

# THEORIES OF ECONOMIC GROWTH

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EE 462 Development Macroeconomics

Semester 1/2014

# Topics

- The Basic Growth Model
- The Harrod-Domar Model
- The Solow (Neoclassical) Growth Model
- Diminishing Returns and the Production Function
- Beyond Solow: New Approaches to Growth

# Introduction

- In previous lectures, we examined the basic process and patterns that characterize economic growth based on empirical approach.
- This topic explores key contributions to the theory of economic growth based on theoretical models.
- Growth models provide consistent frameworks for understanding the growth process (capital accumulation and productivity gain) and theoretical foundation for empirical studies.
  - Need to identify specific mathematical relationship between the quantity of capital and labor, their productivity, and the resulting aggregate output.

# The Basic Growth Model

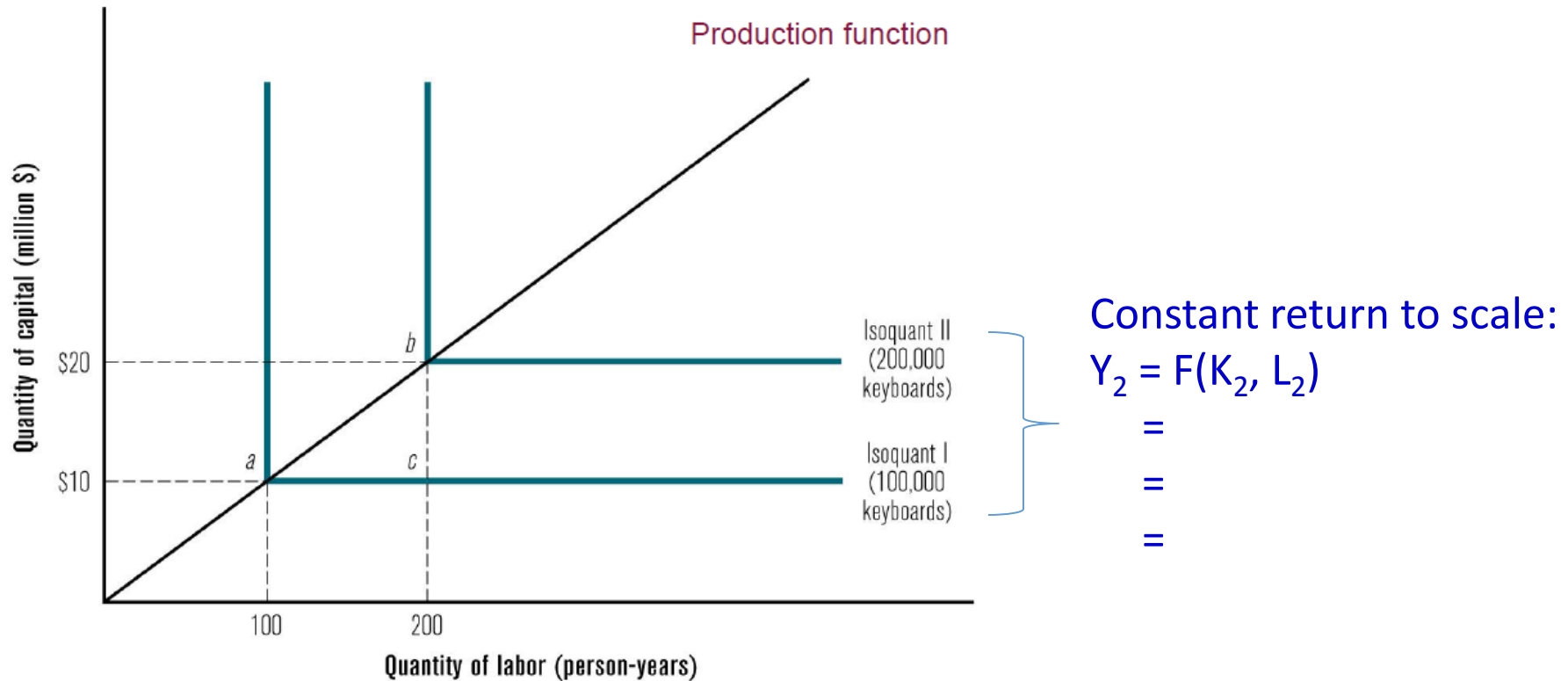
- The **aggregate production function** is based on five equations:
  - Aggregate production function:  $Y = F(K, L)$  -- (1)
  - Saving is a fixed share of income:  $S = sY$  -- (2)
  - Assume a closed economy:  $S = I$  (saving = investment)
  - Change in capital stock:  $\Delta K = I - (d \times K)$ , -- (3)  
where  $d$ =depreciation rate
  - Assume the labor force grows exactly as fast as the total population. Let  $n$  =population growth and  $L$ =Labor force, then:  $\Delta L = n \times L$ . -- (4)
- Based on equations (2), (3), and (4), we have:

$$\Delta K = \text{----} (6)$$

# The Harrod-Domar Growth Model

- Harrod-Domar Growth model is a particular model with basic feature of fixed coefficient production function or “Leontief” production function.
  - Example:  $Y = F(K, L) = \min(K, L)$ 
    - Capital-labor ratio =
- It assumes *no substitution* between labor and capital.
- The production isoquant is L-shaped.
- It also shows constant returns to scale (CRS).
  - i.e. doubling inputs will double output

# Fixed-Coefficient Production Function



- At point  $a$ , capital-labor ratio =
- At point  $b$ , capital-labor ratio =
  - Constant capital-labor ratio

# Harrod-Domar Production Function

- The Harrod-Domar production function:

$$Y = (1/v) \times K \quad \text{or} \quad Y =$$

- $v = K/Y$  : “capital output ratio” or measure of the productivity of capital or investment (indication of capital intensity)

- Incremental capital-output ratio is fixed:  $ICOR = v \rightarrow \Delta Y = \underline{\hspace{2cm}}$

- Growth rate of output:  $g = \Delta Y/Y = \underline{\hspace{2cm}}$

- Since  $\Delta K = sY - dK$ , growth rate of output is equal to:

$$g =$$

$$g =$$

- *K created by I is the main determinant of growth in output.*

# Examples: Harrod-Domar Model

- If  $v=4$ , then how much investment (or capital) will be needed to produce 5 million of output?

→ Based on  $v = K/Y$ , so

$$K = \underline{\hspace{2cm}}$$

- Suppose  $s = 0.24$ ,  $v = 3$ , and  $d = 0.05$ . What is the growth rate of this economy?

→  $g =$

Thus, the economy will grow at \_\_\_\_.

# Strengths and Weaknesses of HD Model

- *Strengths*

- The model works well in the absence of severe economic shocks, or only in a short period of time.
- Focus on the key role of saving – important driver of growth

- *Weaknesses*

- It assumes saving is necessary and *sufficient* for growth.
- The assumptions of fixed K-L, L-Y, and L-Y ratios are too rigid.
  - This is true only when K, L, and Y grow at the same rate (Ex.  $n = g = s/v - d$ ). → Not always true!
  - ICOR is not constant over time (see the case of Thailand).
- Productivity growth plays no role.
  - An increase in factor productivity implies a smaller ICOR → contradiction!

# Case Study: Economic Growth in Thailand

- Thailand in 1960 was an agrarian economy with 75% of population in agriculture, GDP was about \$1000.
- Beginning 1970 Thailand began to save averaging 20%, reaching 35% in 1990s, and falling to 32% in 2000s. This combined with good governance and prudent policies led to rapid economic growth.
- After the financial crisis, GDP fell by 10% in 1998. Then, between 1999-2007, the growth rebounded to 3.9%.
- During 1960-2007, average per capita income grew at 4.5%.
- *Consistent with HD*: High saving rate and  $\uparrow K \rightarrow$  increase in income
- *Contradict to HD*: ICOR rose from 2.6 in 1970s to  $\sim 5$  by early 2000s (due to shifts toward more capital-intensive production).

