

(1) Testing individual regression coefficients

With the normality assumption, we find that estimators are normally distributed with an unknown variance. Therefore, to test statistical significance, we need to rely on t statistics as follows.

$$\bullet t_{cal}(\beta_1) = \frac{\hat{\beta}_1 - \beta_1}{\sigma_{\hat{\beta}_1}} \sim t_{n-3}$$

$$\bullet t_{cal}(\beta_2) = \frac{\hat{\beta}_2 - \beta_2}{\sigma_{\hat{\beta}_2}} \sim t_{n-3}$$

$$\bullet t_{cal}(\beta_3) = \frac{\hat{\beta}_3 - \beta_3}{\sigma_{\hat{\beta}_3}} \sim t_{n-3}$$

where each of the

- $\hat{\beta}$ is the value of estimator,
- β is the value that we would test against and
- $\sigma_{\hat{\beta}}$ is the standard error of the estimator.

Now that the d.f. is $n - 3$ if we have two independent variables in our estimation. Testing procedures are the same which are

Example: Given the data collected at the beginning of this class. We hypothesize that shoe size (Y) is determined by both height (X_{2i}) and weight (X_{3i}), the estimated SRF is

$$\bullet \hat{Y}_i = 2.8667 + 0.2011X_{2i} + 0.0633X_{3i}$$

$$(4.4847) \quad (0.0329) \quad (0.0316)$$

$$n = 24 \quad R^2 = 0.8264 \quad \bar{R}^2 = 0.8099$$

$$F(2,21) = 49.99 \quad Prob > F = 0.000$$

(1) State your hypothesis

Note we usually say that the estimator is statistically significant when we reject a null hypothesis. For instance, testing if our β_2 is significantly different from zero or not, our hypothesis will be.

$$\bullet H_0: \beta_2 = 0$$

$$\bullet H_a: \beta_2 \neq 0$$

To test β_1 and β_3 , the hypothesis statement goes in line.

(1) Testing individual regression coefficients

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Regression results

$$\bullet \hat{Y}_i = 2.8667 + 0.2011X_{2i} + 0.0633X_{3i}$$

$$(4.4847) \quad (0.0329) \quad (0.0316)$$

$$n = 24 \quad R^2 = 0.8264 \quad \bar{R}^2 = 0.8099$$

$$F(2,21) = 49.99 \quad Prob > F = 0.000$$

(2) Calculate test statistics

$$\bullet t_{cal}(\beta_1) = \frac{\hat{\beta}_1 - \beta_1}{\sigma_{\hat{\beta}_1}} =$$

$$\bullet t_{cal}(\beta_2) = \frac{\hat{\beta}_2 - \beta_2}{\sigma_{\hat{\beta}_2}} =$$

$$\bullet t_{cal}(\beta_3) = \frac{\hat{\beta}_3 - \beta_3}{\sigma_{\hat{\beta}_3}} =$$

Don't forget to indicate their d.f.

(3) Pick an α and state decision rules

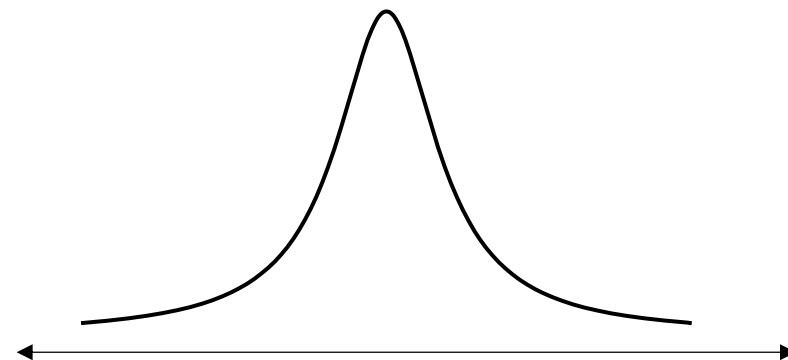
Supposed that we pick $\alpha = 0.05$, in other words we are making sure that 95 percent of the time, our parameters will be different from zero. Since now we are testing against zero, we can directly at the t-table for the critical values which are

$$\bullet t_{lower} =$$

$$\bullet t_{upper} =$$

Illustrate the critical region here.

Two-tails test



(2) Testing the overall significance of the sample regression

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(4) Concluding the test results

Note that when we test a parameter, we are holding other parameters constant, under the separate hypothesis.

