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Group Homework 3 & 4

Semester 2/2022 EE320 Introductory mathematical economics

Due date: Feb 25th 2022 (before midnight /B.E. moodle).

Note: Late homework will not be accepted. Use the format of filename as required; this will cost you two points if you don't follow the instruction.

1. Suppose that $Y = x^3 - 5x^2 + 7x - 5$

a) Show the domain of X where the function exhibits the property of an increasing function.

b. Define the domain set of X. Is the function concave for all over the domain?

2. Suppose that a firm's short-run production function is given by

$$Q(L) = 6L^2 - L^3$$

where *Q(L)* is the output level, and *L* is the number of workers

a. Derive the average product of labor (AP_L) and the marginal product of labor (MP_L).

b. What size of the work force (L^{**}) maximizes the average output per labor, Q(L)/L?

c. Use calculus to show that the MP_L curve must cross the AP_L curve at its maximum point.

d. Given that the firm faces the demand function

$$Q = 100 - 2P$$

derive the marginal revenue product (*MRP*) function.

1. Suppose that $Y = x^3 - 5x^2 + 7x - 5$

a) Show the domain of X where the function exhibits the property of an increasing function.

b. Define the domain set of X. Is the function concave for all over the domain?

a. $\frac{dy}{dx} = 3X^2 - 10X + 7 \Rightarrow (3X-7)(X-1) = 0$

$\therefore X = \frac{7}{3}, 1 > 0$

b. from previous

$\frac{d^2y}{d^2x} = 6X - 10$

$x = \frac{7}{3} ; 6\left(\frac{7}{3}\right) - 10 = 4 > 0$

this concave at $x = \frac{7}{3}$ min. point

$X = 1 ; 6(1) - 10 = 4 < 0$

this concave at $X = 1$ max. point

$\therefore$ No, it is not concave and convex

2. Suppose that a firm's short-run production function is given by

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c. Use calculus to show that the MP_L curve must cross the AP_L curve at its maximum point. → MPL = APL

d. Given that the firm faces the demand function

$$Q = 100 - 2P$$

derive the marginal revenue product (*MRP*) function.

a) Average Product of labor $= \frac{Q}{L}$

Marginal Product of labor $= \frac{dQ}{dL}$

$Q = 6L^2 - L^3$

$APL = \frac{Q}{L}$

$= \frac{6L^2 - L^3}{L}$ *

$= 6L - L^2$

$MPL = \frac{dQ}{dL}$

$= 12L - 3L^2$ *

c) MPL cross APL

$MPL = APL$

$12L - 3L^2 = 6L - L^2$

$12L - 6L = 3L^2 - L^2$

$6L = 2L^2$

$0 = 2L(L-3)$

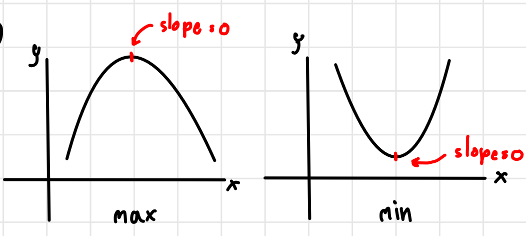
$0 = L(L-3)$

$\therefore L = 0, 3$

$L = 3$ *

MPL will intercept APL at L=3 so it's the point that are maximum point (from question b) *

b)



