EE312 Chapter 10
Long-term Economic Growth

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1 Introduction

- The standards of living in the long term depend on economic growth.

  Short-run fluctuations (boom and bust) tend to cancel out in the long run; average long-term growth rate matter a lot

- Growth outcome has markedly differed across countries; what determines the long-term economic growth, then?

- Differences in growth outcome have also contributed to the disparity of income-per-capita across countries: rich v.s. poor

  Distribution of income distribution within country and across countries

- In this section, we focus on academic explanations towards the determinants and process of long-term growth

  Models of economic growth

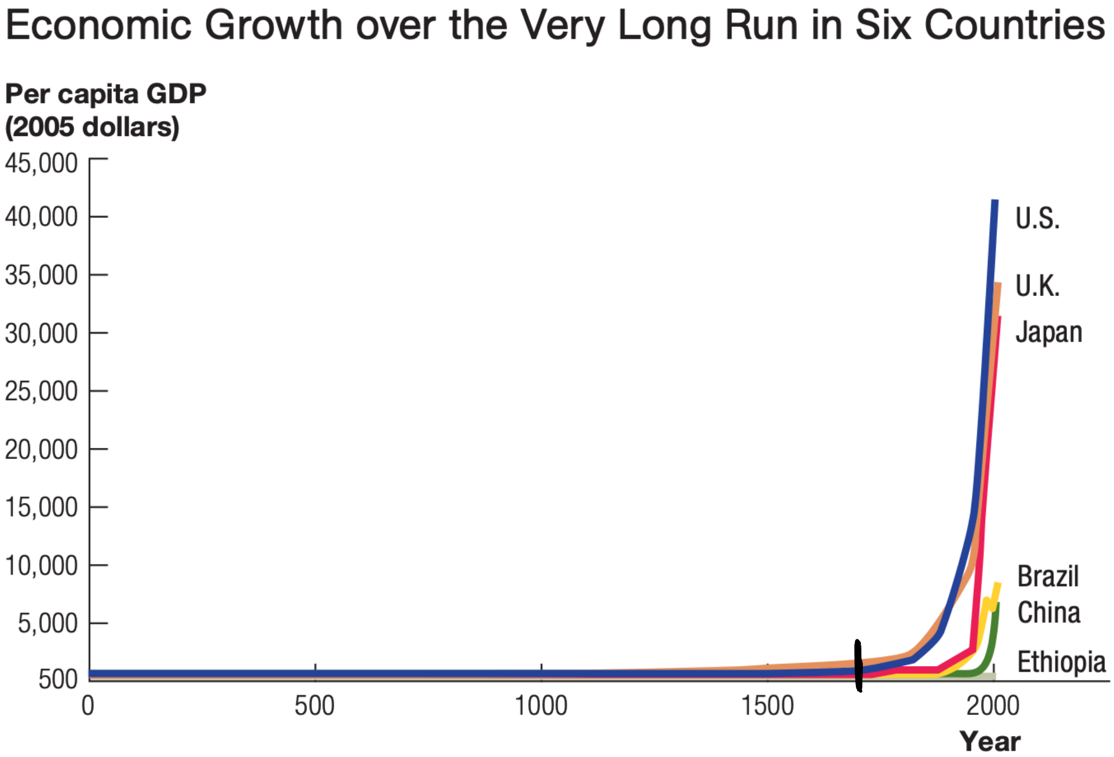
- We begin with documenting some stylized facts of historical growth over several past decades, and draw some issues that economists have aimed to explain

2 Stylized Facts on Long-term Economic Growth

2.1 Facts

1. Before the Industrial Revolution (1800), standards of living differed little over time and across regions.

2. Since the Industrial Revolution, per capita income growth has been sustained in developed countries.



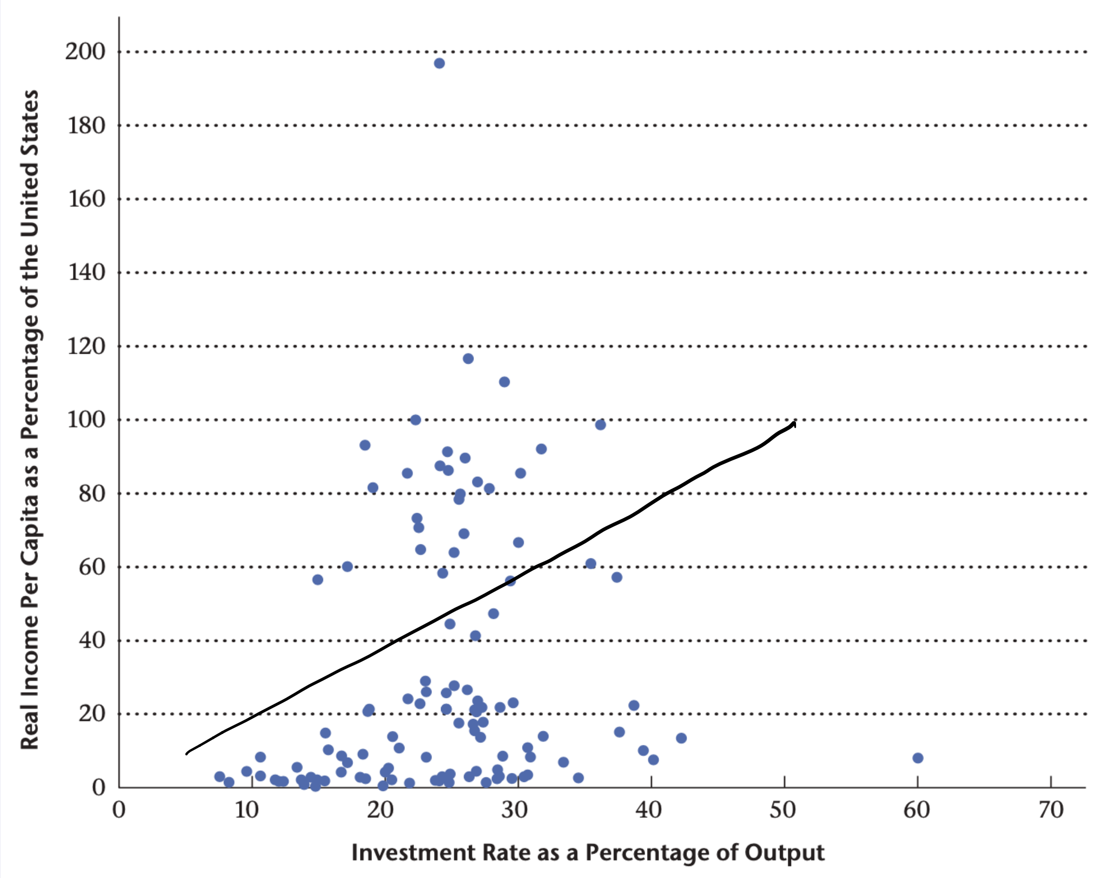
Source: Angus Maddison, "Statistics on World Population, GDP and Per Capita GDP, 1 AD–2006 AD."

3. The higher the rate of investment, the higher output per worker.

Figure 7.2 Real Income Per Capita vs. Investment Rate

The figure shows a positive correlation across the countries of the world, between the output per capita and the investment rate.

Source: Alan Heston, Robert Summers and Bettina Aten, *Penn World Table Version 7.0,* Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania, May 2011. Reprinted with permission.

