Assignment 4

DUE DATE: Tuesday 9^{nd}, March 2021.

I pledge to the Honor Code and to obey all rules for taking and performing homework assignments as specified by the course instructor.

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Question 1 (50 points) Your score.......................

Given the daily log returns : (R_t) can be explained by the AR(2) model as following:

$$(1 - 1.5B + 0.9B^2)R_t = 0.25 + \varepsilon_t$$

where ε_t is distributed as the Gaussian White Noise with mean $(\mu) = 0$ and variance $(\sigma^2) = 0.25$

B lag-operator

$R_t - 1.5BR_t + 0.9B^2R_t = 0.25 + \varepsilon_t$

Question 1.1 (10 points) Your score.......................

From the above AR(2) model, Is the model weakly stationary? Write down the reverse characteristic equation and find out the conditions to support your answer.

$(1-1.5B+0.9B^2)R_t = 0.25+\varepsilon_t$

reverse eq. $\rightarrow$ $x^2-1.5x+0.9=0$

$x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a} = \dfrac{-(-1.5)\pm\sqrt{(-1.5)^2-4(1)(0.9)}}{2(1)}$

$= \dfrac{1.5\pm\sqrt{-1.35}}{2} = \dfrac{1.5\pm 1.162\sqrt{-1}}{2} = 0.75 \pm 0.581i$

$a = 0.75$, $b = 0.581$

modulus $= R = \sqrt{a^2+b^2}$

$= \sqrt{0.75^2+0.581^2} = 0.95$

Since, $|0.95| < 1$

So, The model is weakly stationary

Question 1.2 (10 points)

$R_t - 1.5BR_t + 0.9B^2R_t = 0.25 + \varepsilon_t$

$r_t - 1.5r_{t-1} + 0.9r_{t-2} = 0.25 + \varepsilon_t$

$r_t = 0.25 + 1.5r_{t-1} - 0.9r_{t-2} + \varepsilon_t$

Your score......................

Calculate the unconditional mean: $E(R_t)$ of R_t and the conditional mean: $E(R_t|F_{t-1})$

unconditional mean : $E(r_t)$

$E(r_t) = E(0.25 + 1.5r_{t-1} - 0.9r_{t-2} + \varepsilon_t)$

$= 0.25 + 1.5E(r_{t-1}) - 0.9E(r_{t-2}) + 0$

We assume that r_t is weak stationarity, then $E(r_t) = E(r_{t-1}) = E(r_{t-2})$

$E(r_t) - 1.5E(r_t) + 0.9E(r_t) = 0.25$

$E(r_t) = \frac{0.25}{1-1.5+0.9} = 0.625$

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conditional mean : $E(r_t|F_{t-1})$

$E(r_t|\cdot) = E(0.25 + 1.5r_{t-1} - 0.9r_{t-2} + \varepsilon_t|\cdot)$

$= 0.25 + 1.5E(r_{t-1}|\cdot) - 0.9E(r_{t-2}|\cdot) + 0$

$= 0.25 + 1.5r_{t-1} - 0.9r_{t-2}$

Question 1.3 (10 points) | Your score......................

$$r_t = 0.25 + 1.5 r_{t-1} - 0.9 r_{t-2} + \varepsilon_t$$

Find out the unconditional variance: $Var(R_t)$ of R_t and conditional variance $Var(R_t|F_{t-1})$ of R_t

unconditional var . $var(r_t) = E[(r_t - \mu)^2]$

deviation form $r_t - \mu = \phi_1(r_{t-1} - \mu) + \phi_2(r_{t-2} - \mu) + a_t$

$$(r_t - \mu)^2 = \phi_1^2(r_{t-1} - \mu)^2 + \phi_2^2(r_{t-2} - \mu)^2 + a_t^2 + 2\phi_1\phi_2(r_{t-1} - \mu)(r_{t-2} - \mu) + 2\phi_1 a_t(r_{t-1} - \mu) + 2\phi_2 a_t(r_{t-2} - \mu)$$

$$E(r_t - \mu)^2 = \phi_1^2 E(r_{t-1} - \mu)^2 + \phi_2^2 E(r_{t-2} - \mu)^2 + E(a_t^2) + 2\phi_1\phi_2 E[(r_{t-1} - \mu)(r_{t-2} - \mu)] + 2\phi_1 E(\overset{0}{\cancel{a_t(r_{t-1} - \mu)}}) + 2\phi_2 E(\overset{0}{\cancel{a_t(r_{t-2} - \mu)}})$$

$$var(r_t) = \phi_1^2 var(r_{t-1}) + \phi_2^2 var(r_{t-2}) + \sigma^2 + 2\phi_1\phi_2\gamma_1$$