$APL = 6L - L^2$

$\frac{dAPL}{dL} = 6 - 2L$

$0 = 6 - 2L$ - fide Max Min

$L = 3$ *

d) $MRP = \frac{dTR}{dL} = MPL \cdot MR$

step 1 fide MR $Q = 100 - 2P$

$TR = P \cdot Q$ $P = 50 - 0.5Q$

$TR = (50 - 0.5Q)Q$

$= 50Q - 0.5Q^2$

step 2 $MR = \frac{dTR}{dQ}$

$= 50 - Q$

step 3 $MRP = MPL \cdot MR$

$= (12L - 3L^2) \cdot (50 - Q)$

$Q = 6L^2 - L^3$

$= (12L - 3L^2) \cdot (50 - 6L^2 + L^3)$ *

3. Suppose a monopolist faces with the market demand equation given by,

$$P = 40 + \frac{105}{Q} - \frac{3}{2}Q^2$$

where P is the unit price and Q is the amount of quantity purchased. The monopolist is running the firm using the cost function given as follow,

$$C(Q) = 6Q^3 - 81Q^2 - 175Q + 10.$$

Consider the following questions.

a. Determine the level of revenue-maximizing output, and calculate the value of the elasticity of demand at that level of output?
b. Construct the profit function.
c. Determine the profit-maximizing level of output. Confirm your result that the proposed solution is correct.
d. Discuss the effect that would likely be happening if the government imposes a lump-sum tax on the monopolist.

4. The swimming pool maintenance sector consists of 100 identical firms, each having short-run total costs given by $STC = 0.5q^2 + 10q + 5$, where q is the number of swimming pools serviced per day.

a. What is the short-run supply curve for each pool maintenance firm? What is the short-run supply curve for the market as a whole?
b. Suppose the demand for the maintenance of swimming pools is given by $Q = 1100 - 50P$. What will be the equilibrium in this marketplace? What will *each* firm's total short-run profits be?

3. Suppose a monopolist faces with the market demand equation given by,

$$P = 40 + \frac{105}{Q} - \frac{3}{2}Q^2$$

$40 + \frac{105}{2.98} - \frac{3}{2}(2.98)^2$ $\quad 40 + 35.23 - 13.32 = 61.91$

where P is the unit price and Q is the amount of quantity purchased. The monopolist is running the firm using the cost function given as follow,

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Consider the following questions.

ผลิตที่ขึ้นถึง Revenue มากสุด

a. Determine the level of revenue-maximizing output, and calculate the value of the elasticity of demand at that level of output?

b. Construct the profit function.

c. Determine the profit-maximizing level of output. Confirm your result that the proposed solution is correct.

d. Discuss the effect that would likely be happening if the government imposes a lump-sum tax on the monopolist.

a) $TR = P \cdot Q$

$= \left[40 + \frac{105}{Q} - \frac{3}{2}Q^2\right] \cdot Q$

$TR = 40Q + 105 - \frac{3}{2}Q^3$

$\frac{dTR}{dQ} = 40Q - \frac{9}{2}Q^2 = 0$

$40 = \frac{9}{2}Q^2$

$\frac{80}{9} = Q^2$

$Q = \sqrt{\frac{80}{9}} = 2.98$ #

[illegible] $Q = 2.98$ for Max TR #

$$\boxed{\text{elasticity Demand} = \frac{dQ}{dP} \cdot \frac{P}{Q}}$$

b) Profit function $= TR - TC$

$P = 40 + \frac{105}{Q} - \frac{3}{2}Q^2$

$C(Q) = 6Q^3 - 81Q^2 - 175Q - 10$

$\Pi = TR - TC$

$= P \cdot Q - TC$

$= \left[40 + \frac{105}{Q} - \frac{3}{2}Q^2\right]Q - \left[6Q^3 - 81Q^2 - 175Q - 10\right]$

$= -75Q^3 - 972Q^2 - 2020Q + 90$ *

c) $\Pi = TR - TC$

$= PQ - TC$

$= \left[40 + \frac{105}{Q} - \frac{3}{2}Q^2\right]Q - \left[6Q^3 - 81Q^2 - 175Q - 10\right]$

$= 40Q + 105 - \frac{3}{2}Q^3 - 6Q^3 + 81Q^2 + 175Q + 10$

Max Profit

$\frac{d\Pi}{dQ} = 40 + \frac{9}{2}Q^2 - 18Q^2 + 162Q + 175$

$= -13.5Q^2 + 162Q + 215 = 0$

$= 13.5Q^2 - 162Q - 215 = 0$

$Q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{162 \pm \sqrt{-162^2 - 4(13.5)(-215)}}{2(13.5)}$