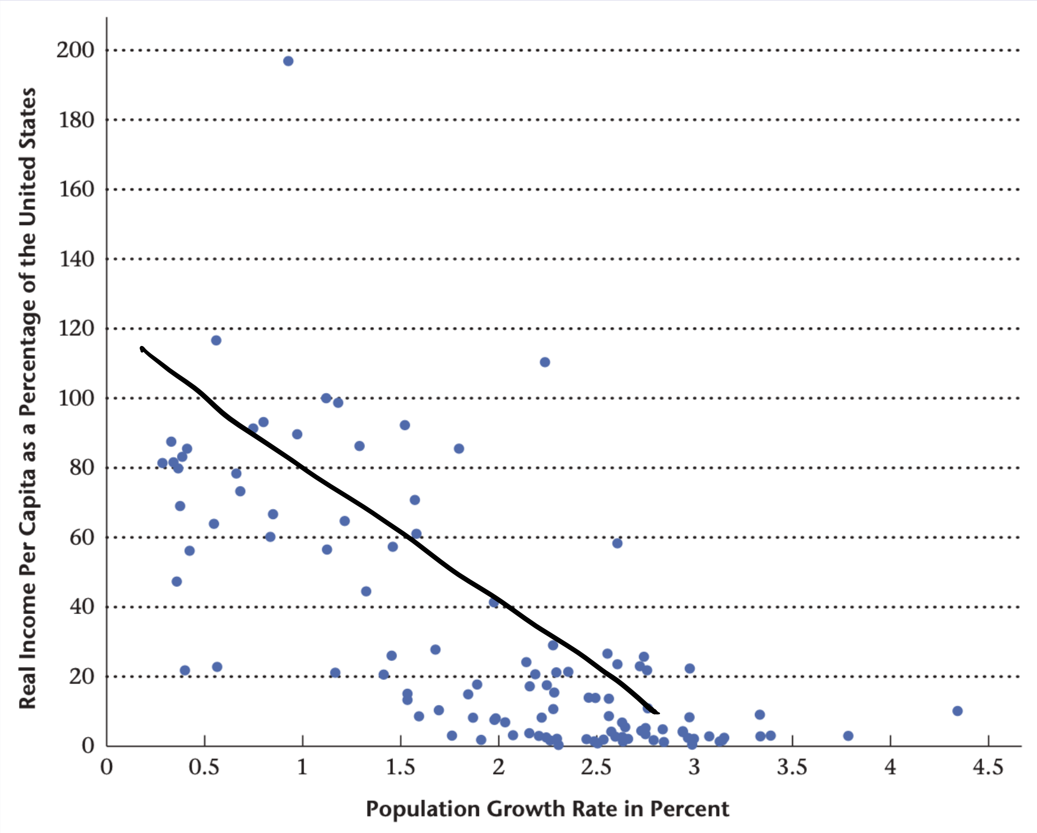


Y/N

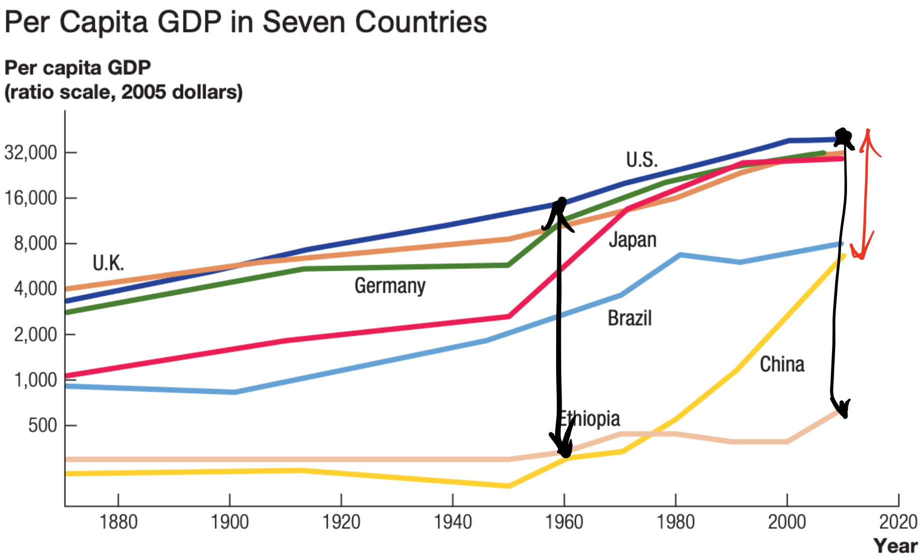
4. High population growth corresponds with low living standards.

Figure 7.3 Real Per Capita Income vs. the Population Growth Rate

Across the countries in the world, real per capita income and the population growth rate are negatively correlated.



5. Huge international differences in living standards increasingly and persistently widen between rich and poor countries



Source: Angus Maddison, "Statistics on World Population, GDP and Per Capita GDP, 1 AD–2006 AD." Observations are presented every decade after 1950 and less frequently before that as a way of smoothing the series.

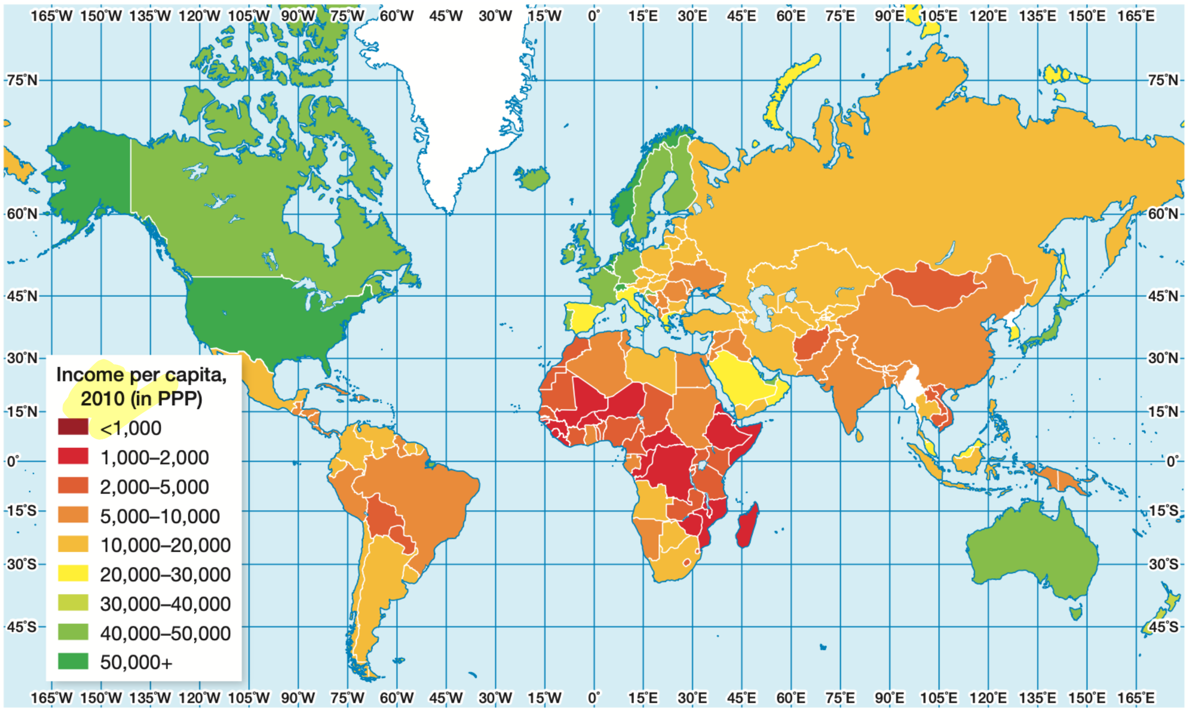


Exhibit 6.2 A Map of Income per Capita Across the World

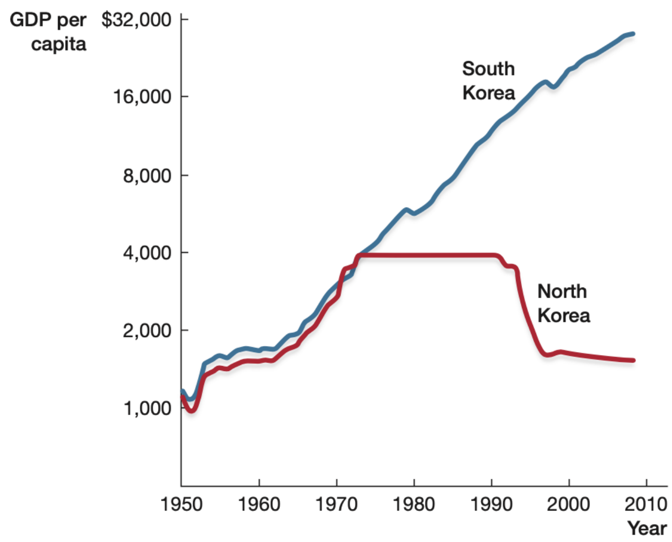
The large disparities in income per capita across countries are easily visible on this map, which also shows that the poorest countries are concentrated in Africa, parts of Asia, Central America, and the Caribbean.

Source: Data from Penn World Table; Alan Heston, Robert Summers and Bettina Aten, Penn World Table Version 7.1, Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania (Nov 2012).

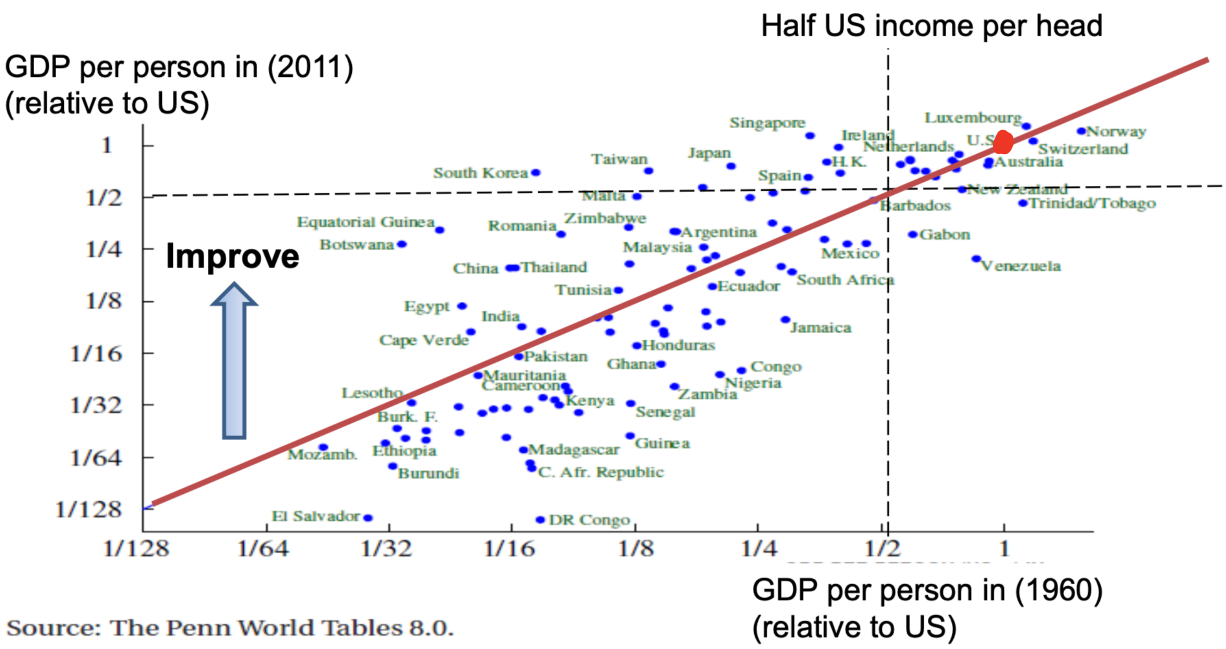
Exhibit 8.2 GDP per Capita in North and South Korea (in PPP-adjusted 2005 Constant Dollars)

The economic fortunes of North and South Korea, starting from parity in the 1940s when they were united, have diverged sharply. South Korea, with institutions mostly based on a market economy, has reached a high level of GDP per capita. In contrast, North Korea, under a communist dictatorship, has failed to grow and has less than 1/16th of the level of the GDP per capita of the South.

Source: Data from Maddison Project (1820–2010); J. Bolt and J. L. van Zanden, "The First Update of the Maddison Project; Re-Estimating Growth Before 1820." Maddison Project Working Paper 4 (2013).



6. Historical transition suggests that some have been successful in closing up the gap, and hence achieving a **convergence.** Those countries had experienced a miraculous growth path over the past several decades



Source: The Penn World Tables 8.0.