Now let consider if we want to test parameters jointly under the same hypothesis, for instance,

- $H_0: \beta_2 = \beta_3 = 0$
- H_a : Not all the slope coefficients are simultaneously zero.

This hypothesis states that “ β_2 and β_3 are jointly or simultaneously equal to zero”. We **cannot** utilize ordinary t-test anymore. We need to rely on specific joint test or the

Analysis of variance (ANOVA)

The ANOVA test is a different approach, as the name suggested, which will only focus on dispersion of each part, namely the ESS and RSS. According to the study of variation from the mean in the coefficient of determination, we have

$$\begin{aligned} \bullet \sum \hat{y}_i^2 &= \hat{\beta}_2 \sum y_i x_{2i} + \hat{\beta}_3 \sum y_i x_{3i} + \sum \hat{u}_i^2 \\ \text{TSS} &= \qquad \qquad \text{ESS} \qquad \qquad + \text{RSS} \end{aligned}$$

Note that each part has a different d.f. as

- TSS has d.f. of $n - 1$
- ESS has d.f. of $k - 1$
- RSS has d.f. of $n - k$

(2) Testing the overall significance of the sample regression

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From the estimation of shoe size, we now look at another information printed from regression table to make use of our joint test.

Regression results

	SS	d.f.	MS
Model	120.6210	2	60.3105
Residual	25.3373	21	1.2065
Total	145.9583	23	6.3460

By the way, we do not need any coefficient report since we are only interested in variance (sum of squares or SS) of each part. Mean squares (MS) is derived from taking SS divide by d.f.

When we do the ‘analysis of variance’, we are comparing between a part that the model can explain (ESS) versus a part it cannot (RSS), therefore the comparison is

• $\frac{ESS/df}{RSS/df} = \frac{\hat{\beta}_2 \sum y_i x_{2i} + \hat{\beta}_3 \sum y_i x_{3i} / (k-1)}{\sum \hat{u}_i^2 / (n-k)} \sim$

Consider the ratio, from the assumption of $u_i \sim N(0, \sigma^2)$,

• $E\left(\frac{\sum \hat{u}_i^2}{n-k}\right) = \sigma^2$

and if we assume that our stated hypothesis

- $H_0: \beta_2 = \beta_3 = 0$ is true, then
- $\sum y_i^2 = \sum u_i^2$

or the effect of X_{2i} and X_{3i} is trivial to the variation in Y_i , the only source of variation in Y_i is the error term u_i .

Therefore, the ratio becomes smaller if $\beta_2 = \beta_3 = 0$. The decision rule for this test is

• $F_{cal} > F_{\alpha}(k - 1, n - k)$ then we can reject H_0

Now let’s to come up with a formal testing procedures.

(2) Testing the overall significance of the sample regression

Regression results

	SS	d.f.	MS
Model	120.6210	2	60.3105
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Total	145.9583	23	6.3460

(1) State your hypothesis

(2) Calculate test statistics

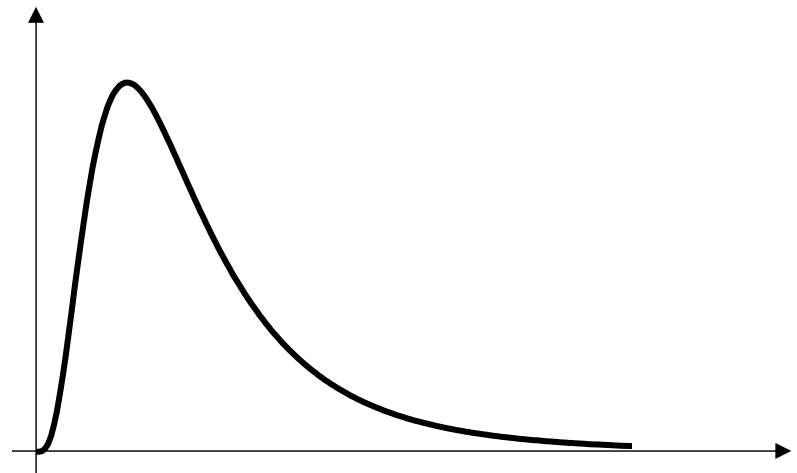
$$\bullet F_{cal} = \frac{ESS/df}{RSS/df} =$$

(3) Pick an α and state decision rules

- $\alpha =$
- $F_{upper,\alpha}(2,21) =$

Illustrate the critical region here.

