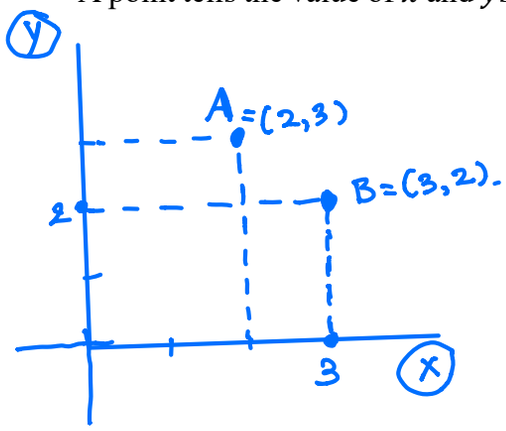
Graphs for Economics

A ***point*** in a graph of xy plane is identified by an ordered pair. $A = (2,3), B = (3,2)$ – coordinates

- A point tells the value of x and $y$simultaneously.



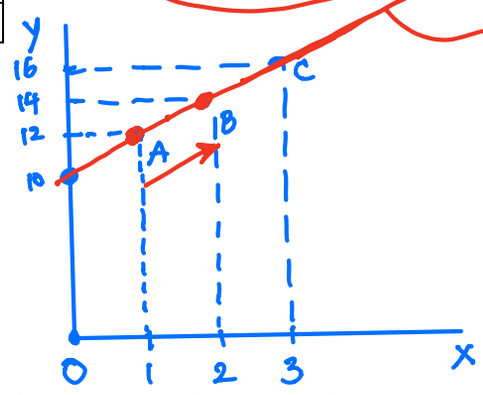
A ***line*** is a continuation of points. It shows the relationship between x and y.

Point	x	Y
	0	10
A	1	12
B	2	14
C	3	16

$\Leftrightarrow y = 10 + 2x$

linear !

described relationship between x and y.



$y = f(x)$

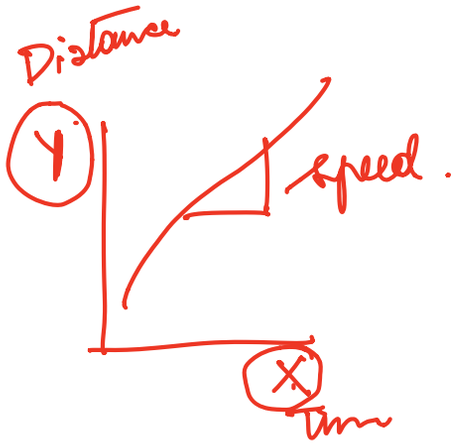
$\Delta x = 1 = 2 - 1.$

$\Delta y = 2 = 14 - 12.$

$\frac{\Delta y}{\Delta x} = \frac{2}{1} = 2 =$ slope

- A line can be thought as a function of x, $y = f(x)$
- A line can be linear or nonlinear.

A linear line is a straight line with constant slope



Slope = rate of change of y per unit change of x

$$= \frac{\Delta Y}{\Delta X}$$

For linear line, from A to B

$$\Delta y = 2 = 14 - 12$$

$$\Delta x = 1 = 2 - 1$$

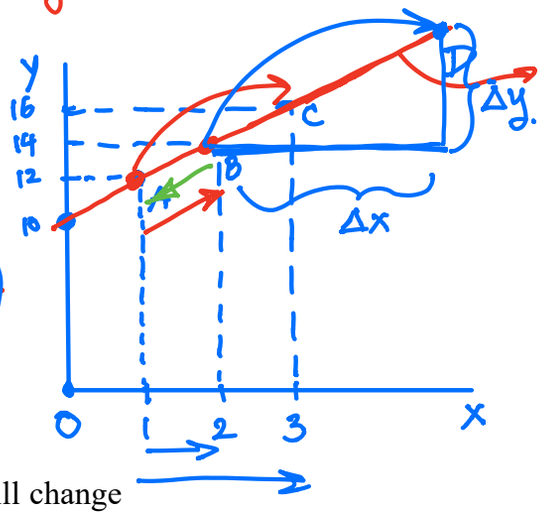
$$Slope = \frac{\Delta y}{\Delta x} = \frac{2}{1}$$ - if x increases by 1, y increases by 2.