2.2 These facts raise questions:

Key motivated questions

- Positive questions:
  - What is the growth process?
  - Do we expect a convergence in growth? Pattern of convergence? All become rich at the end?
  - If not, what causes the divergence, and hence income disparities at the global scale?
- Normative questions:
  - Growth policies $\Rightarrow$ closing the gap!

Growth Models

- Solow growth: sustainable growth based on technological progress
  - Exogenous growth: technology is determined outside the model.
  - Growth convergence among countries.
- Endogenous growth: sustainable growth based on human capital/technology.
  - Growth engine is endogenous.
  - No certainty in growth convergence.

3 Solow Growth Model

3.1 Introduction

- The basis of all modern theories of growth.
- Long-term economic growth depends on one single factor –technological progress.
  - Rising total factor productivity (z).
  - Sustained improvement in living standards (real per capita income or output per worker).

3.2 Population growth

- Assume population grows exogenously at a constant rate.

  N = population (or workers) in the current period.

  N' = population in the future period.

  n = rate of population growth in the current period

$$N' = (1+n)N$$

where $n > -1$

3.3 Consumer

- Consumers = population = workers.
- Consumers supply labor in production.
- Consumers receive real output (Y) as (wage and dividend) income.
- Spend on consumption goods (C) and save a constant fraction (s) of Y as saving (S).

$$\boxed{Y = C + S}$$
$$S = sY$$
$$C = (1-s)Y$$

3.4 The representative firm

3.4.1 The Neoclassical Production Function

- The firm produces output using the current capital stock (K) and the current labor input (N).
- Assume:

  1. Constant returns to scale.

$$F(\lambda K, \lambda N) = \lambda F(K, N); \text{ for all } \lambda > 0$$

2. Positive and diminishing returns to private inputs:
   For all $K > 0$ and $N > 0$, F exhibits positive and diminishing marginal products with respective to each input.
3. Inada conditions:
   The marginal product of capital (or labor) approaches infinity as capital (or labor) goes to 0 and approaches 0 as capital (or labor) goes to infinity.

- Production function:
$$Y = zF(K, N)$$

- Per worker production function:

  Let $y = \frac{Y}{N}$ = output per worker and $k = \frac{K}{N}$ = capital per worker

aggregate

$$Y = z \cdot F(K, N)$$

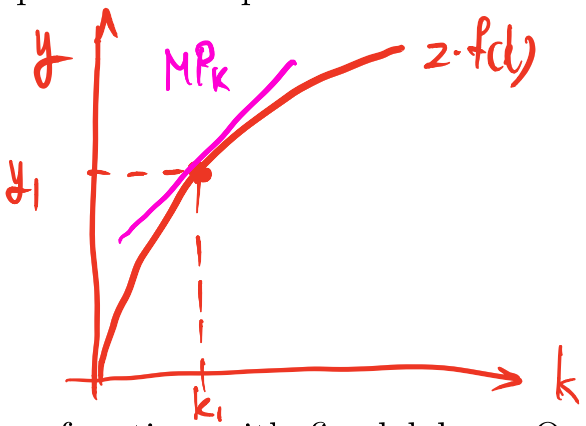
$$\frac{Y}{N} = \frac{1}{N} zF(K, N)$$

$$y = z \cdot F\left(\frac{K}{N}, \frac{N}{N}\right) = z \cdot F(k, 1)$$

per capita

$$y = z \cdot f(k)$$

- Marginal product of capital



Production function with fixed labor: Output per worker (y) increases at a decreasing rate as capital per worker (k) rises. Slope is the marginal product of capital (MP_K).

3.4.2 Growth of capital stock: Capital accumulation

- Assume capital wears out over time at the rate of d (or depreciation), where $0 < d < 1$

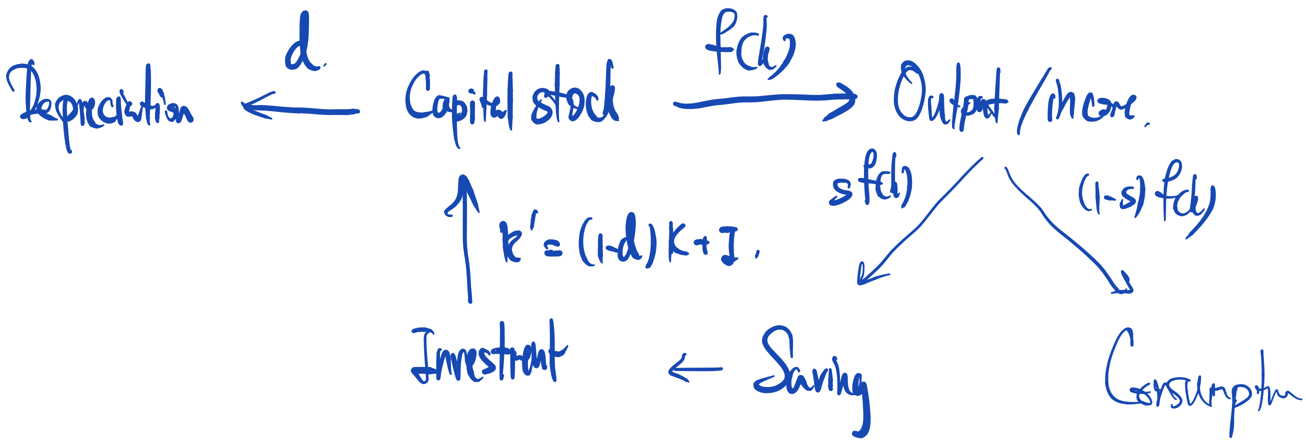
  I = (gross) investment = addition to capital stock.

  K' = capital stock in the future period.

$$K' = (1 - d)K + I \qquad y' = z'f(k)$$

3.4.3 The working of growth process

Capital accumulation drives output.



3.5 Equilibrium output

- At equilibrium, saving is equal to investment so that output consists of consumption and investment.

equilibrium

$$\boxed{S = I}$$
$$S = Y - C \qquad \text{saving = what we have left after consumption}$$
$$\Rightarrow \quad Y = C + I = C + S$$

- Equilibrium condition: The future capital stock is the current capital stock deducted by depreciation and added by investment (= saving).

$k' \to y'$

$$\checkmark\ Y = C + I$$
$$\checkmark\ C = (1 - s)Y$$
$$\checkmark\ I = K' - (1 - d)K$$

Substitute C and I into Y equation and rearrange terms in per capita format:

$\boxed{K'} \to k'$

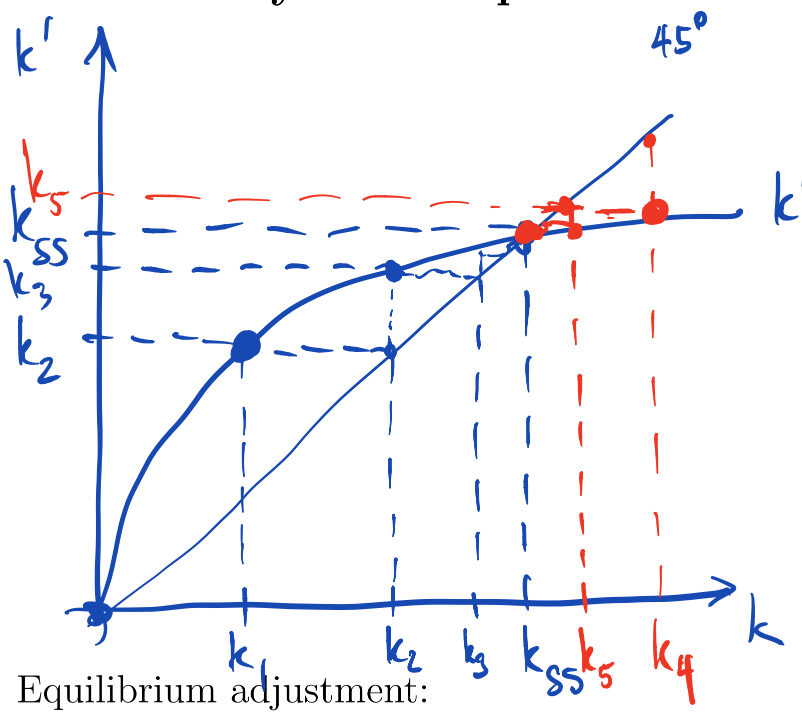
$$Y = C + I$$
$$= (1-s)Y + K' - (1-d)K.$$
$$K' = sY + (1-d)K.$$
But $Y = zF(K,N)$
$$K' = s \cdot z \cdot F(K,N) + (1-d)K.$$
$$\frac{K'}{N} = s \cdot z \cdot F\left(\frac{K}{N}, 1\right) + (1-d)\frac{K}{N}$$

- Future capital per worker function

$$\frac{K'}{N} \cdot \frac{N'}{N'} = s \cdot z \cdot f(k) + (1-d)k.$$
$$k' \cdot \frac{N'}{N} = s \cdot z \cdot f(k) + (1-d)k,$$ where $k' = \frac{K'}{N'}$, $\frac{N'}{N} = 1+n$.
$$k'(1+n) = s \cdot z\, f(k) + (1-d)k.$$
(1) $$k' = \frac{s \cdot z \cdot f(k)}{1+n} + \frac{(1-d)k}{1+n}$$ $\{s, z, d, n\}$ = exogenously determined.

Evolution of per-worker capital stock.

3.6 The steady-state capital of worker



$$k' = \frac{s \cdot z f(k)}{1+n} + \frac{(1-d)k}{1+n}$$

$$= \frac{s \cdot z f(k)}{1+n} + \frac{k}{1+n} - \frac{d}{1+n} k.$$

$\frac{K}{N} = \bar{k}_{ss}$

① $\bar{K}, \bar{N}$

② $K\uparrow$, $N\uparrow$
by n by n

- Equilibrium adjustment:

  At A, $k_2 > k_1$; k is growing.

  At B, $k_3 > k_2$; k is growing.