Simplified F-distribution



(2) Testing the overall significance of the sample regression

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We can also generalize the F-test for more than two-independent variable models.

- $H_0: \beta_2 = \beta_3 = \dots = \beta_k = 0$
- H_a : Not all the slope coefficients are simultaneously zero.

Relationship between R^2 and F

Since we can generalize the F statistics as

- $F_{cal} = \frac{ESS/(k-1)}{RSS/(n-k)}$ then

We then can compute the F statistics directly from R^2

Regression results

- $\hat{Y}_i = 2.8667 + 0.2011X_{2i} + 0.0633X_{3i}$

$$(4.4847) \quad (0.0329) \quad (0.0316)$$

$$n = 24 \quad R^2 = 0.8264 \quad \bar{R}^2 = 0.8099$$

$$F(2,21) = 49.99 \quad Prob > F = 0.000$$

- $F_{cal} = \frac{R^2/(k-1)}{1-R^2/(n-k)} =$

which yields the same result compared to computing from the mean squares.

(3) The marginal contribution of explanatory variables

If we estimate SRF with the same context, but we are not sure if we should add X_{3i} into the model or not, we have two different models.

- Excluding: $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_{2i}$
- Including: $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_{2i} + \hat{\beta}_3 X_{3i}$

The following test is to measure increment or contribution of added independent variable(s), in this case X_{3i} . Hence, the test highlights a comparison of whole models, instead of just a particular variable's significance.

The answer we are looking for from this test is that **“should the incremental variable(s) be added into the model?”**

The F-statistics becomes

$$F_{cal} = \frac{ESS_{new} - ESS_{old} / (\text{number of new regressors})}{RSS_{new} / (n - k_{new})}$$

or if we measure from the R^2 , it would be

$$F_{cal} = \frac{R_{new}^2 - R_{old}^2 / (\text{number of new regressors})}{1 - R_{new}^2 / (n - k_{new})}$$

Note that the R^2 method is applicable only when both models have the same Y_i , while the mean squares method can apply to other types of test, more on that later.

(3) The marginal contribution of explanatory variables

Marginal contribution

$$(1) \hat{Y}_i = -0.7058 + 0.2440X_{2i}$$

$$(4.3872) \quad (0.0266)$$

$$n = 24 \quad R^2 = 0.7932 \quad \bar{R}^2 = 0.7839$$

$$F(2,21) = 84.41 \quad Prob > F = 0.000$$

$$(2) \hat{Y}_i = 2.8667 + 0.2011X_{2i} + 0.0633X_{3i}$$

$$(4.4847) \quad (0.0329) \quad (0.0316)$$

$$n = 24 \quad R^2 = 0.8264 \quad \bar{R}^2 = 0.8099$$

$$F(2,21) = 49.99 \quad Prob > F = 0.000$$

Example: The **first** only includes X_{2i} or *height* as the independent variable while the **second** model adds X_{3i} or *weight* into the model.

(1) State your hypothesis

(2) Calculate test statistics

$$\bullet F_{cal} = \frac{R_{new}^2 - R_{old}^2 / (\text{number of new regressors})}{1 - R_{new}^2 / (n - k_{new})} =$$

(3) Pick an α and state decision rules

$$\bullet \alpha =$$

$$\bullet F_{upper, \alpha}(1, 21) =$$

(4) Conclude the result

The addition of variable X_{3i} or *weight* _____ increases ESS and hence the R^2 value.

Note that the F-test for the contribution is most of the time in line with the individual t-test of those added variable(s) since $t^2 \sim F(1, n)$.

(3) The marginal contribution of explanatory variables

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When to add a new variable

- Choose a model with the highest F value, leading to highest $\bar{R}^2$.
- Absolute term of t value of added variable is more than 1, which will also eventually lead to higher $\bar{R}^2$.

When to add a group of variables

- Choose a model with the highest F value, leading to highest $\bar{R}^2$.

(4) Testing the equality of two regression coefficients

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Since the collected data provide interesting insight to this topic, let's shift to another data provided in the book. We have an ordinary demand model represented here.

$$\bullet Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \beta_4 X_{4i} + u_i$$

where Y_i is quantity demanded for a commodity

X_{2i} is its price

X_{3i} is consumer income

X_{4i} is consumer wealth

A question arises if X_{3i} and X_{4i} , income and wealth, both are representing affordability in the same way or not.

