

1. In Table 1. X_i is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

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Table 1

Student	Y_i	X_i	$X_i Y_i$	X_i^2	$X_i - \bar{X}$
1	2.8	63	176.4	3963	-14.625
2	3.4	72	244.8	5184	-5.625
3	3.0	78	234	6084	0.375
4	3.5	81	283.5	6561	3.375
5	3.6	87	313.2	7569	9.375
6	3.0	75	225	5625	-2.625
7	2.7	75	202.5	5625	-2.625
8	3.7	90	333	8100	12.375
SUM	25.7	621	2012.4	49,713	

$$\bar{Y} = 3.2125, \bar{X} = 7.7625$$

$$E(Y|X_i) = \beta_1 + \beta_2 X_i$$

- 1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of β_1 and β_2 . Interpret the regression.

$$\text{Find } \hat{\beta}_1 \text{ \& } \hat{\beta}_2; \hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}, \hat{\beta}_2 = \frac{n \sum_{i=1}^n X_i Y_i - \sum_{i=1}^n Y_i \sum_{i=1}^n X_i}{n \sum_{i=1}^n X_i^2 - \left(\sum_{i=1}^n X_i \right)^2}; n=8$$

$$\hat{\beta}_2 = \frac{8(2012.4) - 25.7(621)}{8(49,713) - (621)^2}$$

$$= \frac{16099.2 - 15959.7}{397,704 - 385,641}$$

$$\hat{\beta}_2 = 0.0341$$

$$\text{so } \hat{\beta}_1 = 3.2125 - (0.0341)(7.7625)$$

$$= 0.5655$$

$$\therefore \hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$$

$$\hat{Y}_i = 0.5655 + 0.0341(X_i)$$

so every econometrics exam point increase by 1, the GPA will increase by 0.0341

- 1.2 Find $\hat{Y}_i$ and $\hat{u}_i$ and show that $\sum_{i=1}^n \hat{u}_i \approx 0$

$$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i, \hat{u}_i = Y_i - \hat{Y}_i; \hat{\beta}_1 = 0.5655, \hat{\beta}_2 = 0.0341$$

i	$\hat{Y}_i$	$\hat{u}_i$	$\hat{u}_i^2$
1	2.7139	0.0862	0.0074
2	3.0207	0.3793	0.1439
3	3.2253	-0.2253	0.0508
4	3.3276	0.1724	0.0297
5	3.5322	0.0679	0.0046
6	3.123	-0.123	0.0151
7	3.123	-0.423	0.1789
8	3.6345	0.0655	0.0043

$$\sum_{i=1}^8 \hat{u}_i = 0.0862 + 0.3793 - 0.2253 + 0.1724 + 0.0679 - 0.123 - 0.423 + 0.0655$$

$$= -0.0001 \rightarrow \approx 0$$

1.3 Find $\text{var}(\hat{u}_i)$, $\text{var}(\hat{\beta}_1)$, and $\text{var}(\hat{\beta}_2)$

$$\text{var}(\hat{u}_i) = \hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{0.43472597}{8-2} = 0.0725$$

$$\text{var}(\hat{\beta}_1) = \frac{\hat{\sigma}^2 \sum x_i^2}{n \sum (x_i - \bar{x})^2} = \frac{0.0725(48717)}{9(511.875)} = 0.8625$$

$$\text{var}(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\sum (x_i - \bar{x})^2} = \frac{0.0725}{511.875} = 0.000142$$

2. Data is listed in the table

X_i	Y_i	$X_i Y_i$	X_i^2	$X_i - \bar{X}$	
10	0	0	100	-10	
12	2	24	144	-8	
14	5	70	196	-6	
16	6	96	256	-4	
18	7	126	324	-2	
22	10	220	484	2	
24	10	240	576	4	
26	15	390	676	6	
28	16	448	784	8	
30	20	600	900	10	
Σ	200	91	2214	4440	0
mean	$\frac{200}{10} = 20$	$\frac{91}{10} = 9.1$	221.4	444	0
$(\bar{X}) (\bar{Y})$					

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

Find $\hat{\beta}_1$ & $\hat{\beta}_2$; $\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$, $\hat{\beta}_2 = \frac{n \sum x_i y_i - \sum y_i \sum x_i}{n \sum x_i^2 - (\sum x_i)^2}$, $n=10$, $\bar{X}=20$, $\bar{Y}=9.1$

$$\hat{\beta}_2 = \frac{10(2214) - 200(91)}{10(4440) - (200)^2}$$

$$= \frac{22140 - 18200}{44400 - 40000}$$

$$= \frac{3940}{4400} = \frac{197}{220} = 0.8955$$

$$\hat{\beta}_1 = 9.1 - (0.8955)(20) = -8.91$$

$$\therefore Y_i = -8.91 + 0.8955 X_i$$

then when X increase 1 unit

Y will increase 0.8955 unit.