  $k = k^*$: steady-state capital per worker.

- Diminishing returns on k

  At E, $k = k' = k^*$ so that k^* is steady.

  To the left of k^*, $k' > k$ so that k is increasing.

  To the right of k^*, $k' < k$ so that k is decreasing

- As k is increasing, MP_K is falling so that y is increasing at a decreasing rate.

- Finally, investment (or new capital) is just sufficient to keep up with population growth and depreciation, so that k (and y) is ~~stagnant~~. at steady state.

Steady-state aggregates

- With k^* at the steady state, y^*, c^* and $szf(k^*)$ are all at the steady-state.

  – No further improvement in output per worker (y).

At steady state $k = k' = k_{ss}$, $k_{ss} = \frac{K_{ss}}{N}$ $\Rightarrow$ $n\% \uparrow K_{ss} = \bar{k}_{ss} \cdot N \uparrow n\%$
$n\% \uparrow Y_{ss} = \bar{y}_{ss} \cdot N = z \cdot f(k_{ss}) \cdot N \uparrow n\%$
$n\% \uparrow C_{ss} = \bar{c}_{ss} \cdot N = (1-s) \cdot z f(k_{ss}) N \uparrow n\%$
$n\% \uparrow I = sY = s \cdot z \cdot f(k_{ss}) \cdot N \uparrow n\%$

- Given population growth (n), total factor productivity (z) and the saving rate (s), the steady-state growth rate is 'n' for aggregate quantities:
  - Capital stock (K) and output (Y);
  - Consumption (C), saving (S) and investment (I).

3.7 Analysis of Steady State

$$k' = \frac{szf(k)}{1+n} + \frac{(1-d)k}{1+n}$$

Denote. $S = sY$
$S = s z \cdot F(K, N)$
$\underline{s} = s \cdot z \cdot f(k)$

At steady state, $k = k' = k_{ss}$

$$k_{ss} = \frac{s \cdot z \cdot f(k_{ss})}{1+n} + \frac{(1-d) k_{ss}}{1+n}$$

$$(1+n)\, k_{ss} = s \cdot z \cdot f(k_{ss}) + (1-d) k_{ss}$$

$$\underbrace{s z f(k_{ss})}_{\text{saving}} = \underbrace{(n+d) k_{ss}}_{\text{investment.}}$$

$k^* = k_{ss}$

- $szf(k^*)$ = saving per worker
- $(n+d)k^*$ = investment per worker needed to keep up with population growth and depreciation.
- At k^*, the capital stock is still growing, but just sufficient to equip each worker with the same k and depreciation (so k^* is steady).

  - 'Capital widening': growing K just to keep the steady k and y

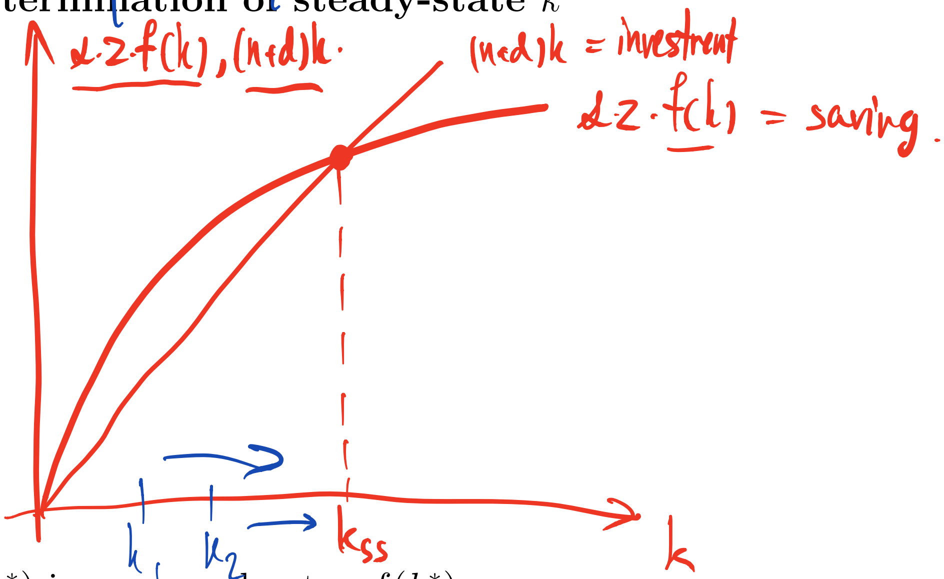
investment $\longrightarrow$ capital stock. per worker $\longrightarrow$ depreciation and pop growth

$\bar{k}_{ss} = \frac{K}{L \uparrow}$

d
$n.$

saving investment.

3.7.1 Determination of steady-state k^*



- $szf(k^*)$ is concave due to $zf(k^*)$.
- $(n + d)k^*$ has the slope $= (n + d)$.

3.7.2 Transitional dynamics

- What happen to "growth of per-capita variables" along the path towards their long- term steady states?
  - Suppose you start from an initial k to the left of k^*, k' will be increasing.
  - Since k' increases, income-per-capita (y) will increase as well.
  - As y increases at a decreasing rate, growth will decline over time.
  - At the steady state, $y = y^*$, i.e. zero income-per-capita growth.
- At the steady state, aggregates grow at the rate of "n". (aka, balanced growth path)
- While directing towards the steady state, aggregate Y will grow at the rate above "n", and slowly decline to consistently keep trace with the growth of income per capita (y).

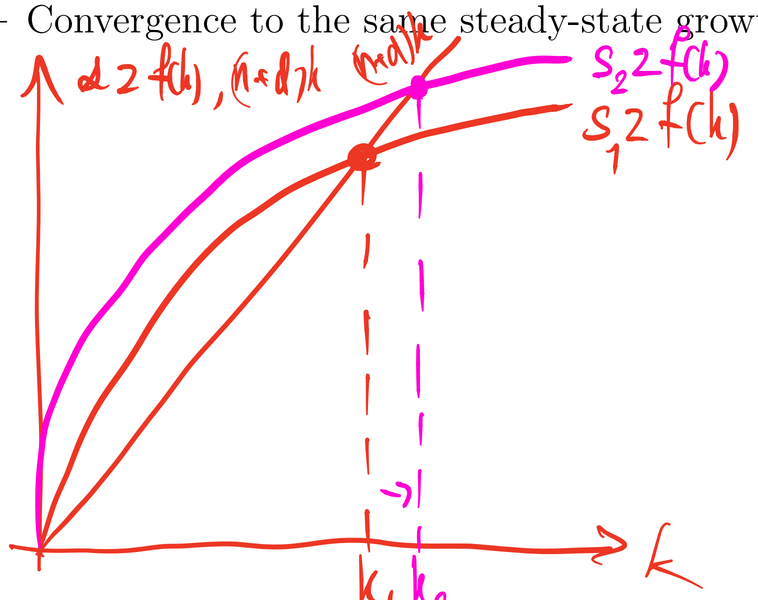
3.7.3 Effect of an increase in s

- s may increase due to changes in consumers' propensity or government policy.

- Assume a permanent increase in s:

$s\uparrow(s_1 \to s_2)$

  - $szf(k^*)$ rotates upwards.
  - Higher steady-state k^* and y^* (on a different 'growth path')
  - Higher growth of K and Y is transitional.
  - Convergence to the same steady-state growth rate of 'n'.



$y_1 = z \cdot f(k_1)$

$\Downarrow$ $k\uparrow(k_1 \to k_2)$

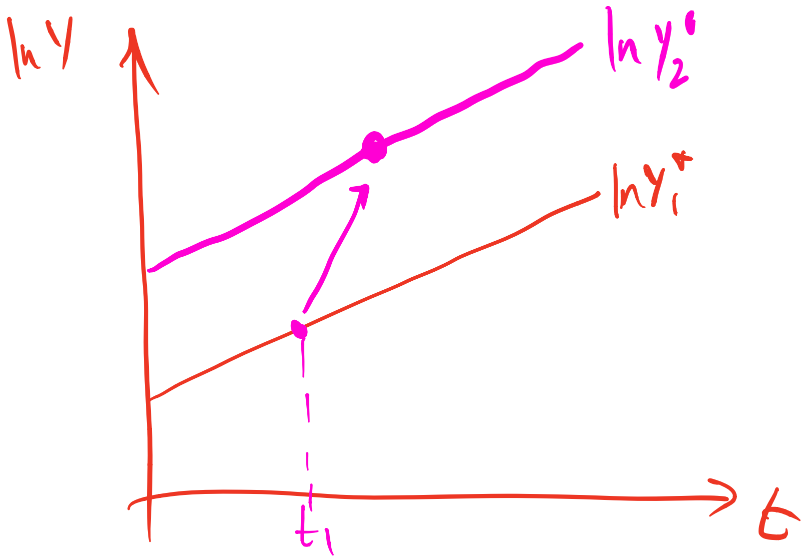
$y_2 = z \cdot f(k_2)$

- Higher saving rate results in a higher k^* and y^*.

match w/ #3.