Here we can use the t-test to answer this question by

$$\bullet t_{cal} = \frac{(\hat{\beta}_3 - \hat{\beta}_4) - (\beta_3 - \beta_4)}{\sigma(\hat{\beta}_3 - \hat{\beta}_4)} \sim t_{n-k} \text{ where}$$

$$\bullet \sigma(\hat{\beta}_3 - \hat{\beta}_4) = \sqrt{\text{var}(\hat{\beta}_3) + \text{var}(\hat{\beta}_4) - 2\text{cov}(\hat{\beta}_3, \hat{\beta}_4)}$$

Example: Given an estimated of cubic cost function below

$$\bullet \hat{Y}_i = 141.7667 + 63.4777X_i - 12.9615X_i^2 + 0.9396X_i^3$$

$$(6.3753) \quad (4.7789) \quad (0.9857) \quad (0.0591)$$

$$\text{cov}(\hat{\beta}_3, \hat{\beta}_4) = -0.0576 \quad R^2 = 0.9983 \quad n = 10$$

where Y_i is total cost and X_i is output.

(1) State your hypothesis

$$\bullet H_0: \beta_3 = \beta_4 \text{ or } \beta_3 - \beta_4 = 0$$

$$\bullet H_a: \beta_3 \neq \beta_4 \text{ or } \beta_3 - \beta_4 \neq 0$$

(2) Calculate test statistics

$$\bullet t_{cal} = \frac{(\hat{\beta}_3 - \hat{\beta}_4) - (\beta_3 - \beta_4)}{\sigma(\hat{\beta}_3 - \hat{\beta}_4)} =$$

(4) Testing the equality of two regression coefficients

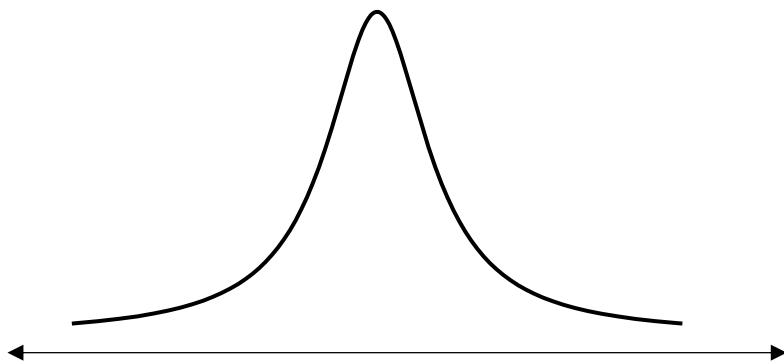
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(3) Pick an α and state decision rules

- $\alpha = 0.05$
- $t_{lower} =$
- $t_{upper} =$

Illustrate critical region below.

Two-tails test

**(4) Conclude the result**

(5) Restricted Least Squares (RLS)

There are some certain models that economic theory suggest that the coefficients in a regression satisfy some linear equality restrictions. Consider a Cobb-Douglas here, where some notations are altered, which is already linearized

$$\bullet \ln Y_i = \ln \beta_1 + \beta_K \ln K_i + \beta_L \ln L_i + u_i$$

β_K and β_L are the elasticity of capital and labor input respectively. We also know that addition of these two parameters reveals returns to scale.

Supposed that we want to test if there are constants returns to scale or not, we can hypothesize

$$\bullet H_0: \beta_K + \beta_L = 1$$

$$\bullet H_a: \beta_K + \beta_L \neq 1$$

which this test is very similar to the testing of equality of two coefficients, relying on the t-test.

$$\bullet t_{cal} = \frac{(\hat{\beta}_K + \hat{\beta}_L) - (\beta_K + \beta_L)}{\sigma(\hat{\beta}_K + \hat{\beta}_L)} = \frac{(\hat{\beta}_K + \hat{\beta}_L) - 1}{\sigma(\hat{\beta}_K + \hat{\beta}_L)} \sim t_{n-k} \text{ where}$$

$$\bullet \sigma(\hat{\beta}_K + \hat{\beta}_L) = \sqrt{\text{var}(\hat{\beta}_K) + \text{var}(\hat{\beta}_L) - 2\text{cov}(\hat{\beta}_K, \hat{\beta}_L)}$$

This is a test to see if we can make sure that the result of addition of two parameters is 1 or not.

