Beta and Single Index Model
R2, Ch7

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Aj.Siraprapa Watakit

Agenda

- Single Index Model
- Estimating Beta
- Estimating Portfolio Beta

Portfolio Analysis Problem

- **Portfolio Analysis** calls for estimations of expected return and risk of a portfolio

$$\bar{R}_P = \sum_{i=1}^{N} X_i \bar{R}_i$$

Harry Markowitz

$$\sigma_P = \left[\sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} X_i X_j \sigma_i \sigma_j \rho_{ij} \right]^{1/2}$$

- Single stock expected return and risk is easy; but **pair wise** correlation is much more complex
- If a financial intuition follows 250 stocks, it will have to estimate 250 expected return and risk and **31,125** correlation coefficient, which means we need at least 31,125 data point

[N(N-1)/2]

Single Index Model

- **Casual observation:** When market goes up, most stocks go up. When market goes down, most stocks go down
- Hence, this market factor must be the core of securities correlation
- **Single Index Model:** an expected return of a stock is a linear function 2 components;

$$\boxed{R_i = a_i + \beta_i R_m}$$

Beta

- Beta measures a market sensitivity of a stock relative to the expected market return(Rm)
- How is beta interpreted?
  - If β = 1.0, stock has average risk.
  - If β > 1.0, stock is riskier than average.
  - If β < 1.0, stock is less risky than average.
- Example of SIM
  - CAPM $[E(R_i) - R_f] = \beta_i[E(R_M) - R_f)]$
  - Market Model $R_i = \alpha_i + \beta_i R_M + e_i$

Important Results of SIM

- The expected return of security

$$E(R_i) = \alpha_i + \beta_i R_M$$

- The expected risk of security

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{ei}^2$$

Systematic Risk

Un-Systematic Risk /Firm-Specific Risk

- The expected correlation of security

$$\sigma_{ij} = \beta_i \beta_j \sigma_m^2$$

[3N+2]

Estimating β Exercise

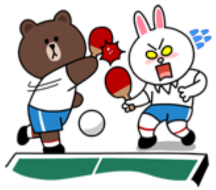
- 3-Method of Calculating β
  - Method 1

$$\beta_i = (\frac{\sigma_i}{\sigma_M})\rho_{iM}$$

  - Method 2

$$\beta_i = (\frac{\sigma_i}{\sigma_M})(\frac{\sigma_{iM}}{\sigma_i \sigma_M}) = \frac{\sigma_{iM}}{\sigma_M^2}$$

  - Method 3: Linear Regression Function
- Calculating GE Beta



Calculating Beta in Practice

- Many analysts use the S&P 500 to find the market return.
- Analysts typically use four or **5 years' of monthly returns** to establish the regression line.
  - Some analysts use **2 years of weekly returns.**
- Portfolio β is simple a weighted average of β_i

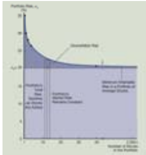
$$\beta_p = \sum_{i=1}^{N} x_i \beta_i$$

Portfolio Beta and Risk

- Portfolio Beta and Alpha can be calculated as follow

$$\beta_p = \sum_{i=1}^{N} x_i \beta_i, \quad \alpha_p = \sum_{i=1}^{N} x_i \alpha_i, \quad Rp = \alpha_p + \beta_p Rm$$

- It can be shown that…



$$\sigma_p^2 = \beta_p^2 \sigma_m^2 + \frac{1}{N}\left[\sum_{i=1}^{N} \frac{1}{N} \sigma_{ei}^2\right]$$

  - By adding more stocks, we can eliminate residual variance, but we still have to bear with beta

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Accuracy of Historical Beta

- On the Assessment of Risk, Marshal E. Blume [1971]
- Can we use historical betas to predict future betas?
  - β is can be used as a measure of risk for both **single security** and **security portfolio**
  - Economics variables, including betas, usually change over time, but we can, at some level , assess future value of risk based on historical value of beta
  - Further question
    - Is it accurate? Do we need any adjustment?