$\uparrow s(\uparrow I) \Rightarrow \uparrow y$

- A rise in s raises k^*



- Temporary gain in growth rate
- K and Y move to new 'growth paths'.
- Higher growth rates of K and Y are transitional, converging to n

3.7.4 Steady-state consumption per worker

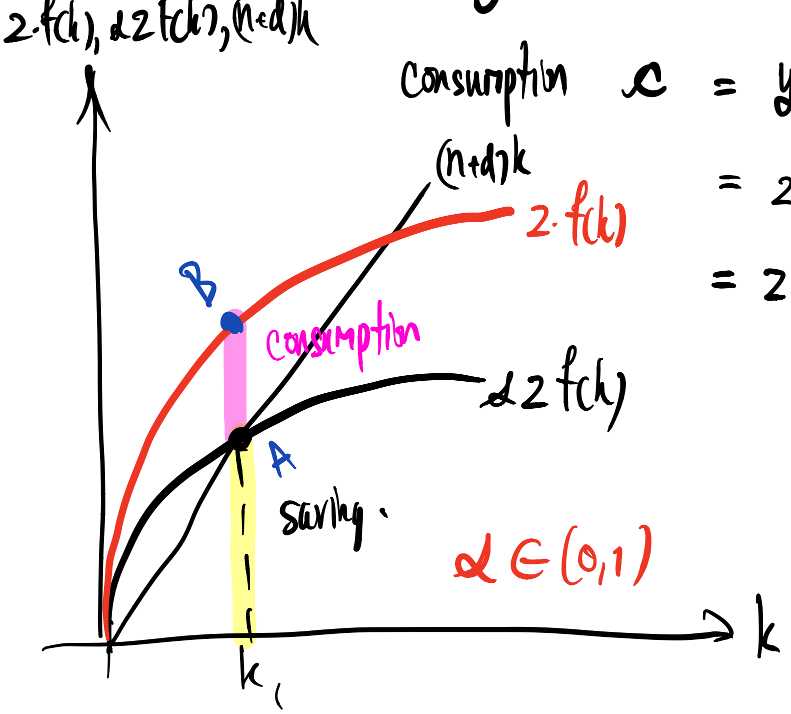
output $y^* = zf(k^*)$

saving $S/N = \alpha \cdot z \cdot f(k)$

consumption $c = y^* - \underline{s}^*$

$= z \cdot f(k) - \alpha z f(k) = (1-\alpha) z \cdot f(k)$

$= z \cdot f(k) - (n+d)k$



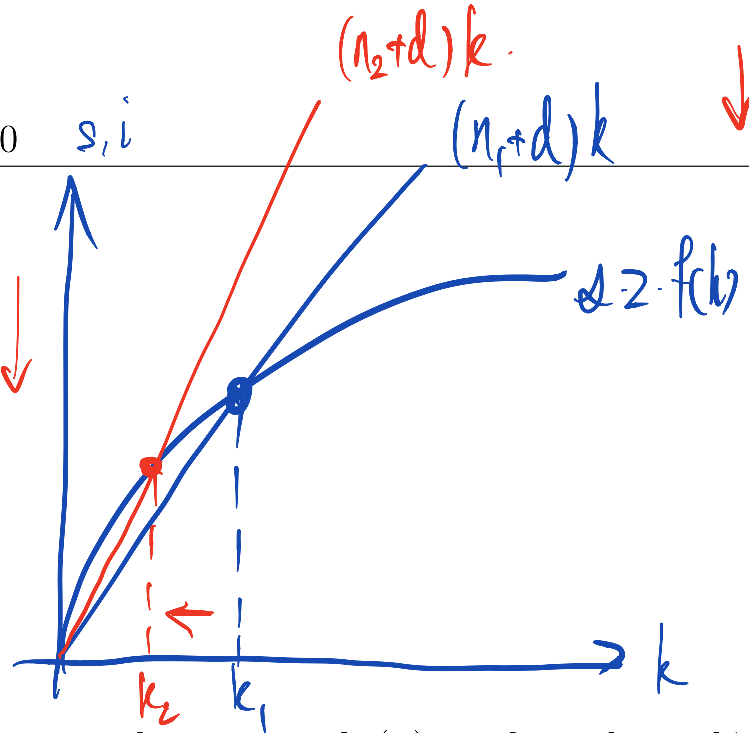
- $c^* = y^* - szf(k^*) = zf(k^*) - (n+d)k^*$
- AB $= c^*$

3.7.5 Effect of an increase in n

- The increase in population growth (n_1 to n_2) rotates $(n+d)k^*$ upwards.
- Decreased steady-state capital (k^*) and output per worker (y^*)
  - More workers (N^*) produce larger output (Y^*).
  - But falling productivity of labor results in lower output per worker (y^*).
- The steady-state growth rate is higher at n_2 for the capital stock (K) and total output (Y).

$y = z \cdot f(k)$

$k = \frac{K}{N}$ ↑ ↓

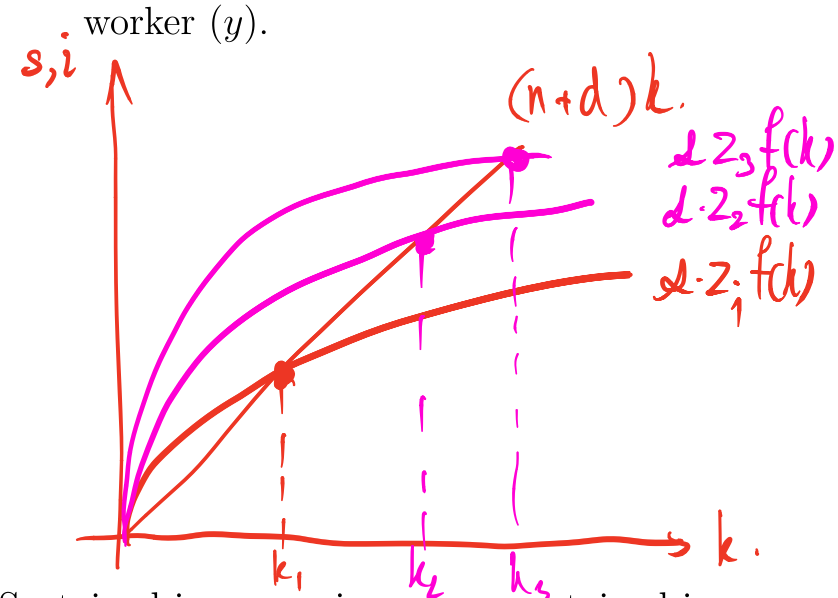


match w/ #4.
$n\uparrow \Rightarrow y\downarrow$.

- Higher population growth (n) results in lower k^* and y^*.
- A higher n with lower k^*

3.7.6 Effect of an increase in z

- A rising s or falling n raises steady-state output per worker (living standards).
  - But the improvement will cease at some point (s cannot exceed 1; n cannot fall indefinitely).
- An increase in total factor productivity (z) raises steady-state capital (k^*) and output per worker (y^*).
  - Sustained increases in z cause sustained increases in output per worker (y).



- Sustained increases in z cause sustained improvements in y*.

$\uparrow y$: $\uparrow s$, $\downarrow n$, $\uparrow z$

Sources of sustained growth

- Growth from increases in productive inputs:
  - Physical capital accumulation, $F(K, N)$.
  - Human capital accumulation, $F(K, H)$.
- Growth from total factor productivity (z):
  - Technical progress, inventions, better management and organization.
  - Weather, improved government regulations, falling input prices.

Solow model predictions

- In the long run, higher savings rate results in higher income per worker.
  - Fact: Positive correlation between GDP per capita and the ratio of investment to GDP.
- An increase in population growth causes a decrease in income per worker.
  - Fact: Negative correlation between population growth and GDP per capita

3.7.7 Golden Rule

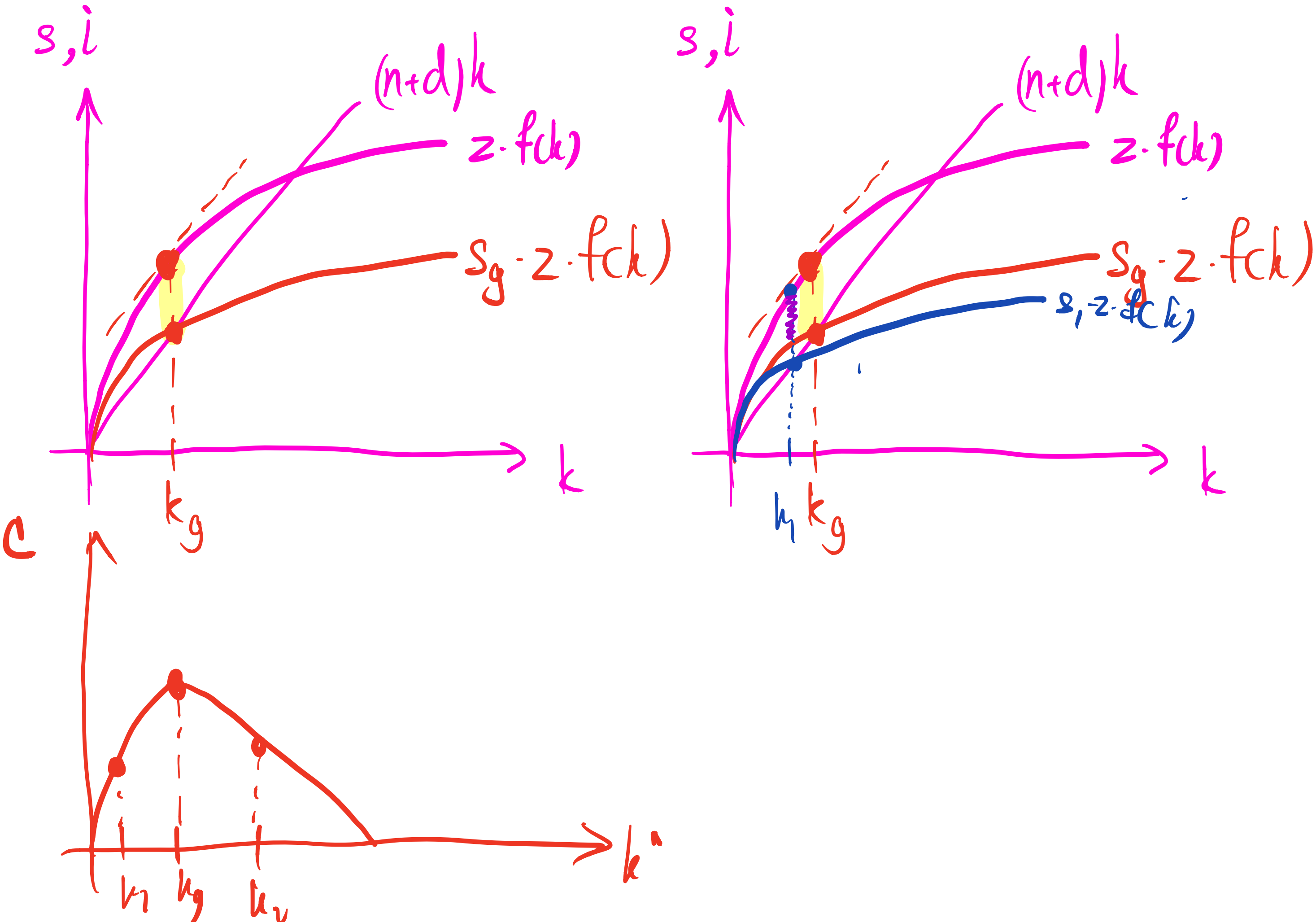
- Consumption per worker in the steady state is $c^* = y^* - szf(k^*) = zf(k^*) - (n + d)k^*$
- Golden rule quantity of capital per worker, k^{**}, gives the maximum consumption per worker, c
- Maximize c^{**} :