However, we can “restrict” the regression or force that returns to scale into the regression directly that either

$$\bullet \beta_K + \beta_L = 1 / \beta_K = 1 - \beta_L / \beta_L = 1 - \beta_K$$

If we rewrite the Cobb-Douglas function with the restriction here, it becomes

$$\bullet \ln Y_i = \ln \beta_1 + (1 - \beta_L) \ln K_i + \beta_L \ln L_i + u_i$$

where $\frac{Y_i}{L_i}$ is output per labor and $\frac{K_i}{L_i}$ is capital per labor.

You may notice that there is one less coefficient to estimate. This is known as “Restricted Least Square” (RLS).

(5) Restricted Least Squares (RLS)

Therefore, the test we are about to perform is a comparison between

- Unrestricted model:

$$\ln Y_i = \ln \beta_1 + \beta_K \ln K_i + \beta_L \ln L_i + u_i$$

- Restricted model:

$$\ln \left(\frac{Y_i}{L_i} \right) = \ln \beta_1 + \beta_L \ln \left(\frac{K_i}{L_i} \right) + u_i$$

which is a test to see if **the restriction imposed is valid or not**. We can set up the same hypothesis.

- $H_0: \beta_K + \beta_L = 1$ or the restriction is valid
- $H_a: \beta_K + \beta_L \neq 1$ or the restriction is not valid

After that we can compute the F statistics, which is

$$F_{cal} = \frac{RSS_R - RSS_{UR}/m}{RSS_{UR}/(n - k_{UR})} \sim F_{(m, n - k_{UR})}$$

where R and UR indicates the value from restricted and unrestricted model respectively, m is the number of linear restriction (1 for this case). We can also derive F_{cal} from R^2 as well.

$$F_{cal} = \frac{(R_{UR}^2 - R_R^2)/m}{(1 - R_{UR}^2)/(n - k_{UR})} \sim F_{(m, n - k_{UR})}$$

Note that $R_{UR}^2 \geq R_R^2$ and the variable on the left-hand side in both models is not the same so **they are not comparable**. In this case, we might want to rely on calculating F statistics from RSS instead.

Example: Supposed we have a result of

- Unrestricted model:

$$\ln \hat{Y}_i = -1.6524 + 0.8460 \ln K_i + 0.3397 \ln L_i$$

$$R_{UR}^2 = 0.9951 \quad RSS_{UR} = 0.0136$$

- Restricted model:

$$\ln \left(\frac{\hat{Y}_i}{L_i} \right) = -0.4947 + 1.0153 \ln \left(\frac{K_i}{L_i} \right)$$

$$R_{UR}^2 = 0.9777 \quad RSS_{UR} = 0.0166$$

Both models has 20 observations. Perform the test from both RSS and R^2 with the hypothesis here.

(1) State your hypothesis

- $H_0: \beta_K + \beta_L = 1$ or the restriction is valid
- $H_a: \beta_K + \beta_L \neq 1$ or the restriction is not valid

(5) Restricted Least Squares (RLS)

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Regression results (n=20)

- Unrestricted model:

$$\ln \hat{Y}_i = -1.6524 + 0.8460 \ln K_i + 0.3397 \ln L_i$$

$$R_{UR}^2 = 0.9951 \quad RSS_{UR} = 0.0136$$

- Restricted model:

$$\ln \left(\frac{\hat{Y}_i}{\hat{L}_i} \right) = -0.4947 + 1.0153 \ln \left(\frac{K_i}{L_i} \right)$$

