

EE431 Economics of Financial Markets and Institutions
 Problem Set 1: Debt Market and Structure of Interest Rate
 Solution

Instructions : Do all questions by using the present value tables.

- Describe the time value of money concept and how it affects your investment programs.

Answer. “The time value of money allows people to invest money over a period of time. At the end of the investment period, people receive not only what they have invested, but also the earnings (dividends and interest) the investment has earned. For most people, accumulation is built on the principle of investing small sums of money over a long period of time. Thus, the time value of money can really help people accumulate wealth.”

- Suppose the market interest rate is 10%, find the present value of the following securities

- Security A will pay \$1100, one year from now (at year 1).
- Security B will pay \$1210, two years from now (at year 2).
- Security C will pay \$1310. three years from now (at year 3).

Answer

$$\text{Present Value of Security A} = \frac{1,100}{(1 + 0.10)} = 1,000$$

$$\text{Present Value of Security B} = \frac{1,210}{(1 + 0.10)^2} = 1,000$$

$$\text{Present Value of Security C} = \frac{1,310}{(1 + 0.10)^3} = 984.22$$

- Suppose the market interest rate is 8%. Find the present value of a 20 years fixed payment loan, which pays 20,000 Baht at the end of each year.

Answer

$$\begin{aligned} \text{Present value of the fixed payment loan} &= \frac{20,000}{(1 + 0.08)} + \frac{20,000}{(1 + 0.08)^2} + \frac{20,000}{(1 + 0.08)^3} + \dots + \frac{20,000}{(1 + 0.08)^{20}} \\ &= 20,000 \times \left(\frac{1}{(1 + 0.08)} + \frac{1}{(1 + 0.08)^2} + \frac{1}{(1 + 0.08)^3} + \dots + \frac{1}{(1 + 0.08)^{20}} \right) \\ &= 20,000 \times \text{Present Value of An Annuity of 1 Baht due at the end of 20 periods} \\ &= 20,000 \times PVIFA_{8\%,20} \\ &= 20,000 \times 9.81815 \\ &= 196,363 \end{aligned}$$

Present value of the fixed payment loans is 196,363 Baht.

4. Suppose the market interest rate is 8%. A 10 years fixed payment loan with present value \$1,342,016. The equal fixed payment is paid annually. Find the annual payment of this fixed payment loan.

Let the annual payment = PMT

Answer.

$$\text{Present value of the fixed payment loans} = PMT \times PVIFA_{8\%,10}$$

$$1,342,016 = PMT \times 6.71008$$

$$\begin{aligned} PMT &= \frac{1,342,016}{6.71008} \\ &= 200,000 \end{aligned}$$

The annual payment is equal to \$200,000.

5. You are buying your first car for 1,250,000 Baht, and are paying \$250,000 as a down payment. You have arranged to finance the remaining 1,000,000 Baht 10-year mortgage with a 6% nominal interest rate and yearly payments. What are the equal yearly payments you must make?

Answer.

$$\begin{aligned} \text{Present Value of Installments} &= \text{installment per year} \times PVIFA_{6\%,10}, \\ 1,000,000 &= PMT \times PVIFA_{6\%,10}, \\ &= PMT \times 7.36009. \end{aligned}$$

Solve for PMT.

$$\begin{aligned} PMT &= \frac{1,000,000}{7.36009}, \\ &\approx 135,867.904. \end{aligned}$$

This is an amortizing loan is a loan where the principal of the loan is paid down over the life of the loan. Consider the following table.

Year	Beginning Balance	PMT	Interest Payment	Principal Payment	Ending Balance
1	1,000,000.00	135,867.90	60,000.00	75,867.90	924,132.10
2	924,132.10	135,867.90	55,447.93	80,419.97	843,712.13
3	843,712.13	135,867.90	50,622.73	85,245.17	758,466.95
4	758,466.95	135,867.90	45,508.02	90,359.88	668,107.07
5	668,107.07	135,867.90	40,086.42	95,781.48	572,325.60
6	572,325.60	135,867.90	34,339.54	101,528.36	470,797.23
7	470,797.23	135,867.90	28,247.83	107,620.07	363,177.16
8	363,177.16	135,867.90	21,790.63	114,077.27	249,099.89
9	249,099.89	135,867.90	14,945.99	120,921.91	128,177.99
10	128,177.99	135,867.90	7,690.68	128,177.22	0.76739

- Interest payment is equal to 5% times the beginning balance.
- Principal payment is equal to PMT minus interest payment.
- The last ending balance is 0.76739. This is because we use the PV table which cannot provide the same level of accuracy as financial calculators or computer software because they use factors that are rounded off. Theoretically, it has to be equal to 0.
- Notice that the payments are the same each year. Interest payment is decreasing. Principal payment is increasing.

6. What is yield to maturity (YTM)?

Answer. Yield to maturity is the rate that equates the price of the bond with the discounted cashflows to maturity.

$$\text{Mathematically, Bond Price} = \sum_t \frac{\text{Cash flow received at time } t}{(1 + \text{discount rate})^t} = \sum_t \frac{CF_t}{(1 + k)^t}.$$

7. Consider a 5.5% coupon bond with the par value of 1,000 Baht, selling for 863.38505 Baht. The bond will mature in 15 years. The coupon payment is made annually.

(a) Calculate the yield to maturity.