$$c^* = z \cdot f(k^*) - \underline{(n+d)} k^*$$

$$\frac{\partial c^*}{\partial k} = z \cdot f'(k') - (n+d) = 0$$

$$z f'(k) = n+d.$$

$$MP_k = n+d$$



- With the golden-rule s_g, c^{**} is maximized and steady in all periods.
  - Other s results in different c^* which is not max.
  - So, an increase in "s" may not lead to an increase if s is already higher than s_g.
- Should we achieve s_g if the current $s \neq s_g$?
  - A sacrifice of current c to build up a larger capital stock in the future is needed; is it worth?
  - s depends on individuals' preference and the market for investment.

3.8 Take-away from Solow Model

- In this economy, there are two markets in the current period.
  1. consumption goods are traded for current labor
  2. consumption goods are traded for capital

- Consumers save by accumulating capital.

  At equilibrium, $S = I$ so that $Y = C + I$.

  Equilibrium condition: The future capital stock is the current capital stock deducted by depreciation and added by investment (= saving)

  Aggregates:

$$\begin{aligned} K &= k^* N \\ Y &= y^* N = zf(k^*)N \\ I &= sY = szf(k^*)N \\ C &= (1-s)Y = (1-s)zf(k^*)N \end{aligned}$$

- Given population growth (n), total factor productivity (z) and the saving rate (s), the steady-state growth rate is 'n' for aggregate quantities.

  "while the growth rate of per worker variable = 0 at steady state"

- Solow model tells us that growth in key macroeconomic aggregates is determined by exogenous labor force growth when the saving rate, the labor force growth rate, and total factor productivity are constant.

- Solow model states that investment in capital cannot drive long run growth in GDP per worker. n

- Policy lesson: don't advise poor countries to invest without due regard for technology and incentives. Capital deepening (an increase in capital per worker) cannot lead to a sustained economic growth in the long run.

3.9 Growth Accounting

- Growth since the Industrial Revolution has come mainly from rising z.

  Will this continue indefinitely into the future?

- Growth accounting: identify sources of growth. Increases in productive inputs (K, N) or in total factor productivity (z).

- Calculation based on the production function and the Solow residual.

Thailand's Production Function:

$\ln Y = \ln z + \alpha \ln K + (1-\alpha) \ln N$

$$Y = zK^{\alpha}N^{1-\alpha}$$

where $0 < \alpha < 1$. Note that $\alpha + (1 - \alpha) = 1$

$\ln Y = \alpha \ln K + (1-\alpha) \ln N + u.$

- Assume constant returns to scale (CRS).
- α = share of the capital input in GDP.
- $1 - \alpha$ = share of the labor input in GDP.

$\ln \hat{Y} = \hat{\alpha} \ln K + (\widehat{1-\alpha}) \ln N$

$$Y = zK^{0.6}N^{0.4}$$

$\hat{u} = \ln Y - \ln \hat{Y} = TFP.$

The Solow residual for Thailand

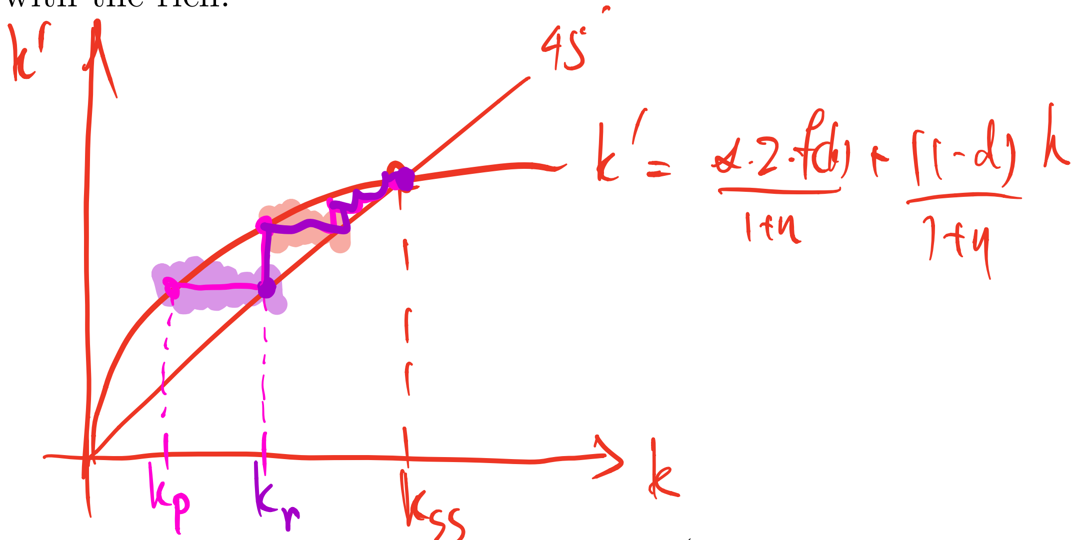
$$Y = zK^{0.6}N^{0.4}$$
$$z = \frac{Y}{K^{0.6}N^{0.4}}$$

- Estimated z = Solow residual.
- It measures the level of total factor productivity for Thailand.

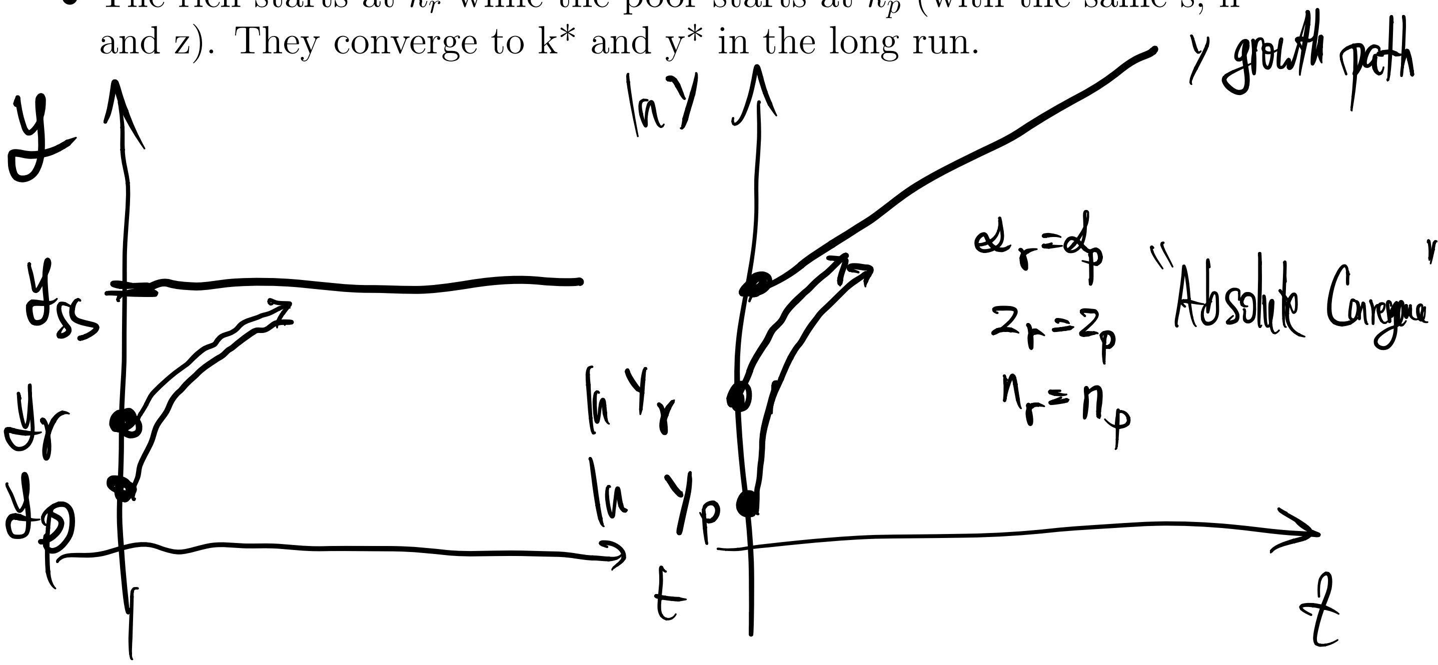
3.10 Solow Growth and Stylized Facts

3.10.1 Solow Growth Prediction

- **Absolute and conditional convergences** are predicted
- Absolute Convergence
  - If two countries start with: The same population growth rate (n), saving rate (s) and total factor productivity (z), but different per capita incomes (y), e.g., rich versus poor countries; they will converge to the same steady-state k^*, y^* and c^* $\Rightarrow$ Absolute convergence.
  - The poor country will have temporary higher growth and catch up with the rich.



- The rich starts at k_r while the poor starts at k_p (with the same s, n and z). They converge to k* and y* in the long run.



