Example of midterm exam questions

3. $f(x,y) = 2x^4 + y^4 - 2x^2 - 2y^2$. Identify all the critical point(s) and classify them.

4. $f(x,y) = x^3 + y^3 + 2x^2 + 2y^2$ subject to $x^2 + y^2 = 1$
   a) Use Lagrange multiplier method to find the maximum value of the function under the given constraint.
   b) If the constraint is changed to $x^2 + y^2 = 0.95$, estimate the new maximum value of the function.

5. $f(x,y) = y(x-1)$ subject to $(x-1)^2 + y^2 \leq 1$ and $y - 2x \leq -1$
   a) Use Lagrange multiplier method to determine the maximum value of the function under given constraints.
   b) If the function is changed to $f(x,y) = y[(0.9)x - 1]$, estimate the new maximum value of the function.
   c) If the first constraint is changed to $(x-1.1)^2 + y^2 \leq 1$, estimate the new maximum value of the function.

6. Determine the **absolute minimum value** of the function $f(x,y) = \frac{16}{x} + \frac{16}{y} + 2xy$ subject to $x \leq 8$, $y \geq 0$ and $y \leq \frac{1}{2}x$ using the **Extreme-Value theorem**.

7. Prices of houses is dependent on the size of usable area from the data collected in Pathumthani as shown in table below.

Size of usable area (m^2)	Price of the house (THB)
80	700,000
110	930,000
140	1,310,000
150	1,400,000
180	1,620,000
240	2,200,000
270	2,610,000
300	2,990,000
350	3,630,000

Use the **least square approximation** to predict the price of a house with the usable area of 210 m^2.

4. Find maximum value of the function

$f(x,y,z) = x^2 - y^2 + 2xy - \frac{z^2}{2} + 6$

subject to $2x + 2y - z = 1$ and $x + y + z \leq 2$

If the constraint is changed to $2x + 2y - z = 0.9$, estimate the new maximum value.

5.1 Find maximum value of the function $f(x,y) = x^2 - 6x + y^2 + 9$ subject to $x - y \leq 1$ and $x^2 + y^2 \leq 4$

5.2 If one of the constraints in 5.1 is changed to $x^2 + 0.8y^2 \leq 4$, use envelope theorem to estimate the new maximum value of the function

5.3 If the function in 5.1 is changed to $f(x,y) = x^2 - 5.6x + y^2 + 9$, use envelope theorem to estimate the new maximum value of the function

7. A market research study conducted for Century Communications provided the following data based on the project monthly sales x (in thousands) of Century's DVD version of a box-office hit adventure movie with a proposed wholesale unit price of p dollars.

x	2.2	5.4	7.0	11.5	14.6
p	38	36	34.5	30	28.5

Use linear regression to estimate the wholesale unit price if the project monthly sales are 22.2