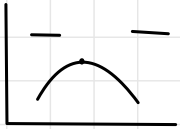
$= -1.21, 13.21$

Second order Condition

$\frac{d^2\Pi}{dQ^2} = 9Q - 36Q + 162$

$= -27(13.21) + 162$ ⊖

Max*



d) lumpsum tax $= \Pi = TR - TC - [Tax]$

$\frac{d\Pi}{dQ} = \frac{d}{dQ}[TR - TC - Tax] = 0$ − Tax

$\frac{dTR}{dQ} - \frac{dTC}{dQ} - \frac{dTax}{dQ} = 0$

$MR - MC - 0 = 0$

$MR = MC$

$\Pi = TR - TC$ − with out tax

$\frac{d\Pi}{dQ} = \frac{d}{dQ}[TR - TC] = 0$

$MR = MC$

Produce at $MR = MC$

∴ not effective *

c. Suppose the government imposed a €3 tax on chemicals per pool maintained. How would this tax change the market equilibrium?

d. How would the burden of this tax be shared between owners of swimming pools and the firms that offer pool maintenance services?

e. Calculate the total loss of producer surplus as a result of the new tax. Show that this loss equals the change in total short-run profits in this industry.

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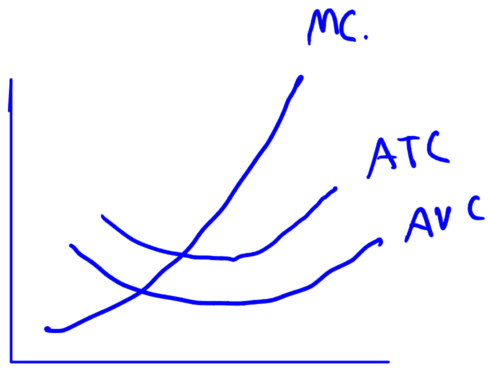
4. The swimming pool maintenance sector consists of 100 identical firms, each having short-run total costs given by $STC = 0.5q^2 + 10q + 5$, where q is the number of swimming pools serviced per day.

a. What is the short-run supply curve for each pool maintenance firm? What is the short-run supply curve for the market as a whole?



$STC = 0.5Q^2 + 10Q + 5$

$MC = \frac{\Delta TC}{\Delta Q}$

$= \frac{dTC}{dQ}$

$MC = 1Q + 10$

Supply $P = MC$

$P = Q + 10$

$Q = -10 + P$ → supply curve

Market supply

$Q_m = \sum_{i=1}^{n} Q_{mi}$

$n = 100$

$Q_m = 100(P - 10)$

$Q^s_m = 100P - 1000$

b. Suppose the demand for the maintenance of swimming pools is given by $Q = 1100 - 50P$. What will be the equilibrium in this marketplace? What will *each* firm's total short-run profits be?

$Q^s_m = Q^D_m$

$100P - 1000 = 1100 - 50P$

$150P = 2100$

$P = 14$

substitute $P = 14$ in Q^s_m

$Q_m = 400$

∴ Market price is 14 units

market quantity 400 units by 100 firms.