From A to C,

$$\Delta y = 4 = 16 - 12$$

$$\Delta x = 2 = 3 - 1$$

$$Slope = \frac{\Delta y}{\Delta x} = \frac{4}{2} = 2$$

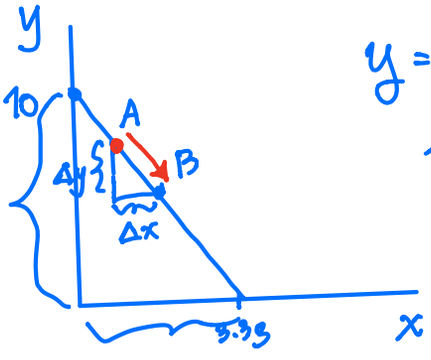


Linear line ⟹ Slope is constant

- x can change by any amount and y will change in such a way that we have same slope
- Slope is the same if we change from B to A.

$\Delta x = -1$
$\Delta y = -2$
$slope = \frac{\Delta y}{\Delta x} = \frac{-2}{-1} = 2$

- Slope > 0 ⟹ when x increases y also increases
  ⟹ x and y have positive relationship



$y = 10 - 3x$.

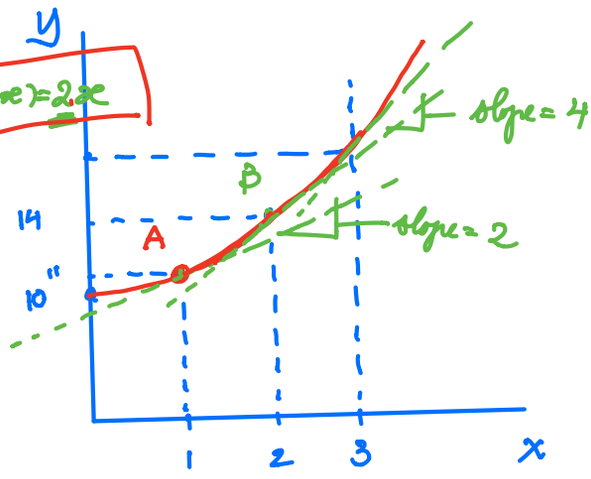
$slope = -3 < 0$

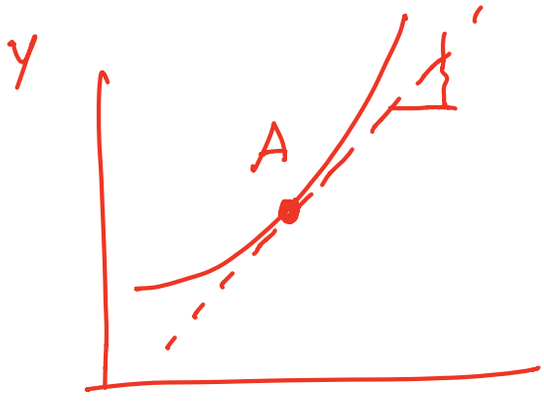
when x increases y decreases.

Nonlinear Line—Slope is not constant.

Example $y = 10 + x^2$

Point	x	y	slope
	0	10	0
A	1	11	2
B	2	14	4
C	3	19	6





slope of the curve x at A = slope of tangent line.

- We can find slope by drawing a straight line to be tangent to the point we want to find the slope. This line is called a ***tangent line***.
- Slope changes with the value of x. Thus slope can be called instantaneous rate of change because it is the rate of change at the instant at a particular value of x.

We can find the slope by taking derivative of the function:

$$y = f(x) = 10 + x^2$$
$$\frac{dy}{dx} = f'(x) = 2x$$

slope changes depending on the value of x.

- Note that in this example, the slope increases as x increases

Approximation of change of y as a result of change of x

- When $x_1 = 2, y_1 = 14$. If $\Delta x = 0.1$, we can approximate

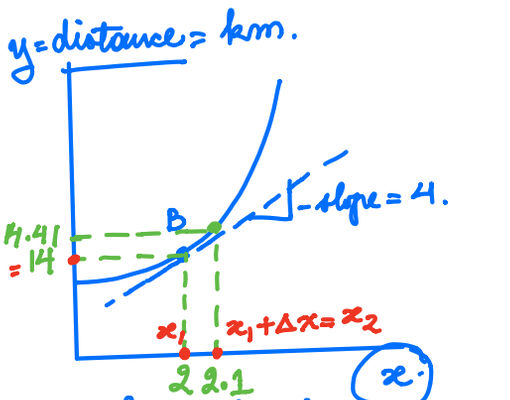
$$\Delta y \approx f'(x_1).\Delta x$$
$$= f'(2).0.1$$
$$= 2(2) \cdot 0.1 = 0.4$$