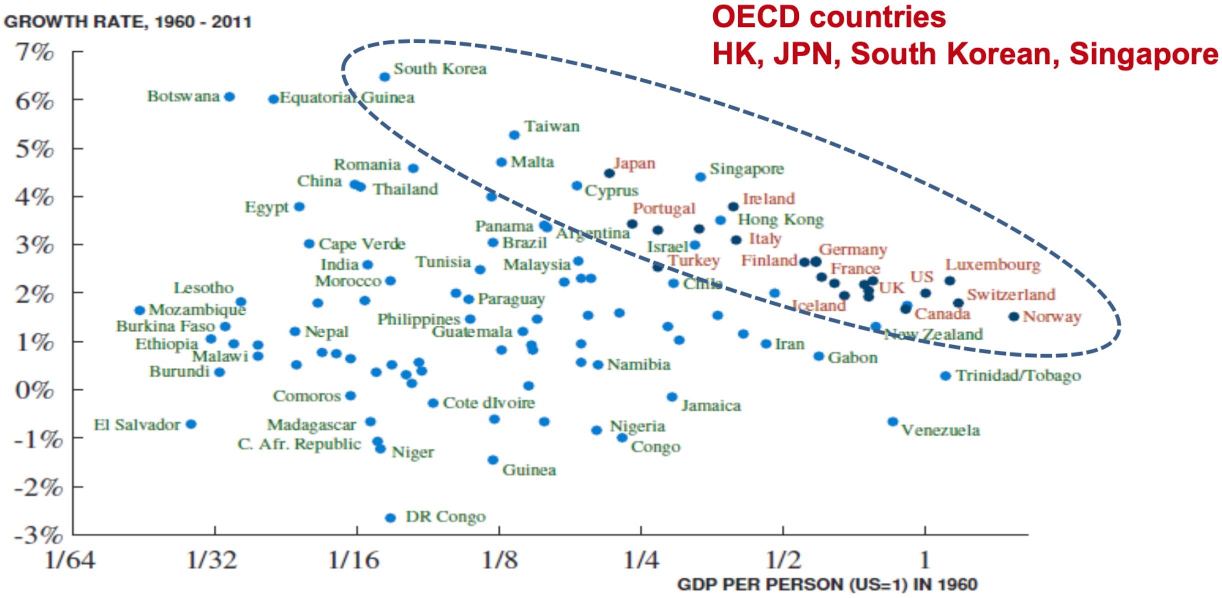
- Conditional Convergence
  - With differences in n, z and s, the steady-state k^*, y^*, c^* are different. Each country has its own steady state.

    The steady-state growth rate of aggregates (K, Y) is still n for each country.
  - Countries are predicted to converge to their own steady state.
  - Disparity among countries due to different values of n, z and s.

3.10.2 Solow Vs Data

- Absolute convergence has occurred among rich countries.
- No absolute convergence between rich and poor countries. Exception is East Asia.
- No absolute convergence among poor countries.
- Great diversity among poor countries.

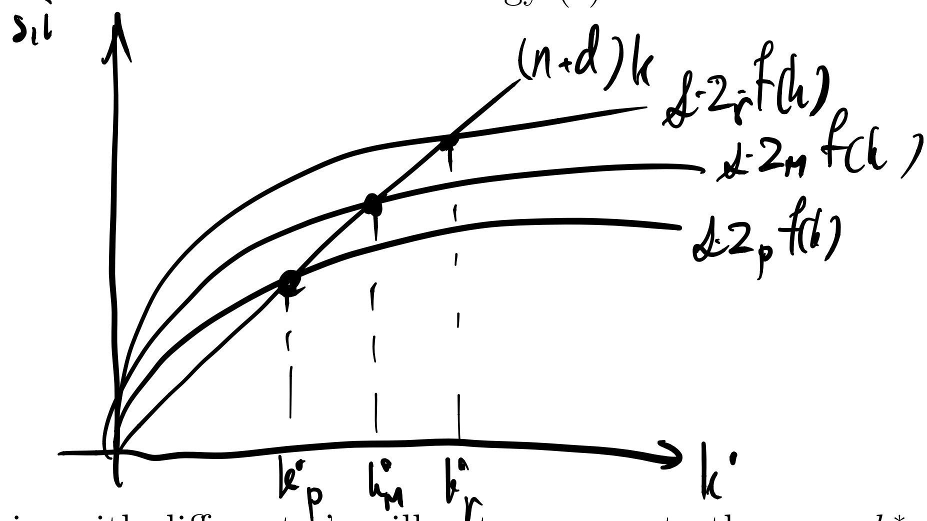
Figure 26: The Lack of Convergence Worldwide



Source: The Penn World Tables 8.0.

- Why no absolute convergence?

- Countries have different s, n and z.
- Each country has different steady-state k*, y*, c*.
- Each country is moving towards its own steady-state: Conditional convergence.
- But differences in s and n are not large enough to explain all international disparity.
- Difference in access to technology (z)?



Countries with different z's will not converge to the same k^* and y^*

- Disparity due to different z?s
  - Different levels of total factor productivity (z) will perpetuate differences in capital per worker (k*), per capita income (y*), ...
  - despite the same saving rate (s) and population growth (n).
- Barriers to technology adoption
  - Labor legislation: strong labor unions obstruct adoption of new technology.
  - Trade protectionism: domestic firms with market power lack incentives to in- novation.
  - Political corruption: government?s protection of inefficient firms.
  - Undeveloped financial system: poor resource allocation mechanism.
- How to catch up?

- Promotion of more competition among firms.

  Liberalization and competition policy.

  More pressure and incentive for firms to innovation.

- Free trade for greater international competition.
- Privatization of state enterprises.

  State enterprises guarantee employment at the expense of efficiency.

(simplified)

4 Endogenous Growth Model

4.1 Introduction

- Explanation of growth within the model.
- Economic growth depends on 'human capital accumulation'.
- Human capital: the accumulated stock of skills and education workers have at a point in time.
- Higher human capital; more production; more production of new human capital — faster growth over time.
- Human capital is an investment.
  - Opportunity cost of education and training: sacrifice of current consumption.
  - Benefits: more future production and consumption.

-Rivalry.

-Public goods.
- non rivalry.
- non excludable

physical capital

- Knowledge is 'non-rivalry': one's acquisition of knowledge does not reduce others' ability to acquire the same knowledge.
- Human capital accumulation is NOT subject to diminishing marginal returns.
- No limit on how productive a person can become, given increasing knowledge and skills.
- Unbounded growth in endogenous models.

- Growth in Solow model is limited:

  Diminishing returns on physical capital accumulation: rivalry in resource uses.

4.2 The representative consumer

- The consumer allocates time between work and accumulating human capital.

  H^S = efficiency units of current human capital;

  u = time allocated to work;

  w = the real wage;

  C = current consumption;

- The budget constraint is total labor earnings:

$$C = uwH^s$$

Exp. Revenue

4.3 Accumulation of human capital

- The consumer trades off current consumption for future consumption by accumulating human capital:

  $H^{s'}$ = future human capital

  $(1 - u)$ = time allocated to human capital accumulation

  b = efficiency of human capital accumulation technology; $b > 0$.

$$H^{s'} = (1 - u)bH^s$$

4.4 The representative firm

- The firm's production function using efficiency units of labor:

  Y = current output;

  z = marginal product of efficiency units of labor, where $z > 0$;

  $u \cdot H^d$ = current input of efficiency units of labor:

$$Y = zuH^d$$

4.5 The firm's profit function

- uH^d is also the firm's demand for the efficiency units of labor.
- The function is characterized by constant returns to scale (CRS): only one input.

$$\begin{aligned}\pi &= Y - wuH^d \\ &= zuH^d - wuH^d \\ &= (z-w)uH^d\end{aligned}$$

Demand for efficiency units of labor:

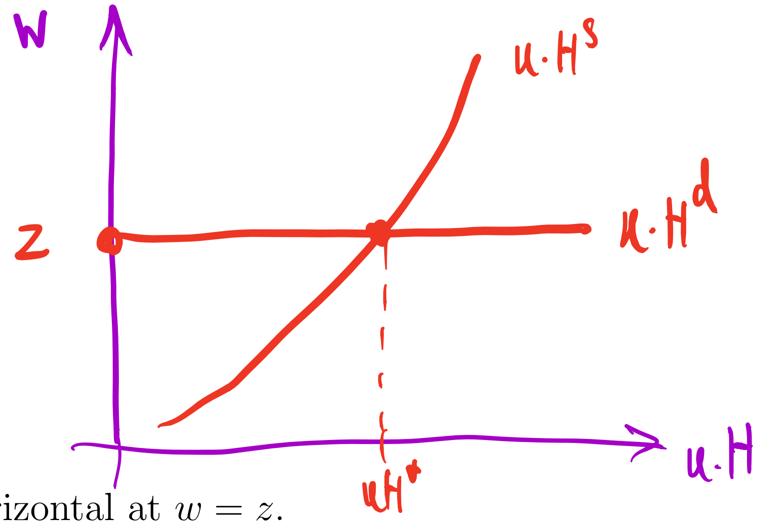
$$\pi = (z-w)uH^d$$

$(z-w) < 0 \Rightarrow \pi < 0$; the firm hires no units of labor; or $uH^d = 0$.

$(z-w) > 0 \Rightarrow \pi > 0$; the firm hires infinite units.

$z = w \Rightarrow \pi = 0$; the firm is indifferent.

The demand curve is infinitely elastic at $w = z$.



u·Hd ~~u^Hd~~ is horizontal at $w = z$.

The real wage equals z, the marginal product of uH^s

Assume uH^s with slope > 0.