(Q of 1 firm $= \frac{400}{100} = 4$ unit)

short-run profit

$\pi = TR - TC$

$= P \cdot Q - (0.5(4)^2 + 10(4) + 5)$

$\pi = 3$

c. Suppose the government imposed a €3 tax on chemicals per pool maintained. How would this tax change the market equilibrium?

imposed a €3 tax producers

$STC = 0.5Q^2 + 10Q + 5 + 3Q$

$= 0.5Q^2 + 13Q + 5$

$MC = \frac{dSTC}{dQ} = 1Q + 13$

$MC = P = Q + 13$

$Q = -13 + P \rightarrow$ firm's supply

Market supply

$Q_s = 100(-13 + P)$

$Q_s = -1300 + 100P$

new equilibrium

$Q_D = Q_s^{new}$

$1100 - 50p = -1300 + 100p$

$p = 16$

substitute $P = 16$ in new market supply equation

$Q_s^{new} = -1300 + 100(16)$

$Q = 300$

$\therefore$ new price equilibrium €16

new quantity equilibrium = 300 units

d. How would the burden of this tax be shared between owners of swimming pools and the firms that offer pool maintenance services?

4d consumer burden

$= \tilde{P} - P^{o}$

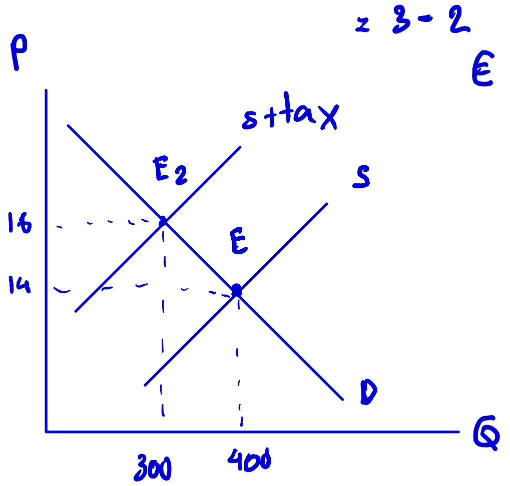
$= 16 - 14$

$=$ € 2 per unit $\rightarrow \frac{2}{3} \times 100 = 66.67\%$

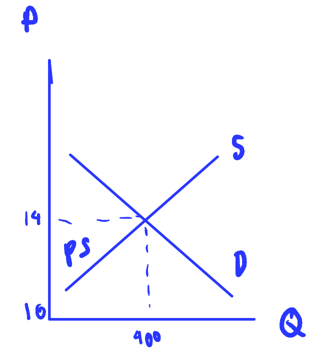
producer burden $=$ tax $-$ consumer pay

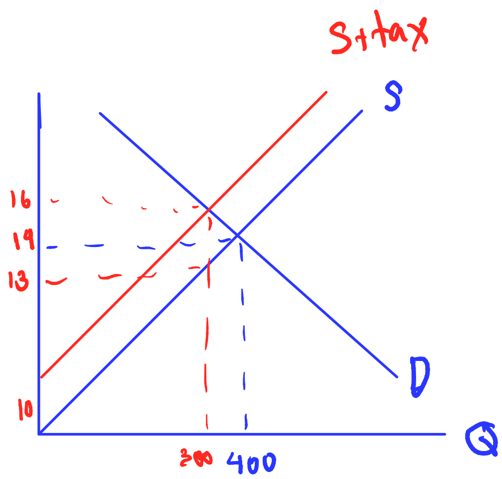
$= 3 - 2$

€ 1 per unit $\rightarrow \frac{1}{3} \times 100 = 33.34\%$



e. Calculate the total loss of producer surplus as a result of the new tax. Show that this loss equals the change in total short-run profits in this industry.





$Q_s = -10 + P$

$Q = 0;\ P = 10$

Producer Surplus $= \frac{1}{2} \times$ base $\times$ height

$= 800$ units

new PS $= \frac{1}{2} \times 3 \times 300$

$= 450$ unit

change in PS

$= -350$ units

Change in total shot-run profit

$$q = \frac{300}{100 \text{ firm}} = 3 \text{ units}$$

new $\pi = PQ - (TC + tax)$

$= 16Q - (0.5Q^2 + 13Q + 5)$

$= -0.5$

before tax profit $=$ €3

change in shot-run profit $= -0.5 - 3$

$= -3.5$

total profit $= -350$

$\therefore$ Total profit decreased €350