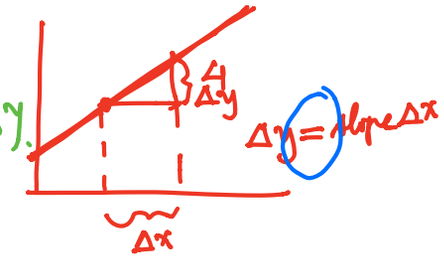
- What is the real Δy?

$$y_2 = f(2.1) = 10 + (2.1)^2 = 14.41$$
$$\Delta y = y_2 - y_1 = 14.41 - 14 = 0.41$$

- We underestimate the real change of y.
- What if $\Delta x = -0.2$? Approximate the change of y.

$$\Delta y \approx f'(x_1)\Delta x$$





HW Given $y = 10 + \sqrt{x}$,

a) Find the derivative $f'(x)$.

b) Fill in the table

Point	X	Y	$f'(x)$
	0	10	
A	1	11	
B	2	14	
C	3	19	

c) Does the slope increase as x increases?

d) Approximate the change in Y when $\Delta x = 0.2$ at $x_1 = 3$. Is the approximation under- or over-estimate?

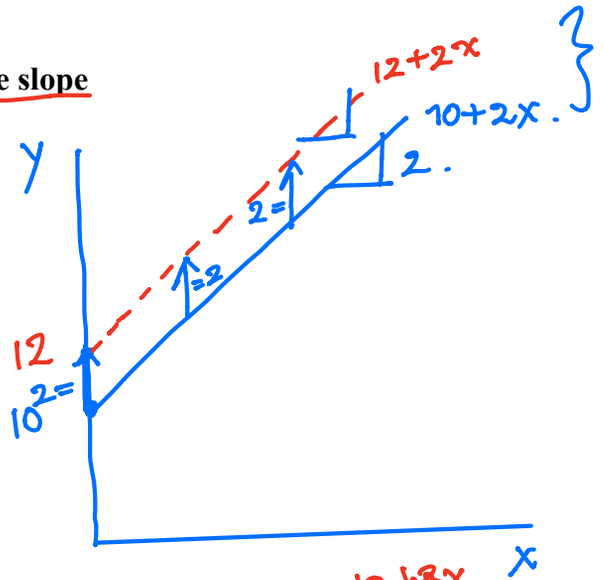
Note: If the function $f(x)$ is linear, the approximation is exact.