$$R_{UR}^2 = 0.9777 \quad RSS_{UR} = 0.0166$$

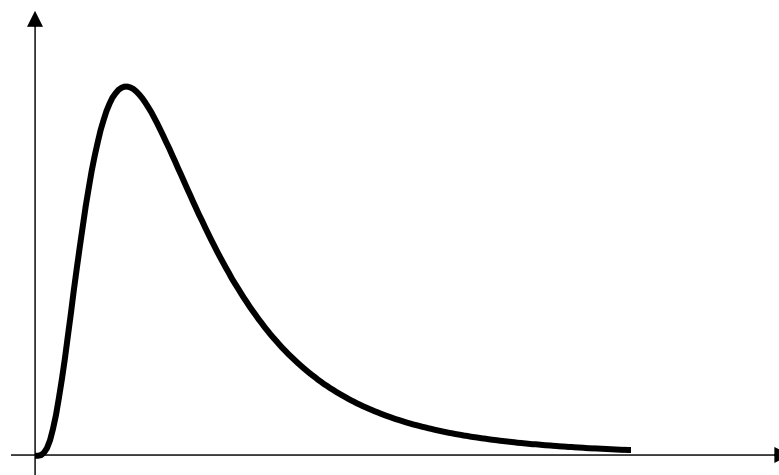
(2) Calculate test statistics

$$\bullet F_{cal} = \frac{RSS_R - RSS_{UR}/m}{RSS_{UR}/(n-k_{UR})} =$$

$$\bullet F_{cal} = \frac{(R_{UR}^2 - R_R^2)/m}{(1 - R_{UR}^2)/(n - k_{UR})} =$$

(3) Pick an α and state decision rules

- $\alpha = 0.05$
- $F_{upper, \alpha}(1, 7) =$

Simplified F-distribution

(4) Conclude the result

(6) General F testing

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As we can see that the F test can be utilize in multiple ways of comparing two competing models, we can also set up the test for “**constrained**” and “**unconstrained**” models as seen in the example below. First, we start from the “unconstrained” model

$$\bullet \ln Y_i = \ln \beta_1 + \beta_2 \ln X_{2i} + \beta_3 \ln X_{3i} + \beta_4 \ln X_{4i} + \beta_5 \ln X_{5i} + u_i$$

where Y_i is per capita consumption of chicken

X_{2i} is real disposable per capita income

X_{3i} is real retail price of chicken

X_{4i} is real retail price of pork

X_{5i} is real retail price of beef

β_4 and β_5 refer to cross-price elasticity of chicken and pork and chicken and beef. Thus,

- If $\beta_4 / \beta_5 > 0$, chicken and pork / beef are substitutable products.
- If $\beta_4 / \beta_5 < 0$, chicken and pork / beef are complementary products.
- If $\beta_4 / \beta_5 = 0$, chicken and pork / beef are unrelated products.

If we want to test that how chicken and pork / beef are related, we can set up a hypothesis as such.

$$\bullet H_0: \beta_4 = \beta_5 = 0$$

$$\bullet H_a: \text{otherwise}$$

If we cannot reject the null hypothesis, the model become “constrained” as

$$\bullet \ln Y_i = \ln \beta_1 + \beta_2 \ln X_{2i} + \beta_3 \ln X_{3i} + u_i$$

The test is be similar to the restricted and unrestricted model. Moreover, we can use the R^2 approach here since the Y_i is the same for both the constrained and unconstrained model. Test statistics can be calculated as follows.

$$\bullet F_{cal} = \frac{RSS_R - RSS_{UR}/m}{RSS_{UR}/(n-k_{UR})} \sim F_{(m, n-k_{UR})}$$

$$\bullet F_{cal} = \frac{(R_{UR}^2 - R_R^2)/m}{(1 - R_{UR}^2)/(n-k_{UR})} \sim F_{(m, n-k_{UR})}$$

Note that we use the same notations compared to the restricted model testing so that we don’t have to create another complication. Hence, m is number of coefficient constrained. R and UR denote values from constrained and unconstrained model respectively.

(6) General F testing

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Given the regression results are as follows, we proceed the test as usual.

Regression results (n=23)

• Unconstrained model:

$$\ln \hat{Y}_i = 2.19 + 0.34 \ln X_{2i} - 0.50 \ln X_{3i} + 0.15 \ln X_{4i} + 0.09 \ln X_{5i}$$

$$R_{UR}^2 = 0.9823$$

• Constrained model:

$$\ln \hat{Y}_i = 2.03 + 0.45 \ln X_{2i} - 0.38 \ln X_{3i}$$

$$R_R^2 = 0.9801$$

(1) State your hypothesis

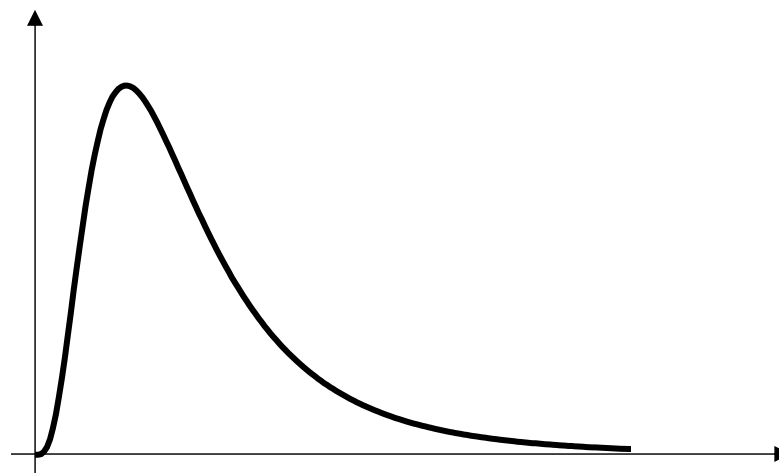
- $H_0: \beta_4 = \beta_5 = 0$
- H_a : otherwise

(2) Calculate test statistics

$$\bullet F_{cal} = \frac{(R_{UR}^2 - R_R^2)/m}{(1 - R_{UR}^2)/(n - k_{UR})} =$$

(3) Pick an α and state decision rules

- $\alpha = 0.05$
- $F_{upper, \alpha}(2, 18) =$

Simplified F-distribution**(4) Conclude the result**

(7) Testing for structural change: The Chow Test

Many times in our history, we had always faced with “**structural change**”, a major shift in the basic how market works and how agents interact. The shift can be both internal or external.

In Thailand, we may consider opposing party getting elected as one of them, with a different approach or policy set. Major flood in 2011 may alter how business choose location established.

In econometrics term, it refers to a change in parameter, either the intercept or slopes, due to socio-political shift.

Consider when there is a pool of data, ranging a long periods of time, estimation the whole data can be misleading as it reveals the average values of estimator within that range.

The Chow Test attempts tackling this problem, to test that **if there is any structural shift sometime in our data or not.**

For this example, we consider saving-income relationship between the second oil shock in the 1980s. Data cover 1970-1995. It is assumed that we can separate between pre and post 1982, when the unemployment rate is at the highest.

Setup

SRF from 1970-1981 (Eq. 1)

$$\bullet Y_t = \lambda_1 + \lambda_2 X_t + u_{1t} \quad n_1 = 12$$

SRF from 1982-1995 (Eq. 2)

$$\bullet Y_t = \gamma_1 + \gamma_2 X_t + u_{2t} \quad n_2 = 14$$

SRF from pooled data 1970-1995 (Eq. 3)

$$\bullet Y_t = \beta_1 + \beta_2 X_t + u_t \quad n = (n_1 + n_2) = 26$$

where Y is savings and X is income. The slope parameter represents

Assumptions

(1) $u_{1t} \sim N(0, \sigma^2)$ and $u_{2t} \sim N(0, \sigma^2)$

(2) u_{1t} and u_{2t} are independently distributed.

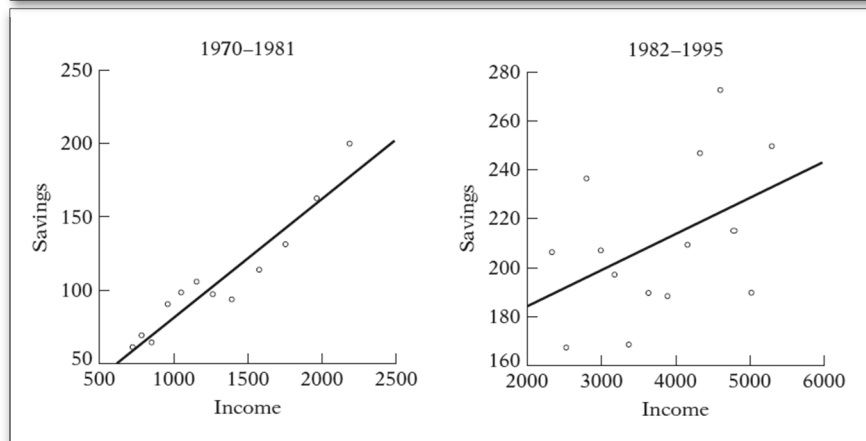
The Chow Test still relies on ANOVA, comparing a restricted model with an unrestricted model.

In this case, the **restricted** one assumes $\lambda_1 = \gamma_1$ and $\lambda_2 = \gamma_2$, or the Eq.3, while the **unrestricted** one allows different values of λ and γ .

