9.1 - 0.8955(20)$

$\hat{\beta}_1 = -8.81$

The final model is $\hat{Y}_i = -8.81 + 0.8955X_i$

$\therefore$ If $X_i = 0$ then, $\hat{Y}_i = -8.81$

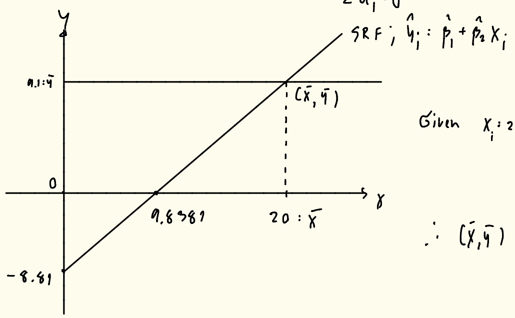
and If X_i increase 1 unit in average, $\hat{Y}_i$ will increase by 0.8955 unit.

2.2)

i	X_i	$\hat{Y}_i$	$Y_i - \hat{Y}_i = \hat{u}_i$	$\hat{u}_i^2$
1	10	0.145	0 - 0.145 = -0.145	0.021
2	12	1.936	2 - 1.936 = 0.064	0.0041
3	14	3.727	5 - 3.727 = 1.273	1.6205
4	16	5.518	6 - 5.518 = 0.482	0.2323
5	18	7.309	7 - 7.309 = -0.309	0.0955
6	22	10.891	10 - 10.891 = -0.891	0.7939
7	24	12.682	10 - 12.682 = -2.682	7.1931
8	26	14.473	15 - 14.473 = 0.527	0.277
9	28	16.264	16 - 16.264 = -0.264	0.0697
10	30	18.055	20 - 18.055 = 1.945	3.7836

$\sum \hat{u}_i = 0$

2.3



Given $X_i = 20$, $\hat{Y}_i = -8.81 + 0.8955(20)$

$\hat{Y}_i = 9.1 = \bar{Y}$

$\therefore (\bar{X}, \bar{Y})$ is a member of SRF.

2.4.) Given $X_i = 18$

$$\hat{Y}_i = -8.41 + 0.8955(18)$$

$$\hat{Y}_i = 7.709$$

2.5.) $$Var(\hat{u}_i) = \frac{\Sigma \hat{u}_i^2}{n-2}$$

$$= \frac{14.0908}{10-2}$$

$$= 1.7614 = \hat{\sigma}^2$$

$$Var(\hat{\beta}_2) = \frac{\sigma^2}{\Sigma x_i^2}$$

$$= \frac{1.7614}{440}$$

$$= 0.004$$

$$Var(\hat{\beta}_1) = \frac{\Sigma X_i^2}{n \Sigma x_i^2} \cdot \sigma^2$$

$$= \frac{4440}{10(440)} \cdot 1.7614$$

$$\simeq 1.7774$$

3.) We need to find $\hat{\beta}_2$ first

$\hat{\beta}_2$ is an unbiased estimator of β_2 when $E(\hat{\beta}_2) = \beta_2$

$$\hat{\beta}_2 = \sum_{i=1}^{n} k_i Y_i$$

$$= \sum_{i=1}^{n} k_i (\beta_1 + \beta_2 X_i + u_i)$$

$$= \sum_{i=1}^{n} (k_i \beta_1 + k_i \beta_2 X_i + k_i u_i)$$

$$= \beta_1 \sum_{i=1}^{n} k_i + \beta_2 \sum_{i=1}^{n} k_i X_i + \sum_{i=1}^{n} k_i u_i, \quad \sum_{i=1}^{n} k_i = 0, \quad \sum_{i=1}^{n} k_i X_i = 1$$

$$\hat{\beta}_2 = \beta_2 + \sum_{i=1}^{n} k_i u_i$$

$$E(\hat{\beta}_2) = E(\beta_2) + E\left(\sum_{i=1}^{n} k_i u_i\right) \rightarrow \text{Assumption 3 } E(u_i | X_i) = 0$$

$$E(\hat{\beta}_2) = \beta_2$$

$\therefore$ $\hat{\beta}_2$ is an unbiased estimator of β_2

And $\hat{\beta}_1$ is an unbiased estimator of β_1

when $E(\hat{\beta}_1) = \beta_1$

since $\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$

$$E(\hat{\beta}_1) = E(\bar{Y} - \hat{\beta}_2 \bar{X})$$

$$= E(\bar{Y}) - \bar{X} E(\hat{\beta}_2)$$

And $$E(\bar{Y}) = E\left[\frac{1}{n} \sum_{i=1}^{n} Y_i\right]$$

$$= \frac{1}{n} \sum_{i=1}^{n} E(Y_i)$$

$$= \frac{1}{n} \sum_{i=1}^{n} (\beta_1 + \beta_2 X_i)$$

$$= \frac{n \beta_1}{n} + \frac{\beta_2 \sum_{i=1}^{n} X_i}{n}$$

$$E(\bar{Y}) = \beta_1 + \beta_2 \bar{X}$$

From $E(\hat{\beta}_1) = E(\bar{Y}) - \bar{X} E(\hat{\beta}_2)$

If $E(\bar{Y}) = \beta_1 + \beta_2 \bar{X}$ and $\beta_2 = E(\hat{\beta}_2)$

Then $E(\hat{\beta}_1) = \beta_1 + \beta_2 \bar{X} - \bar{X} \beta_2$

$$E(\hat{\beta}_1) = \beta_1$$

$\hat{\beta}_1$ is an unbiased estimator