We assume that r_t is weak stationarity, then $var(r_t) = var(r_{t-1}) = var(r_{t-2})$

$$var(r_t) = \frac{\sigma^2 + 2\phi_1\phi_2\gamma_1}{1 - \phi_1^2 - \phi_2^2} = \frac{0.25 + 2(1.5)(-0.9)\gamma_1}{1 - (1.5)^2 - (-0.9)^2} = \frac{0.25 - 2.7\gamma_1}{-2.06} = 1.31\gamma_1 - 0.121$$

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conditional var . $var(r_t|\cdot)$

รู้อยู่แล้วเป็น constant

cov(constant) = 0

$$var(r_t|\cdot) = var(0.25 + 1.5 r_{t-1} - 0.9 r_{t-2} + \varepsilon_t|\cdot)$$

$$= var(\overset{0}{\cancel{0.25}}|\cdot) + 1.5^2 var(\overset{0}{\cancel{r_{t-1}}}|\cdot) + (-0.9)^2 var(\overset{0}{\cancel{r_{t-2}}}|\cdot) + \underbrace{var(\varepsilon_t|\cdot)}_{\sigma_a^2 = 0.25} + 2(1.5)(-0.9)cov(\overset{0}{\cancel{\overset{\gamma_1}{r_{t-1}}, r_{t-2}}}|\cdot) + 2cov(1.5\overset{0}{\cancel{r_{t-1}}}, a_t) + 2cov(-0.9\overset{0}{\cancel{r_{t-2}}}, a_t)$$

$$= 0.25$$

unconditional var (ลองทำ)

deviation form $r_t - \mu = \phi_1(r_{t-1} - \mu) + \phi_2(r_{t-2} - \mu) + a_t$

$$(r_t - \mu)(r_t - \mu) = \phi_1(r_{t-1} - \mu)(r_t - \mu) + \phi_2(r_{t-2} - \mu)(r_t - \mu) + a(r_t - \mu)$$

$$var(r_t) = \phi_1 E(r_{t-1} - \mu)(r_t - \mu) + \phi_2 E(r_{t-2} - \mu)(r_t - \mu) + E(a(r_t - \mu))$$

$$= \phi_1\gamma_1 + \phi_2\gamma_2 + \sigma_a^2$$

ทำวิธีนี้ได้มั้ยคะอาจารย์, แบบนี้ถือว่าถูกมั้ย

Question 1.4 (10 points) Your score......................

Calculate the autocorrelation: ρ_l for l=1 and 2 of R_t. Also, write down the autocorrelation: ρ_l when l $\geq$ 2.

$$\rho_1 = \frac{\gamma_1}{\gamma_0} = \frac{cov(r_t, r_{t-1})}{var(r_t)}$$

deviation form $r_t - \mu = \phi_1(r_{t-1} - \mu) + \phi_2(r_{t-2} - \mu) + a_t$

$$(r_t - \mu)(r_{t-1} - \mu) = \phi_1(r_{t-1} - \mu)(r_{t-1} - \mu) + \phi_2(r_{t-2} - \mu)(r_{t-1} - \mu) + a_t(r_{t-1} - \mu)$$

$$E(r_t - \mu)(r_{t-1} - \mu) = \phi_1 E(r_{t-1} - \mu)(r_{t-1} - \mu) + \phi_2 E(r_{t-2} - \mu)(r_{t-1} - \mu) + E(a_t(\cancelto{0}{r_{t-1}} - \mu))$$

$$\gamma_1 = \phi_1 \gamma_0 + \phi_2 \gamma_1$$

$$\frac{\gamma_1}{\gamma_0} = \phi_1 + \phi_2 \frac{\gamma_1}{\gamma_0}$$

$$\rho_1 = \phi_1 + \phi_2 \rho_1$$

$$\rho_1 = \frac{\phi_1}{1-\phi_2} = \frac{1.5}{1-(-0.9)} = 0.79 \; \#$$

$$\rho_2 = \frac{\gamma_2}{\gamma_0}$$

$$E(r_t - \mu)(r_{t-2} - \mu) = \phi_1 E(r_{t-1} - \mu)(r_{t-2} - \mu) + \phi_2 E(r_{t-2} - \mu)(r_{t-2} - \mu) + E(a_t(\cancelto{0}{r_{t-2}} - \mu))$$

$$\frac{\gamma_2}{\gamma_0} = \phi_1 \frac{\gamma_1}{\gamma_0} + \phi_2 \frac{\gamma_0}{\gamma_0}$$

$$\rho_2 = \phi_1 \rho_1 + \phi_2$$

$$= (15)(0.79) - 0.9 = 0.285 \; \#$$

AS $\underbrace{E[(r_t - \mu)(r_{t-j} - \mu)]}_{\gamma_j} = \phi_1 E[\underbrace{(r_{t-1} - \mu)(r_{t-j} - \mu)}_{\gamma_{j-1}}] + \phi_2 E[\underbrace{(r_{t-2} - \mu)(r_{t-j} - \mu)}_{\gamma_{j-2}}] + E[\cancelto{0}{a_t(r_{t-j} - \mu)}]$