Answer

Yield to maturity is defined as the discount rate that equates present value of cashflows from bonds to its market price.

$$\begin{aligned} \text{Bond Price} &= \sum_t \frac{CF_t}{(1 + k)^t}, \\ 863.38505 &= \frac{55}{(1 + YTM)} + \frac{55}{(1 + YTM)^2} + \dots + \frac{55}{(1 + YTM)^{15}} + \frac{1,000}{(1 + YTM)^{15}}, \\ &= 55 \times PVIFA_{YTM\%,15} + 1,000 \times PVIF_{YTM\%,15}. \end{aligned}$$

The yield to maturity usually cannot be solved directly. It is determined by using trial and error or iterative method. The method consists of a sequence of iterations, involving repeated calculations.

The bond in the question is sold at discount. Hence, the coupon rate is lower than the discount rate.

Therefore, guess the initial value of yield to maturity as 6%. Then,

$$\begin{aligned} \text{Bond Price} &= 55 \times PVIFA_{6\%,15} + 1,000 \times PVIF_{6\%,15}, \\ &= (55 \times 9.71225) + (1,000 \times 0.41727), \\ &= 534.17375 + 417.27, \\ &= 951.44375. \end{aligned}$$

$951.44375 > 863.38505$. Therefore, we increase the trial discount rate to 7% and recalculate.

$$\begin{aligned} \text{Bond Price} &= 55 \times PVIFA_{7\%,15} + 1,000 \times PVIF_{7\%,15}, \\ &= (55 \times 9.10791) + (1,000 \times 0.36245), \\ &= 500.93505 + 362.45, \\ &= 863.38505. \end{aligned}$$

Therefore, yield to maturity of this bond is equal to 7%.

(b) Later in the same year, the bond interest rate is fallen to 5%. What is the price of the 5.5% coupon bond?

Answer. Bond price is equal to its present value of cashflows. Therefore, the price of the bond can be calculated as follows.

$$\begin{aligned} \text{Bond Price} &= 55 \times PVIFA_{5\%,15} + 1,000 \times PVIF_{5\%,15}, \\ &= (55 \times 10.37966) + (1,000 \times 0.48102), \\ &= 570.8813 + 481.02, \\ &= 1,051.9013. \end{aligned}$$

Therefore, the price of the 5.5% coupon bond becomes 1,051.9013, when the interest rate is 5%.

We can see that when the interest rate is 7%, the bond price is 863.38505 Baht.

When the interest rate is 5%, the bond price is 1,051.9013.

Thus, when interest rate increases, the bond price decreases.

(When interest rate decreases, the bond price increases.)

8. Suppose an investor who has 1,000 Baht is considering buying one of 3 years bonds:

- Bond A has 1,000 Baht face value, 12% annual coupon, 10% yield to maturity.
- Bond B has 100 Baht face value, 10% annual coupon, 12% yield to maturity.

Assume that the investor can buy fractions of a bond. If the investor plans to hold the bond until its maturity, which bond the investor should buy to get more returns? Explain the reason.

Answer. Yield to maturity is the rate of return on a bond if held until maturity date. Bond A and Bond B have the same time to maturity. Bond B has a higher yield to maturity than Bond A. Since the investor plans to hold the bond until its maturity, the investor should buy Bond B to get more returns.

Note:

It is quite difficult to answer this question numerically. I do not expect a student to answer this question numerically. Basically, the rate of return on a financial asset is defined as the discount rate that equates its present value of cashflows with its market price. When we calculate yield to maturity, we assume that the bond is held to its maturity date. Therefore, yield to maturity is the the rate of return on a bond if held until maturity date. The question assumes that investor can buy any fraction of a bond. This means that investors can always use up his or her 1,000 Baht to the bond regardless of what the price of the bond is.

Bond A's price is equal to $120 \times PVIFA_{10\%,3} + 1,000 \times PVIF_{10\%,3} = 1,049.732$. With 1,049.732 Baht, you can buy 1 of Bond A. With 1,000 Baht, you can buy $\frac{1,000}{1,049.732} \approx 0.9526$ of Bond A.

You buy $\frac{1,000}{1,049.732}$ of Bond A, then you would receive $\left(\frac{1,000}{1,049.732}\right) \times 120$ at the end of the year for 3 years and $\left(\frac{1,000}{1,049.732}\right) \times 1,000$ at the end of year 3. You pay 1,000 Baht for these cashflows. The rate of return on this investment is the discount rate that solves this following equation.

Price (Baht invested) = Present Value of Cashflows,

$$1,000 = \left\{ \left[\left(\frac{1,000}{1,049.732} \right) \times 120 \right] \times PVIFA_{k\%,3} \right\} + \left\{ \left[\left(\frac{1,000}{1,049.732} \right) \times 1,000 \right] \times PVIF_{k\%,3} \right\}.$$

Rearrange,

$$1,000 = \left(\frac{1,000}{1,049.732} \right) \{120 \times PVIFA_{k\%,3}\} + \left(\frac{1,000}{1,049.732} \right) \{1,000 \times PVIF_{k\%,3}\},$$

$$\left(\frac{1,049.732}{1,000} \right) \times 1,000 = \{120 \times PVIFA_{k\%,3}\} + \{1,000 \times PVIF_{k\%,3}\},$$

$$1,049.732 = \{120 \times PVIFA_{k\%,3}\} + \{1,000 \times PVIF_{k\%,3}\}.$$

Then, the discount rate that solves the equation is equal to the yield to maturity of the bonds. In other words, *you get the same rate of return on the bond no matter how much (how many units) you buy the bond.*

The rate of return on Bond B from 1,000 Baht investment in Bond B can be calculated in the same way.

The rate of return from investing in a bond if held to maturity is equal to the bond's yield to maturity.

Therefore, for this question, the investor just simply goes for the one with the higher yield to maturity.