Shift of Graph

1) Linear with positive slope

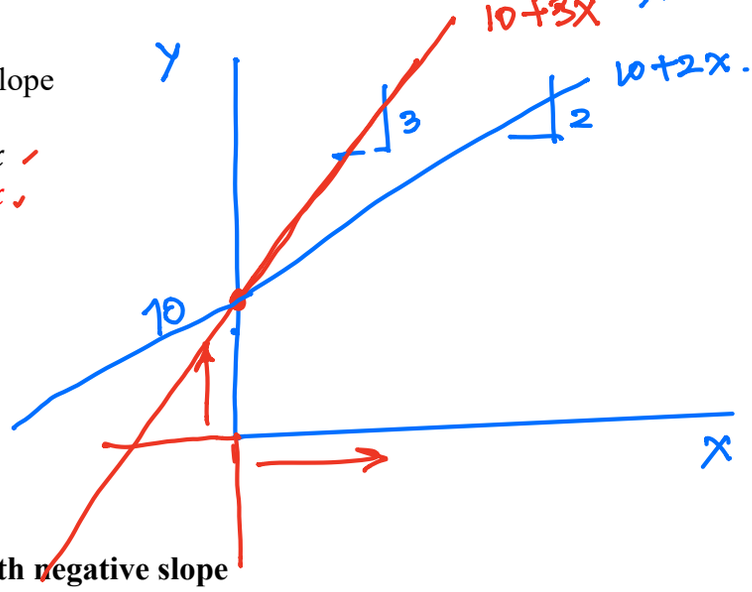
- Change of intercept

$y = 10 + 2x$
$y = 12 + 2x$



- Change of slope

$y = 10 + 2x$
$y = 10 + 3x$

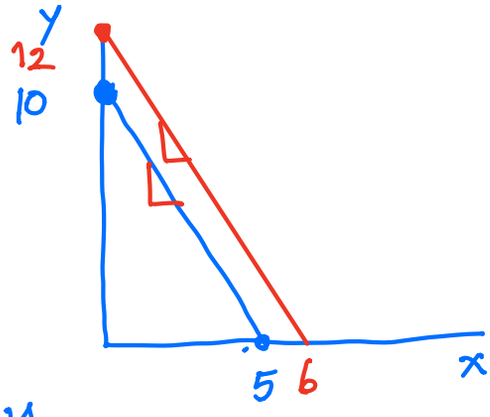


2) Linear with negative slope

- Change of intercept

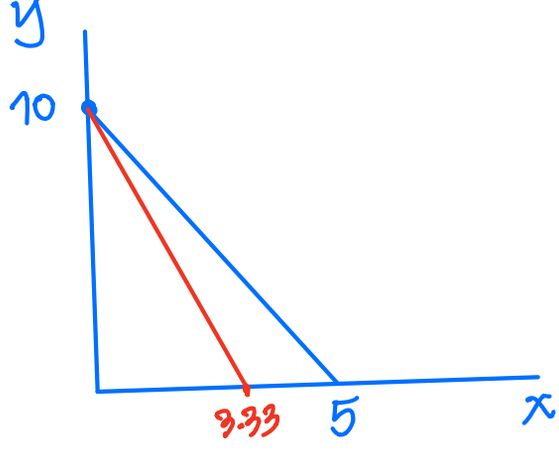
$y = 10 - 2x$
$y = 12 - 2x$

x=0, y=10
y=0, x=5



- Change of slope

$y = 10 - 2x$
$y = 10 - 3x$

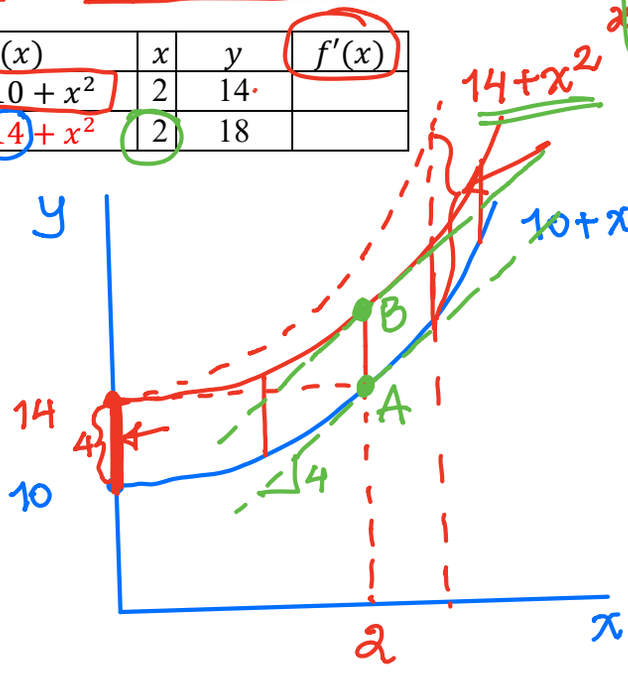


$f(x) = 10 + x^2$ $f'(x) = 2x$ $x=2$ $f'(2) = 4$

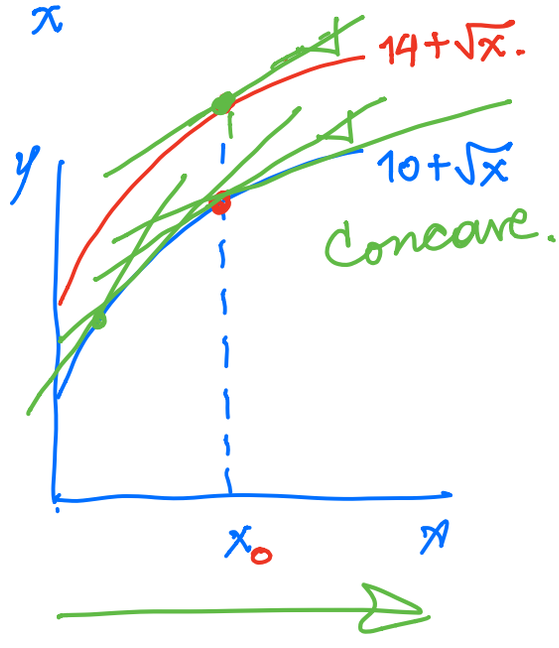
$f(x) = 14 + 2x^2$ $f'(x) = 2x$ $x=2, f'(2) = 4$