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Accuracy of Historical Beta

- Blume shows that, by forming a many-stock-portfolio, we can see that there is a strong correlation between betas in one period and betas in the next adjacent period

TABLE 2
PRODUCT MOMENT AND RANK ORDER CORRELATION COEFFICIENTS OF BETAS FOR PORTFOLIOS OF N SECURITIES

Number of Securities per Portfolio	7/26-6/33 and 7/33-6/40		7/33-6/40 and 7/40-6/47		7/40-6/47 and 7/47-6/54		7/47-6/54 and 7/54-6/61		7/54-6/61 and 7/61-6/68	
	P.M.	Rank	P.M.	Rank	P.M.	Rank	P.M.	Rank	P.M.	Rank
1	0.63	0.69	0.62	0.73	0.59	0.65	0.65	0.67	0.60	0.62
2	0.71	0.75	0.76	0.83	0.72	0.79	0.76	0.76	0.73	0.74
4	0.80	0.84	0.85	0.90	0.81	0.89	0.84	0.84	0.84	0.85
7	0.86	0.90	0.91	0.93	0.88	0.93	0.87	0.88	0.88	0.89
10	0.89	0.93	0.94	0.95	0.90	0.95	0.92	0.93	0.92	0.93
20	0.93	0.99	0.97	0.98	0.95	0.98	0.95	0.96	0.97	0.98
35	0.96	1.00	0.98	0.99	0.95	0.99	0.97	0.98	0.97	0.97
50	0.98	1.00	0.99	0.98	0.98	0.99	0.98	0.98	0.98	0.97

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Accuracy of Historical Beta

- He also show that we can improve the estimates of beta by using ***linear regression*** → these betas are called ***adjusted betas***

TABLE 4
MEASUREMENT OF REGRESSION TENDENCY OF ESTIMATED BETA COEFFICIENTS FOR INDIVIDUAL SECURITIES

Regression Tendency Implied Between Periods	$\beta_2 = a + b\beta_1$
7/33-6/40 and 7/26-6/33	$\beta_2 = 0.320 + 0.714\beta_1$
7/40-6/47 and 7/33-6/40	$\beta_2 = 0.265 + 0.750\beta_1$
7/47-6/54 and 7/40-6/47	$\beta_2 = 0.526 + 0.489\beta_1$
7/54-6/61 and 7/47-6/54	$\beta_2 = 0.343 + 0.677\beta_1$
7/61-6/68 and 7/54-6/61	$\beta_2 = 0.399 + 0.546\beta_1$

TABLE 5
MEAN SQUARE ERRORS BETWEEN ASSESSMENTS AND FUTURE ESTIMATED VALUES

Number of Sec./Port.	Assessments Based Upon 7/33-6/40		7/40-6/47		7/47-6/54		7/54-6/61	
	unadjusted	adjusted	unadjusted	adjusted	unadjusted	adjusted	unadjusted	adjusted
1	0.1929	0.1808	0.1747	0.1261	0.1203	0.1087	0.1305	0.1013
2	0.0915	0.0813	0.1218	0.0736	0.0729	0.0614	0.0827	0.0535
4	0.0538	0.0453	0.0958	0.0483	0.0495	0.0381	0.0587	0.0296
7	0.0323	0.0247	0.0631	0.0276	0.0387	0.0281	0.0523	0.0231
10	0.0243	0.0174	0.0535	0.0220	0.0305	0.0189	0.0430	0.0169
20	0.0160	0.0090	0.0328	0.0106	0.0258	0.0139	0.0291	0.0089
35	0.0120	0.0055	0.0266	0.0080	0.0197	0.0101	0.0302	0.0089
50	0.0096	0.0046	0.0192	0.0046	0.0122	0.0097	0.0237	0.0064
75	0.0081	0.0035	0.0169	0.0067	0.0112	0.0078	0.0193	0.0056
100	0.0084	0.0020	0.0157	0.0035	0.0114	0.0084	0.0195	0.0056

- Comparing between unadjusted beta and adjusted betas, the adjusted betas outperform unadjusted betas in every period
- **Note**: besides Blume technique, there are also others i.e Vasicek technique

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Conclusion

- Markowitz Optimization Techniques provide the best and accurate optimization solution but the requirement is high
- Single Index Model offers a simpler calculation by assuming that each security is driven by a single factor i.e. **market factor**
  - What to be used as market index is still debatable
  - Empirical shows that market factor is not the most and only factor; some study suggest adding fundamental factor such as growth, d/e ratio, SMB, HML and etc..

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Conclusion

- SIM assumed there is no effect beyond market index(**therefore no industry effect**)→$E(e_i,e_j)=0$
  - Empirical shows that residual between firm in the same industry are usually **positively correlated**
  - Adding positively correlated stock will increase un-systematic portfolio risks→ Resulting in **sub-optimal solution**
- Beta measures the sensitivity between Ri and Rm
- Beta can be used as a risk measurement for both single stock and portfolio
- Historical betas can be used to predict future betas but adjustment must also be taken into account

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