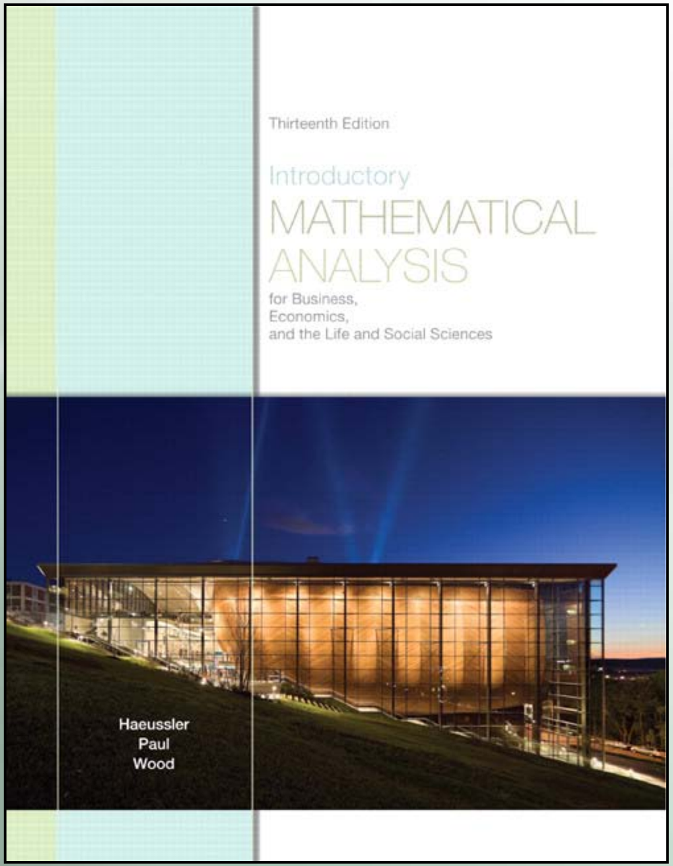
INTRODUCTORY MATHEMATICAL ANALYSIS

For Business, Economics, and the Life and Social Sciences

Chapter 3
Lines, Parabolas, and Systems



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Chapter Objectives

- To develop the notion of slope and different forms of equations of lines.
- To develop the notion of demand and supply curves and to introduce linear functions.
- To sketch parabolas arising from quadratic functions.
- To solve systems of linear equations in both two and three variables by using the technique of elimination by addition or by substitution.
- To use substitution to solve nonlinear systems.
- To solve systems describing equilibrium and break-even points.

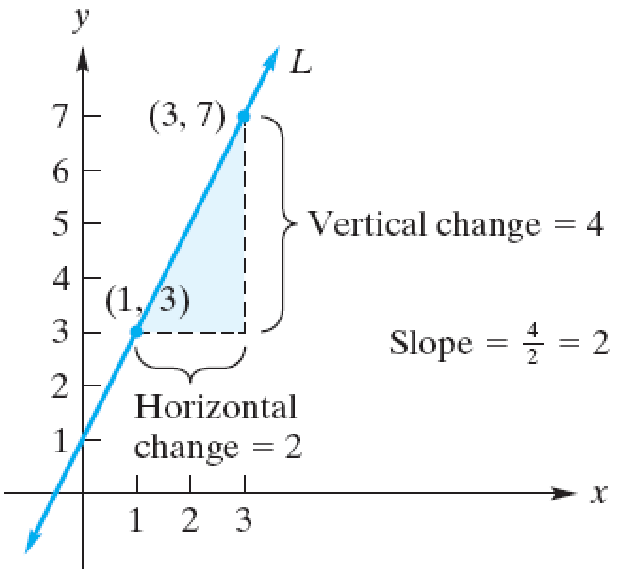
Chapter Outline

3.1 Lines

Slope of a Line

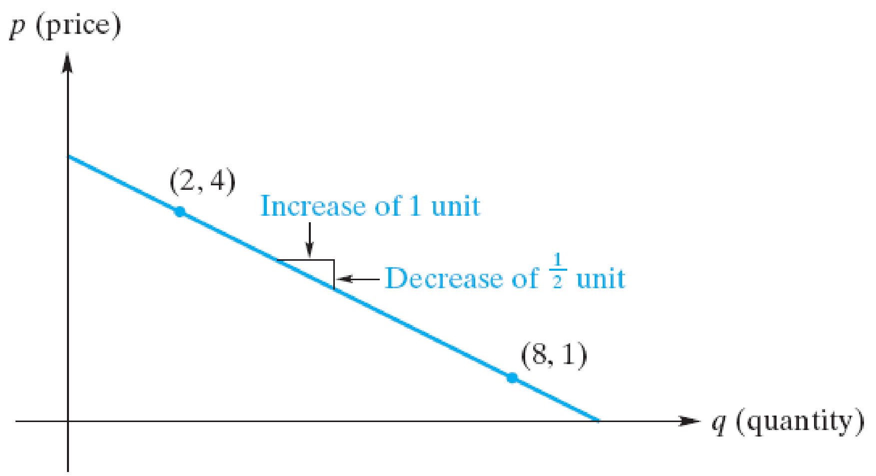
- The slope of the line is for two different points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} \left(= \frac{\text{vertical change}}{\text{horizontal change}} \right)$$