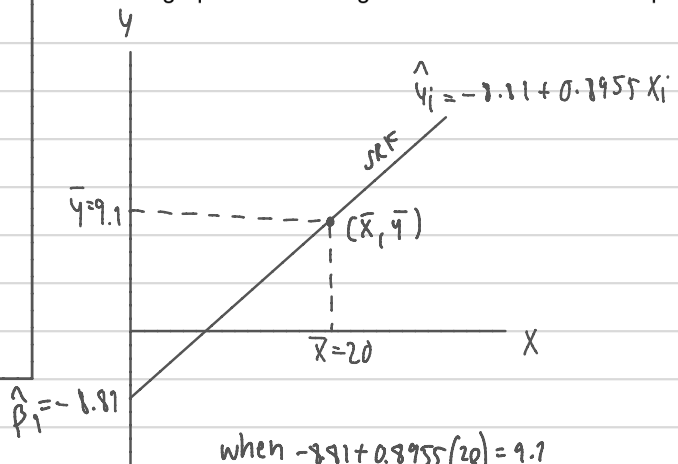
2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

$$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i, \hat{u}_i = Y_i - \hat{Y}_i; \hat{\beta}_1 = -8.91, \hat{\beta}_2 = 0.8955$$

i	$\hat{Y}_i$	$\hat{u}_i$	$\hat{u}_i^2$
1	0.145	-0.145	0.021
2	9.36	0.064	0.0041
3	3.727	1.273	1.6205
4	5.518	0.482	0.2323
5	7.309	-0.309	0.0955
6	10.691	-0.891	0.7939
7	12.682	-2.682	7.1931
8	14.473	0.527	0.2777
9	16.264	-0.264	0.0697
10	18.055	1.945	3.7830

$$\sum \hat{u}_i = 0.145 \approx 0$$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?



$$\text{when } -8.91 + 0.8955(20) = 9.1$$

$\therefore \hat{Y}_i = \bar{Y}$ so this line pass $(\bar{X}, \bar{Y})$
(be a member of SRF)

2.4 If $X_i = 18$, what is the predicted Y ?

$$\begin{aligned} X_i = 18, \hat{y}_i &= \hat{\beta}_1 + \hat{\beta}_2 X_i \\ &= -8.81 + (0.8955)(18) \\ &= 7.309 \end{aligned}$$

2.5 Find $\text{var}(\hat{u}_i)$, $\text{var}(\hat{\beta}_1)$, $\text{var}(\hat{\beta}_2)$

$$\text{var}(\hat{u}_i) = \frac{\sum \hat{u}_i^2}{n-2} = \frac{14.0908}{10-2} = 1.7614$$

$$\text{var}(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\sum (x_i - \bar{x})^2} = \frac{1.7614}{440} = 0.004$$

$$\text{var}(\hat{\beta}_1) = \frac{\hat{\sigma}^2 \sum x_i^2}{n \sum (x_i - \bar{x})^2} = \frac{1.7614 (440)}{10 (440)} = 1.7774$$

3. Consider the below regression function: consider the two-variable model

$$Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim \text{NIID}(0, \sigma^2)$$

Find an OLS estimator of β_1 . Then, provide a proof that this is an unbiased estimator.

Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

$$\begin{aligned} \text{Find } \hat{\beta}_2 \text{ that unbiased} &\rightarrow E(\hat{\beta}_2) = \beta_2 \\ \text{from } \hat{\beta}_2 &= \sum_{i=1}^n k_i y_i \\ &= \sum k_i (\beta_1 + \beta_2 X_i + u_i) \\ &= \sum (k_i \beta_1 + \beta_2 k_i X_i + k_i u_i) \\ &= \beta_1 \sum_{i=1}^n k_i + \beta_2 \underbrace{\sum k_i X_i}_1 + \sum k_i u_i \\ \hat{\beta}_2 &= \beta_2 + \sum_{i=1}^n k_i u_i \quad E(u_i | x_i) = 0 \\ E(\hat{\beta}_2) &= E(\beta_2) + E(\sum k_i u_i) \\ &= \beta_2 \end{aligned}$$

$$\begin{aligned} \text{from } \hat{\beta}_1 &= \bar{y} - \hat{\beta}_2 \bar{x} \\ \text{so, } E(\hat{\beta}_1) &= E(\bar{y} - \hat{\beta}_2 \bar{x}) \\ &= E(\bar{y}) - \bar{x} E(\hat{\beta}_2); \quad E(\bar{y}) = E\left(\frac{\sum y_i}{n}\right) \\ &= \frac{1}{n} E\left(\sum y_i\right) \\ &= \frac{1}{n} \sum \beta_1 + \beta_2 X_i \\ &= \frac{n \beta_1}{n} + \frac{\beta_2 \sum X_i}{n} \\ &= \beta_1 + \beta_2 \bar{x} \\ \text{Then } E(\hat{\beta}_1) &= \beta_1 \rightarrow \text{unbiased!} \end{aligned}$$

so $\hat{\beta}_2$ is unbiased estimator of true β_2