(7) Testing for structural change: The Chow Test

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Income-saving from two periods



Plotting the graphs above seems to suggest that there is a structural change in people saving behavior, due to the slope change.

Procedures

(1) State a hypothesis

- $H_0: \lambda_1 = \gamma_1$ and $\lambda_2 = \gamma_2$
- H_a : otherwise

(2) Estimate Eq. 3 or the restricted model. Retrieve

- $RSS_R = RSS_3$ or restricted sum of squares.
- D.f. is straightforwardly $n_1 + n_2 - k$

(3) Estimate Eq. 1 and Eq.2. Retrieve

- $RSS_{UR} = RSS_1 + RSS_2$ or unrestricted sum of squares.
- D.f. for this model is $n_1 + n_2 - 2k$

Note that k is the number of parameter estimated.

(4) Calculate F statistics by

$$F_{cal} = \frac{RSS_R - RSS_{UR}/k}{RSS_{UR}/(n_1 + n_2 - 2k)} \sim F_{(k, n_1 + n_2 - 2k)}$$

(5) Concluding the results,

- We **can** reject the null hypothesis if $F_{cal} > F_{upper}$, meaning that it is either $\lambda_1 \neq \gamma_1$ or $\lambda_2 \neq \gamma_2$ or both. Therefore, it implies a structural change within these two periods.
- We **cannot** reject the null hypothesis if $F_{cal} < F_{upper}$, meaning that $\lambda_1 = \gamma_1$ and $\lambda_2 = \gamma_2$. So, we cannot make sure that there is a structural change within this period.

(7) Testing for structural change: The Chow Test

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Example: Given the regression results are

- Eq. 1: $\hat{Y}_t = 1.0161 + 0.0803X_t$

$$R_1^2 = 0.9021 \quad RSS_1 = 1785.032 \quad n_1 = 12$$

- Eq. 2: $\hat{Y}_t = 153.4947 + 0.0148X_t$

$$R_2^2 = 0.2971 \quad RSS_2 = 10005.22 \quad n_2 = 14$$

- Eq. 3: $\hat{Y}_t = 62.4226 + 0.0376X_t$

$$R_3^2 = 0.7672 \quad RSS_3 = 23248.30 \quad n_3 = 26$$

Let's skip hypothesis statement and calculate each value we need.

(1) Restricted and unrestricted sum of squares

- $RSS_R =$

- $RSS_{UR} =$

(2) F statistics

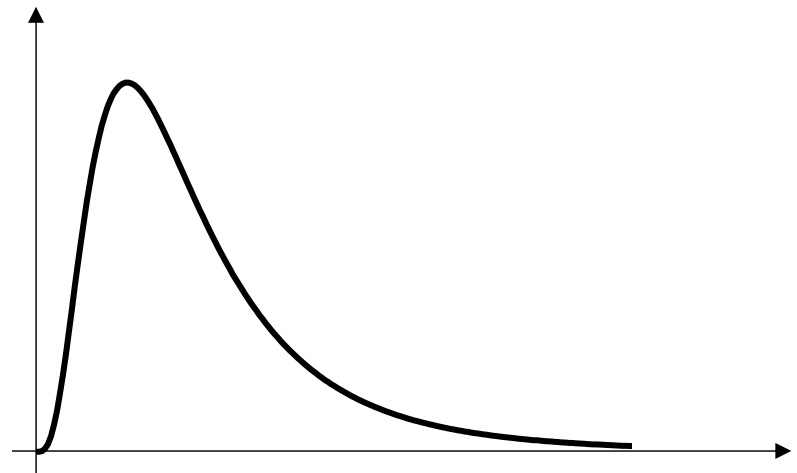
- $F_{cal} = \frac{RSS_R - RSS_{UR}/k}{RSS_{UR}/(n_1 + n_2 - 2k)} =$

(3) Pick an α and state decision rules

- $\alpha = 0.05$

- $F_{upper,\alpha}(2,22) =$

Simplified F-distribution



(4) Concluding the results,

(7) Testing for structural change: The Chow Test

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Limitations of The Chow Test

- The assumption of u_{1t} and u_{2t} being independently distributed should (or must) be tested, see the further explanation on the test on page 258.
- The Chow Test only reveals difference between two models, which means that there might be a difference either in the intercept, slope or both. (The next topic of dummy variables will address this issue)
- We need to assume that we know exactly where or when is the structural break point. If the speculation is not completely correct, the result of the test maybe controversial.