Example 1 – Price-Quantity Relationship

The line in the figure shows the relationship between the price p of a widget (in dollars) and the quantity q of widgets (in thousands) that consumers will buy at that price. Find and interpret the slope.



Solution:

Equations of lines

- A **point-slope form** of an equation of the line through (x_1, y_1) with slope *m* is

$$\frac{y_2 - y_1}{x_2 - x_1} = m$$

$$y_2 - y_1 = m(x_2 - x_1)$$

Example 3 – Determining a Line from Two Points

Find an equation of the line passing through $(-3, 8)$ and $(4, -2)$.

Solution:

- The **slope-intercept form** of an equation of the line with slope *m* and *y*-intercept *b* is $y = mx + c$.

Example 5 – Find the Slope and y-intercept of a Line

Find the slope and y-intercept of the line with equation y = 5(3 – 2x).

Solution:

Example 7 – Converting Forms of Equations of Lines

a. *Find a general linear form of the line whose slope-intercept form is*

$$y = -\frac{2}{3}x + 4$$

Solution:

b. Find the slope-intercept form of the line having a general linear form

$$3x + 4y - 2 = 0$$

Solution:

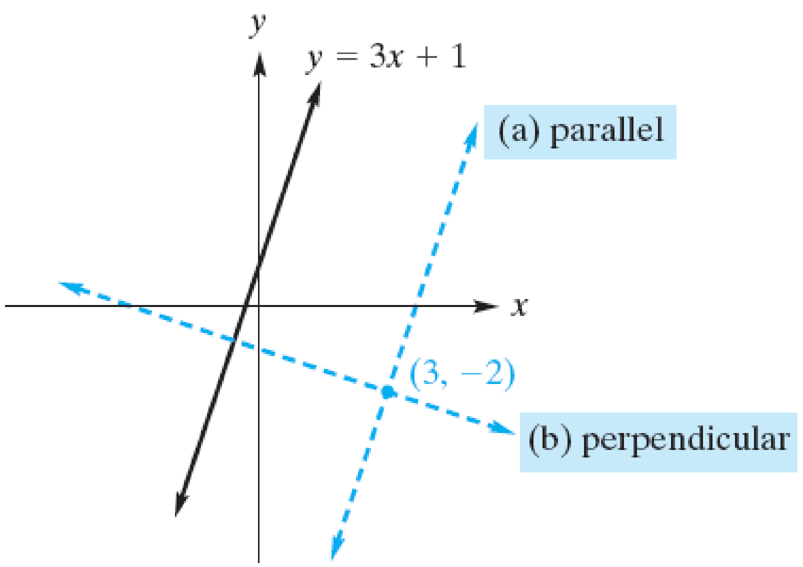
Parallel and Perpendicular Lines

- **Parallel Lines** are two lines that have the same slope.
- **Perpendicular Lines** are two lines with slopes m_1 and m_2 perpendicular to each other only if

$$m_1 = -\frac{1}{m_2}$$

Example 9 – Parallel and Perpendicular Lines

The figure shows two lines passing through (3, −2). *One is parallel to the line* $y = 3x + 1$, *and the other is perpendicular to it. Find the equations of these lines.*



Solution:

3.2 Applications and Linear Functions

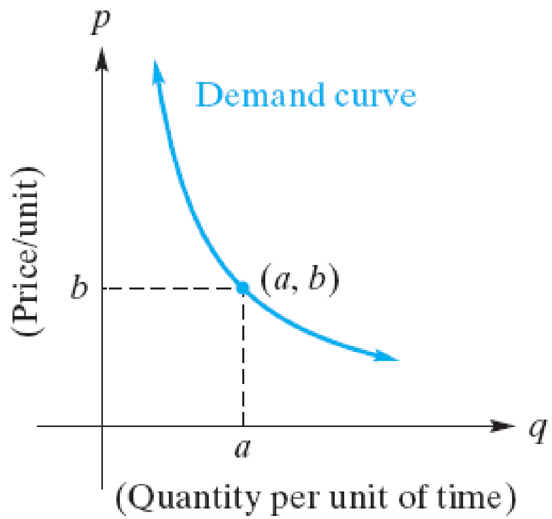
Example 1 – Production Levels

Suppose that a manufacturer uses 100 lb of material to produce products A and B, which require 4 lb and 2 lb of material per unit, respectively.