3) Nonlinear with change in the intercept

$f(x)$	x	y	$f'(x)$
$y = 10 + x^2$	2	14	
$y = 14 + x^2$	2	18	



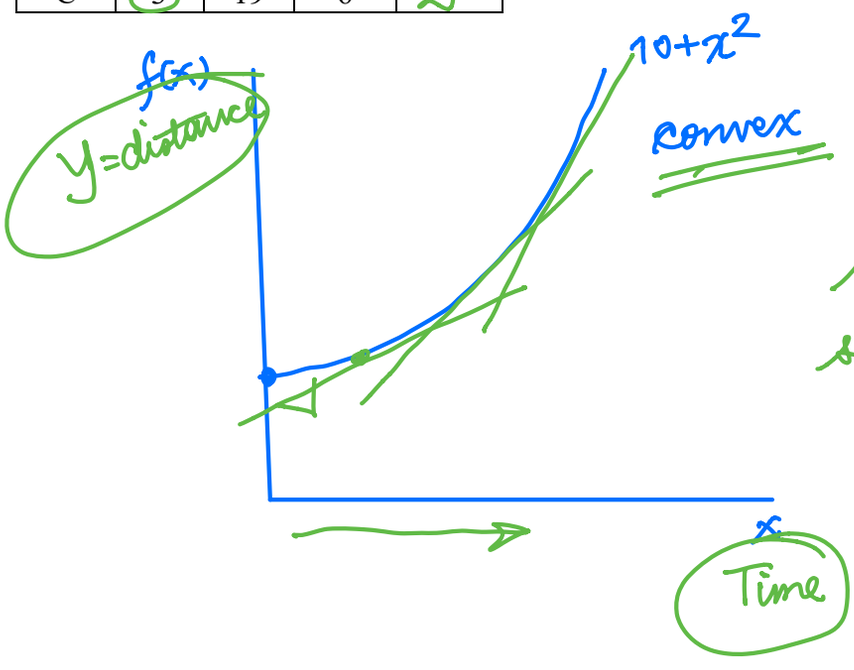
$f(x)$	x	y	$f'(x)$
$y = 10 + \sqrt{x}$	2	14	
$y = 14 + \sqrt{x}$	2	18	



Second-order Derivative: Slope of Slope

$$y = f(x) = 10 + x^2$$
$$f'(x) = 2x$$
$$f''(x) = 2$$

Point	x	y	$f'(x)$	$f''(x)$
	0	10	0	2
A	1	11	2	2
B	2	14	4	2
C	3	19	6	2



Slope is increasing as x increases.

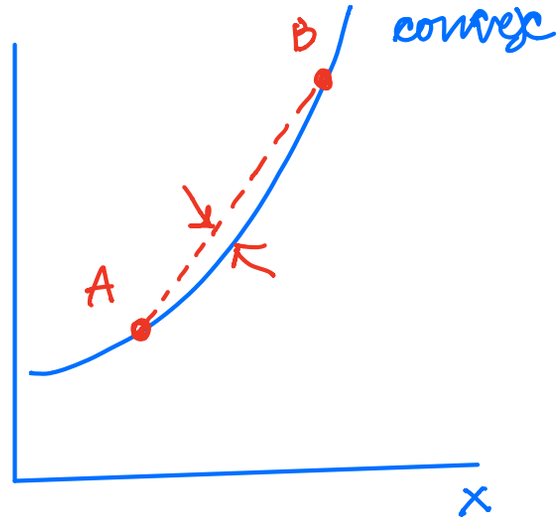
slope = speed
slope of slope = acceleration.

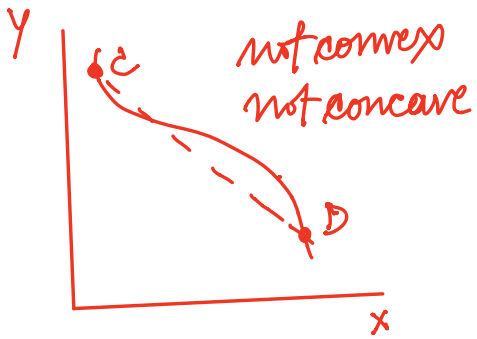
- The 2nd -order derivative indicates the curvature of the graph to be convex or concave.

HW Find the 2nd -order derivative of $y = f(x) = 10 + \sqrt{x}$ and fill in the table:

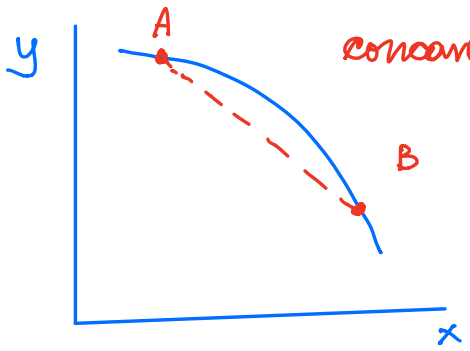
Point	x	y	$f'(x)$	$f''(x)$
	0	10		
A	1	11		
B	2	14		
C	3	19		

Plot the graph of y and $f'(x)$. Is $f'(x)$ linear?





not convex
not concave



concave.