

Example of midterm exam questions

1) Consider the following system of linear equations

$$3x_1 - 6x_2 - 3x_3 = 3$$

$$x_3 + 2x_4 = 1$$

$$6x_1 - 12x_2 + 2x_3 + Rx_4 = C$$

- a) Write down a matrix equation that represents the above system of linear equations.
- b) Write down the augmented matrix.
- c) What are the values of **R** and **C** that make this linear system **Inconsistent**?
- d) If **R= 16**, what will be the value of **C** that makes this linear system **Consistent**? Also solve this system of linear equations and give a general solution in **a vector form**
- e.) If **R= 17** and **C=14**, solve this system of linear equations and give a general solution in **a vector form**

2. Suppose

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 7 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 4 & 5 \\ 0 & 1 & 2 & 2 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

- a) Find a basis for the null space of A .
- b) Find a basis for the column space of A .

c) Give the complete solution to $Ax = \begin{bmatrix} 3 \\ 3 \\ 21 \end{bmatrix}$.

3. Let $A = \begin{bmatrix} 2 & 2 & 2 \\ 4 & 3 & 1 \\ -2 & -1 & 4 \end{bmatrix}$

(a) Compute an LU factorization of A if one exists.

(b) Give all solutions to $Ax=b$ where $b = \begin{bmatrix} 2 \\ -3 \\ 11 \end{bmatrix}$

4. By performing row eliminations on 4×8 matrix A

$$\begin{bmatrix} 1 & 2 & 0 & 3 & -1 & 1 & 1 & -2 \\ -3 & -6 & 2 & -7 & 7 & 0 & -6 & 3 \\ 1 & 2 & 2 & 5 & 3 & 3 & -1 & 0 \\ 2 & 4 & 0 & 6 & -2 & 1 & 3 & 0 \end{bmatrix}$$

we got the following matrix B :

$$\begin{bmatrix} 1 & 2 & 0 & 3 & -1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- a) What is the rank of A ?
 - b) Are the rows of A linearly independent? Why?
 - c) Do columns 4, 5, 6 and 7 form a basis of \mathbb{R}^4 ? Why?
 - d) Give a basis of null space of A
 - e) Give a basis of Column space of A
5. a) If A is a 64 by 17 matrix of rank 11 , how many independent vectors satisfy $Ax=0$?
How many independent vectors satisfy $A^T y = 0$?
- b) If v_1, v_2, v_3, v_4 span \mathbb{R}^3 , give all possible ranks for the matrix with those four columns.
- c) If v_1, v_2, v_3 form a basis for \mathbb{R}^3 then the matrix with those three columns is singular. Is this statement **TRUE** or **FALSE** ? **Why?**