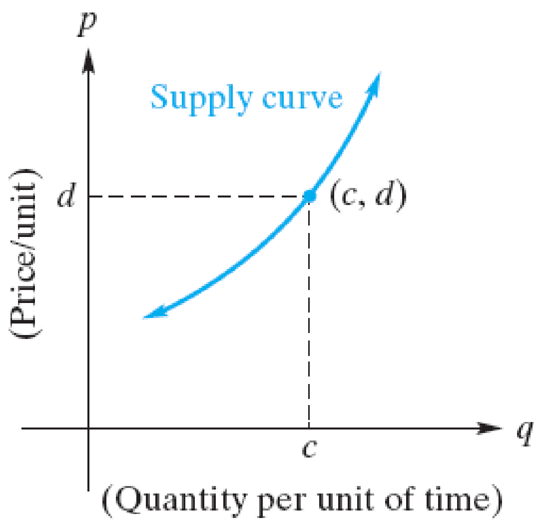
Solution:

Demand and Supply Curves

- Demand and supply curves have the following trends:



(a)



(b)

Linear Functions

- A function *f* is a *linear function* which can be written as $f(x) = ax + b$ where $a \neq 0$

Example 3 – Graphing Linear Functions

Graph $f(x) = 2x - 1$ *and* $g(t) = \dfrac{15 - 2t}{3}$.

Solution:

Example 5 – Determining a Linear Function

If $y = f(x)$ is a linear function such that $f(-2) = 6$ and $f(1) = -3$, find $f(x)$.

Solution:

3.3 Quadratic Functions

- **Quadratic function** is written as $f(x) = ax^2 + bx + 2$ where *a*, *b* and *c* are constants and $a \neq 0$

Example 1 – Graphing a Quadratic Function

Graph the quadratic function $f(x) = -x^2 - 4x + 12$*.*

Solution:.

Example 3 – Graphing a Quadratic Function

Graph the quadratic function $g(x) = x^2 - 6x + 7$.

Solution:

Example 5 – Finding and Graphing an Inverse

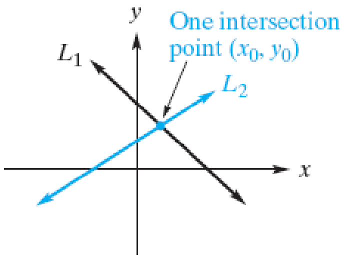
From $y = f(x) = ax^2 + bx + c$ *determine the inverse function for a = 2, b = 2, and c = 3.*

Solution:

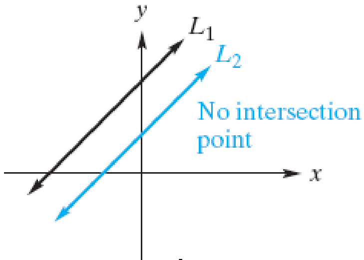
3.4 Systems of Linear Equations

Two-Variable Systems

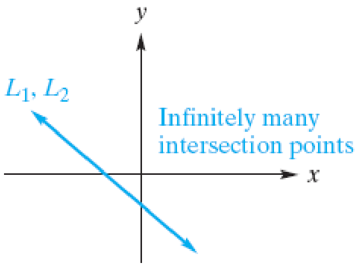
- There are three different linear systems:



Linear system
(one solution)



Linear system
(no solution)



Linear system
(many solutions)

- Two methods to solve simultaneous equations:

a) elimination by addition

b) elimination by substitution

Example 1 – Elimination-by-Addition Method

Use elimination by addition to solve the system.

$$\begin{cases} 3x - 4y = 13 \\ 3y + 2x = 3 \end{cases}$$

Solution: .

Example 3 – A Linear System with Infinitely Many Solutions

Solve
$$\begin{cases} x + 5y = 2 \\ \frac{1}{2}x + \frac{5}{2}y = 1 \end{cases}$$

Solution:

Example 5 – Solving a Three-Variable Linear System

Solve

$$\begin{cases} 2x + y + z = 3 \\ -x + 2y + 2z = 1 \\ x - y - 3z = -6 \end{cases}$$

Solution:

Example 7 – Two-Parameter Family of Solutions

Solve the system

$$\begin{cases} x + 2y + z = 4 \\ 2x + 4y + 2z = 8 \end{cases}$$

Solution:

3.5 Nonlinear Systems

- A system of equations with at least one nonlinear equation is called a **nonlinear system.**