4.6 Competitive Equilibrium

- A competitive equilibrium is an allocation of $\{H_t^d, H_t^s\}_{t=1}^{\infty}$ given the exogenous variables u, b, and z that satisfy
  - Consumers' utility maximization problem
  - Firm's profit maximization problem
  - Human capital accumulation equation
  - The market clearing condition for human capital at $w = z$ where $uH^d = uH^s$.
- Equilibrium consumption and growth of human capital accumulation:

$\frac{H'}{H} - 1 = \frac{H' - H}{H} = \%$ growth

$$C = zuH$$
$$H' = b(1-u)H$$
$$\frac{H'}{H} - 1 = b(1-u) - 1$$

where $b(1-u) - 1$ is constant

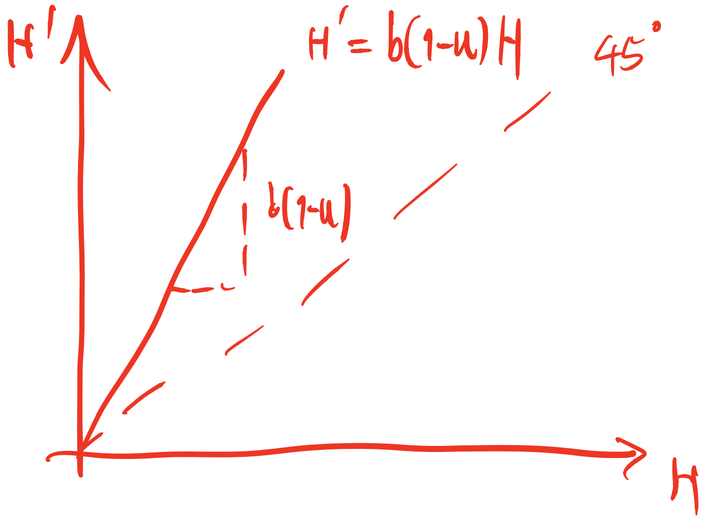
ex. $H = 100$, $H' = 110$

$\frac{H'}{H} - 1 = 1.1 - 1 = 0.1$

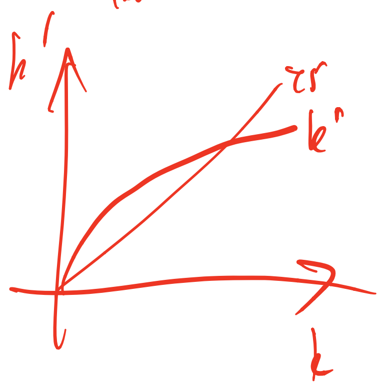
$= 10\%$

4.7 Economics Growth

4.7.1 Growth of human capital



Solow



- H' is a function of H where $H' > H$.
- Slope $= b(1-u) =$ 1+rate of growth of human capital $= \frac{H'}{H}$

4.7.2 Factors in human capital growth

$$\frac{H'}{H} - 1 = \underline{b}(\underline{1-u}) - 1$$

- $\frac{H'}{H}$ is higher if b increases or u decreases
  - b = efficiency of human capital accumulation technology (or efficiency of the education sector).
  - u = time spent on current output production.
  - Falling u (or rising $1-u$) = more time spent on human capital accumulation.

4.7.3 Consumption and output growth

- Current consumption $C = zuH$ also holds for future consumption $C' = zuH'$.

  - So consumption grows at the same rate of b(1-u) as human capital.

- Output also grows at the same rate as Y = C in every period.

$C = zuH$
$C' = zuH'$

$$\boxed{\frac{C'}{C} - 1} = \frac{zu\cancel{C}H'}{zu\cancel{C}H} - 1 = \frac{H'}{H} - 1 = b(1-u) - 1$$

4.7.4 Source of growth

- b and z are fixed: constant technology.
- No population growth
- Growth is determined inside the model, by the value of b and u.
- Growth is unbounded because human capital accumulation is not subject to diminishing returns to scale.

  Output grows in proportion to human capital, given u.

$\frac{H'}{H} - 1 = b(1-u) - 1$

4.8 Government policy on growth

- Government can increase growth:
  - Increases in b, the efficiency of human capital accumulation technology (education policy).
  - Reduction in u, taxes or subsidies to education.
  - Higher $b(1-u)$, higher growth of human capital, consumption and output.

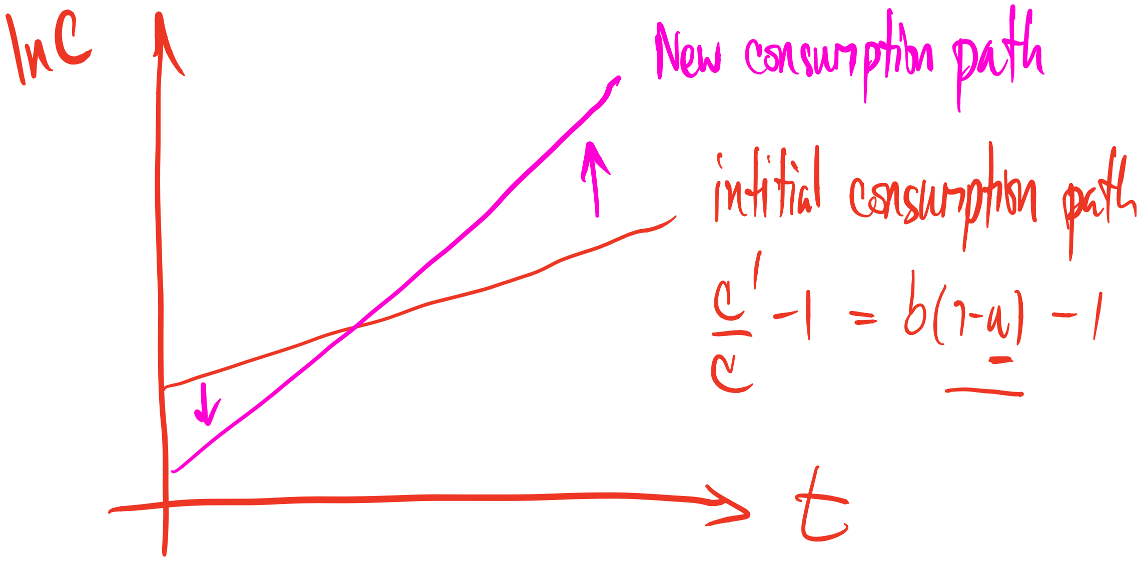
$C = 2uH. \downarrow$

- But current consumption must be sacrificed as u is lower, given initial human capital (H).

$u\downarrow \Rightarrow (1-u)\uparrow \Rightarrow \frac{H'}{H} - 1 \uparrow$

4.8.1 Lower u and consumption

$u\downarrow$



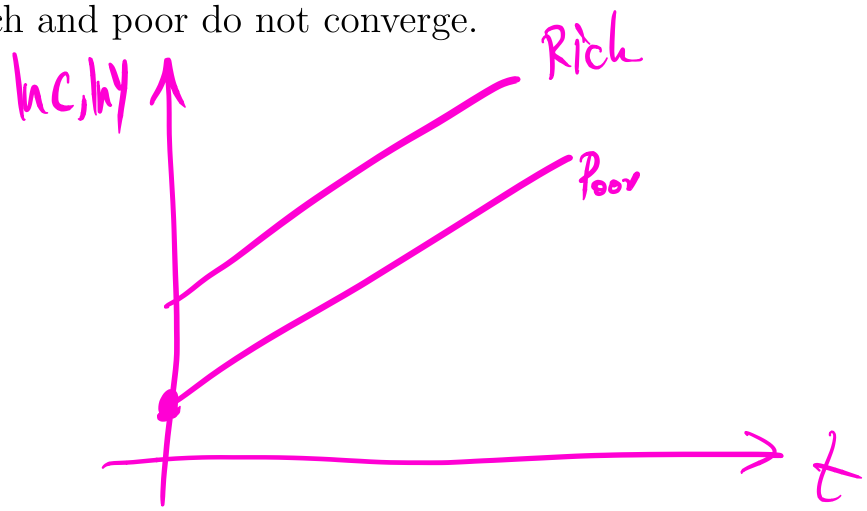
- A lower u results in lower current consumption but higher consumption in the long run.

4.8.2 Consumer preference

- Government's education policy (raising b) involves expenses of current resources and lower current consumption.
- Higher long-run growth is desirable?
  - This depends on the consumer's preference on current and future consumption.
  - The consumer may be worse off if current consumption is actually preferred.

4.9 No convergence

- Countries with all identical characteristics except differences in initial human capital will not converge on the levels of consumption and income.
  - Poor countries: low $Y = C = zuH_p$
  - Rich countries: high $Y = C = zuH_r$
  - But their C and Y grow at the same rate of $b(1-u)$.
- Rich and poor do not converge.



- The Y and Y time paths do not converge despite the same growth rate of $b(1-u)$.

4.10 Human capital externalities

- The endogenous model explains the lack of convergence among poor countries and between rich and poor countries.
- But convergence occurs among rich countries, why? = Human capital externalities.
  - Contact with others with higher human capital increases our own human capital.
  - Capital and labor are highly mobile; skills are more easily transferred in rich countries.
- More opportunities and contact make levels of human capital in rich countries converge.

  - Convergence of income per worker.

- Lack of human capital externalities in poor countries.

  - Less contact with developed countries.
  - People with high human capital move to developed countries (i.e., brain drains).
  - Differences in human capital persist.

5 Comparison between Solow and simplified Endogenous Growth Model

Solow's model	**An endogenous growth model** **(A Simplified version of Lucas's)**
production function : $Y = zF(K, N)$ $y = zf(k)$ Production function is **subject to diminishing marginal returns** in capital per worker.	$Y = uzH^d$ Production function **does not subject to diminishing marginal returns** in human capital.
Source of growth is **physical capital accumulation.**	**Source of growth is** **human capital accumulation**
There is **a limit for growth** as physical capital per worker increases. Output will go to steady state output. After it reaches the steady state, it will stay there. Eventually, growth of per capita output goes down to zero.	**Output continues to grow at a constant rate.** as human capital increases. There is **no limit** for growth. Output growth is always positive. (growth rate is equal to $b(1-u)-1$).
Economic reason for bounded growth : Physical capital is **rivalry** (once one company is using it the other cannot). production function is **subject to diminishing marginal returns** in capital per worker As phyical capital increases, output increases at a decreasing rate. Therefore, we cannot always generate growth by raising physical capital.	Economic reason for unbounded growth : This is because knowledge is '**non-rivalry**', production function **does not subject to diminishing marginal returns** in human capital As human capital increases, output increases at a constant rate. Therefore, we can always generate growth by raising human capital.