$$\gamma_j = \phi_1 \gamma_{j-1} + \phi_2 \gamma_{j-2}$$

$$\frac{\gamma_j}{\gamma_0} = \phi_1 \frac{\gamma_{j-1}}{\gamma_0} + \phi_2 \frac{\gamma_{j-2}}{\gamma_0}$$

$$\rho_j = \phi_1 \rho_{j-1} + \phi_2 \rho_{j-2} \; \#$$

$\uparrow$

ρ_l when $l \geq 2$

$j=1$, $\rho_1 = \phi_1 \rho_0 + \phi_2 \rho_1$ and $\rho_0 = 1$

$\rho_1 = \phi_1 + \phi_2 \rho_1$

$\rho_1 = \frac{\phi_1}{1-\phi_2}$

$j=2$, $\rho_2 = \phi_1 \rho_1 + \phi_2 \rho_0$

$\rho_2 = \phi_1 \rho_1 + \phi_2$

$j=3$, $\rho_3 = \phi_1 \rho_2 + \phi_2 \rho_1$

$j=4$, $\rho_4 = \phi_1 \rho_3 + \phi_2 \rho_2$

$j=5$, $\rho_5 = \phi_1 \rho_4 + \phi_2 \rho_3$

Question 1.5 (10 points) Your score......................

$r_t = 0.25 + 1.5 r_{t-1} - 0.9 r_{t-2} + \varepsilon_t$

Given $R_{1000} = 0.01$ $R_{999} = 0.02$ $R_{998} = 0.03$ $\varepsilon_{1000} = -0.01$ $\varepsilon_{999} = -0.02$ $\varepsilon_{998} = -0.03$ Obtain 1-step, 2-setp 95 % interval forecasts for R_t at the forecast origin $\overset{h}{t} = 1000$. Also the ∞-step 95 % interval forecasts for R_t. Draw these intervals.

1 step $r_{h+1} = \phi_0 + \phi_1 r_h + \phi_2 r_{h-1} + \varepsilon_{h+1} = 0.25 + 1.5 r_h - 0.9 r_{h-1} + \varepsilon_{h+1}$

$E(r_{h+1}|F_h) = 0.25 + 1.5E(r_h|F_h) - 0.9E(r_{h-1}|F_h) + \overset{0}{\cancel{E(\varepsilon_{h+1})}}$

$\hat{r}_h(1) = 0.25 + 1.5 r_h - 0.9 r_{h-1} = 0.25 + 1.5(0.01) - 0.9(0.02) = 0.247$

error $= r_{h+1} - \hat{r}_h(1) = \varepsilon_{h+1}$ and var $=$ var$(\varepsilon_{h+1}|\cdot) = \sigma_a^2 = 0.25$

interval $= 0.247 \pm 1.96\sqrt{(0.25)} = -0.733, 1.227$ #

2 step : $r_{h+2} = \phi_0 + \phi_1 r_{h+1} + \phi_2 r_h + \varepsilon_{h+2}$

$E(r_{h+2}|F_h) = 0.25 + 1.5E(r_{h+1}|\cdot) - 0.9E(r_h|\cdot) + \overset{0}{\cancel{E(\varepsilon_{h+2})}}$

$\hat{r}_h(2) = 0.25 + 1.5(0.247) - 0.9(0.01) = 0.6115$

error $= r_{h+2} - \hat{r}_h(2) = 1.5(r_{h+1} - \hat{r}_h(1)) + \varepsilon_{h+2} = 1.5\varepsilon_{h+1} + \varepsilon_{h+2}$

and var $=$ var$(1.5\varepsilon_{h+1} + \varepsilon_{h+2}|\cdot) = 1.5^2$ var$(\varepsilon_{h+1}|\cdot) +$ var$(\varepsilon_{h+2}|\cdot) + \overset{0}{\cancel{2(1.5)\text{cov}(\varepsilon_{h+1}, \varepsilon_{h+2}|\cdot)}}$

$= 1.5^2(0.25) + 0.25 = 0.8125$

interval $= 0.6115 \pm (1.96)\sqrt{(0.8125)} = -1.155, 2.378$ #

∞ step · $r_{h+\ell} = \phi_0 + \phi_1 r_{h+\ell-1} + \phi_2 r_{h+\ell-2} + \varepsilon_{h+\ell}$

$\hat{r}_h(\ell) = E(r_t)$

var$(e(\infty)) =$ var(r_t)

interval $= E(r_t) \pm 1.96\, SD(r_t)$ #