Practice problem set 8

Integration and its application in economics.

Instructor: Kittichai Saelee

Question 1:

In the manufacture of a product, the marginal cost of producing x units is C'(x) and fixed cost are C(0). Find the total cost function C(x) when:

a. *C'(x) = 3x + 4, C(0) = 40*.

b. *C'(x) = ax + b, C(0) = C_0*.

Question 2

Let *K(t)* denote the capital stock of an economy at time *t*. Then net investment at time *t*, denoted by *I(t)*, is given by the rate of increase $\frac{dK}{dt}$ of K(t).

a. If $I(t) = 3t^2 + 2t + 5, t \geq 0$, what is the total increase in the capital stock during the interval from t = 0 to t = 5?

b. If $K(t_0) = K_0$, find an expression for the total increase in the capital stock from time *t* = t_0 to *t* = *T* when the investment function *I(t)* is as in part (a).

Question 3:

Given the following demand and supply curves, compute the consumer and producer surplus.

a. Demand: $P = 200 - 0.2Q$; Supply: $P = 20 + 0.1Q$.

b. Demand: $P = \frac{6000}{Q+50}$; Supply: $P = Q + 10$.

Question 4

Suppose that the profit of a firm as a function of its output *x* is given by

$f(x) = 4000 - x - \frac{3000000}{x},\ x > 0$

a. Find the level of output that maximizes profit. Sketch the graph of f.

b. The actual output varies between 1000 and 3000 units. Compute the average profit $I = \frac{1}{2000}\int_{1000}^{3000} f(x)dx.$

Question 5:

Evaluate the following integrals by using integrations by substitution:

a. $\int_0^1 x\sqrt{1+x^2}\,dx$

b. $\int_1^e \frac{lny}{y}dy$

Question 6:

Evaluate the following integrals by using integrations by using integrations by parts (r ≠ 0).

a. $\int_0^T bte^{-rt}dt$

b. $\int_0^T (a+bt)e^{-rt}dt$

Question 7:

a. Evaluate $\int_0^1 x^p(x^q + x^r + x^s)dx$ where $p,\ q,\ r,\ and\ s$ are positive numbers.

b. Let $F(x) = \int_0^x (t^2 + 2)dt$ and $G(x) = \int_0^{x^2} (t^2 + 2)dt.$ Find F'(x) and G'(x).

Question 8:

Let the demand and supply of goods Q in a perfectly competitive market be the followings;

Demand Function : $P = 25 - Q^2$

Supply Function : $P = 2Q + 1$

a. Determine the consumer surplus as the equilibrium.
b. If the government imposes tax on consumers for $4 per unit of production, calculate the deadweight loss.

Question 9:

Let $P = 274 - Q^2$ be the demand function in a monopoly market. Suppose further that marginal cost of the monopolist is given by $MC = 4 + 3Q$.

a. Determine consumer's surplus at the profit-maximizing production level.
b. Calculate deadweight loss under monopoly.

Question 10:

A company has MC = 80 where the demand function is P = 1400-6Q. At zero production, the company faces loss by $1,500. Determine the maximum profit by using integral calculus and prove your answer.