Example 1 – Solving a Nonlinear System

Solve

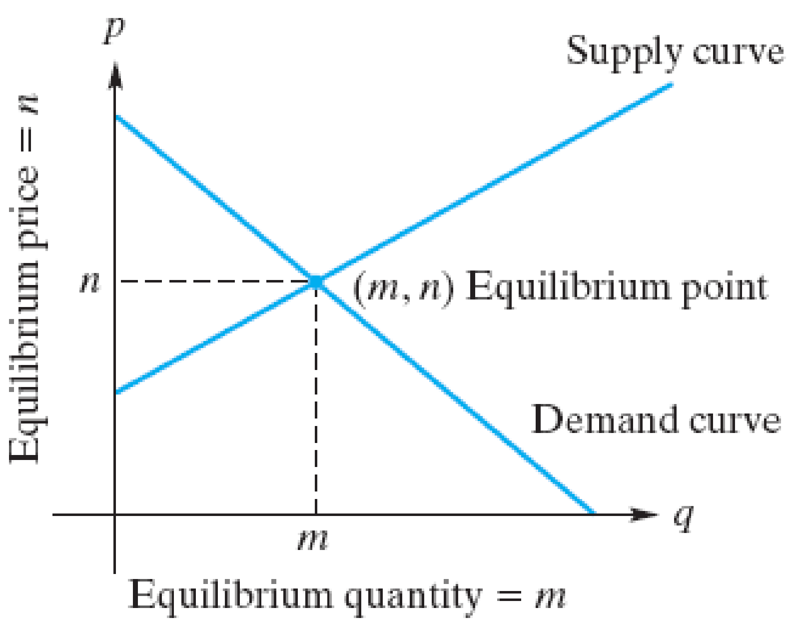
$$\begin{cases} x^2 - 2x + y - 7 = 0 & (1) \\ 3x - y + 1 = 0 & (2) \end{cases}$$

Solution:

3.6 Applications of Systems of Equations

Equilibrium

- The **point of equilibrium** is where demand and supply curves intersect.



Example 1 – Tax Effect on Equilibrium

Let $p = \frac{8}{100}q + 50$ *be the supply equation for a manufacturer's product, and suppose the demand equation is* $p = -\frac{7}{100}q + 65$*.*

a. If a tax of $1.50 *per unit is to be imposed on the manufacturer, how will the original equilibrium price be affected if the demand remains the same?*

b. Determine the total revenue obtained by the manufacturer at the equilibrium point both before and after the tax.

Example 1 – Tax Effect on Equilibrium

Solution:

a.

Example 1 – Tax Effect on Equilibrium

Solution:

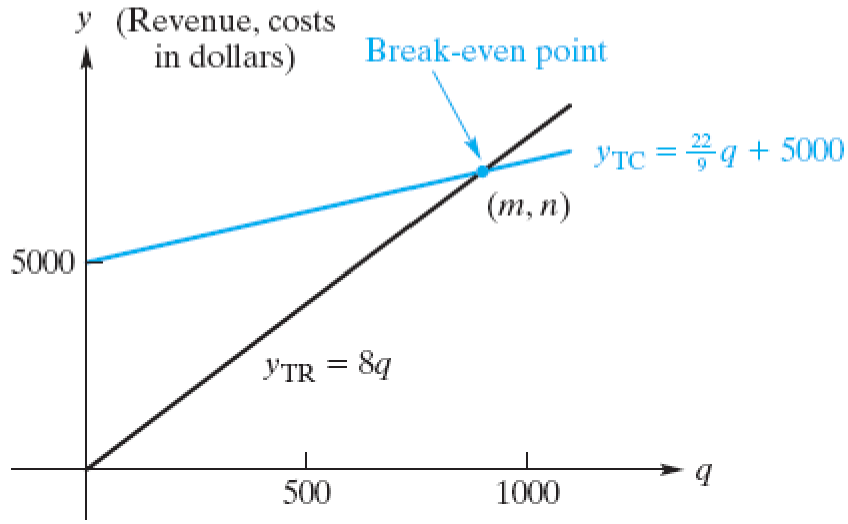
b.

Break-Even Points

- Profit (or loss) = total revenue(TR) – total cost(TC)
- Total cost = variable cost + fixed cost

$$y_{TC} = y_{VC} + y_{FC}$$

- The **break-even point** is where TR = TC.



Example 3 – Break-Even Point, Profit, and Loss

A manufacturer sells a product at $8 *per unit, selling all that is produced. Fixed cost is* $5000 *and variable cost per unit is* 22/9 *(dollars)*.

a. *Find the total output and revenue at the break-even point.*

b. *Find the profit when 1800 units are produced.*

c. *Find the loss when 450 units are produced.*

d. *Find the output required to obtain a profit of $10,000.*