# The Solow Model

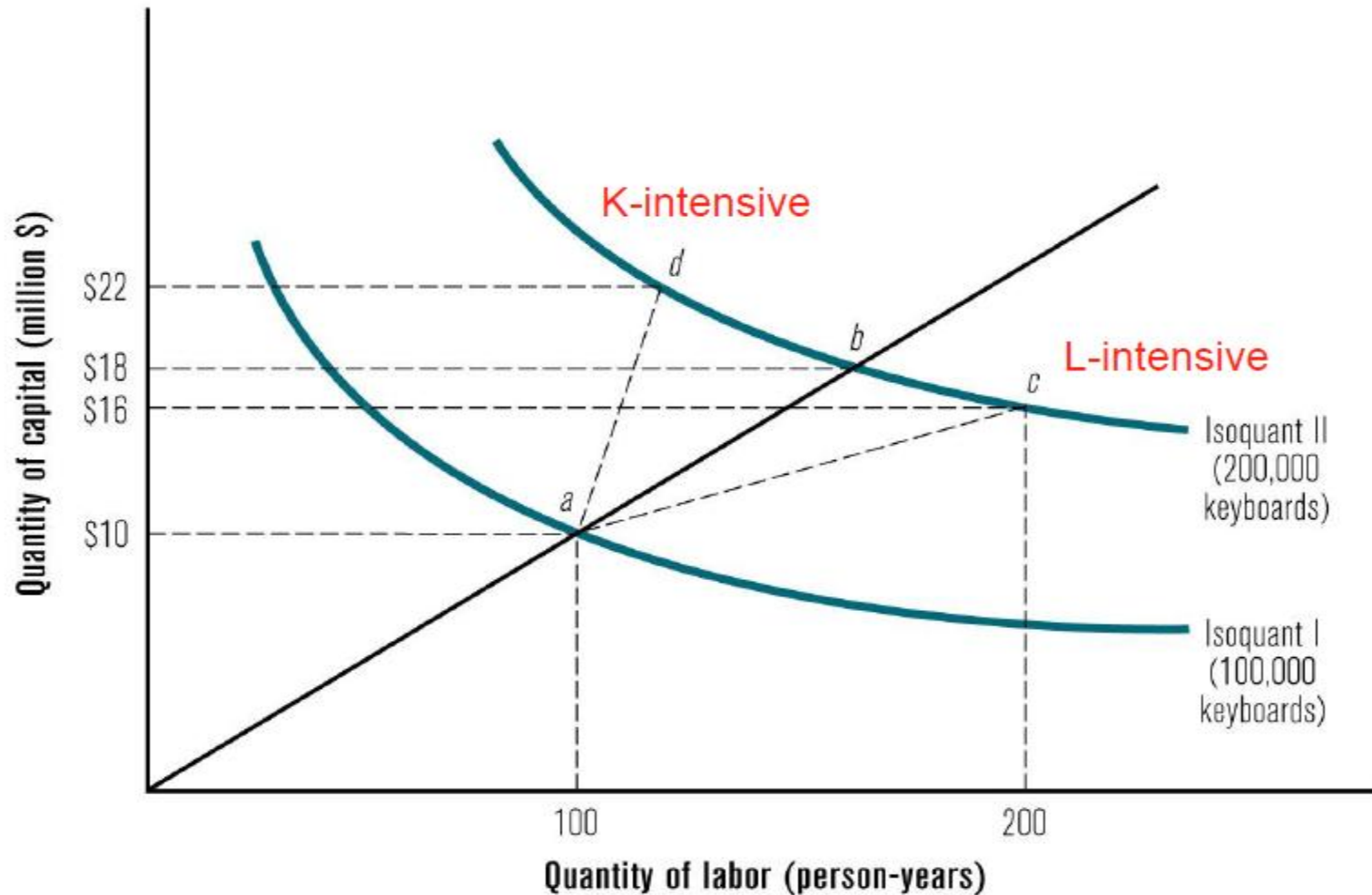
- Introduced in 1956 by **Robert Solow**, this model drops fixed-coefficient production function and replaces with a **neoclassical production function**:

$$Y = F(K, L)$$

Where **labor and capital are substitutable**.

- The **isoquants are curved** (convex and smooth) rather than L-shaped.
  - Output can be achieved with different combinations of K and L.
- This model still assumes **constant returns to scale** of the production function.

# Isoquants for a Neoclassical Production Technology



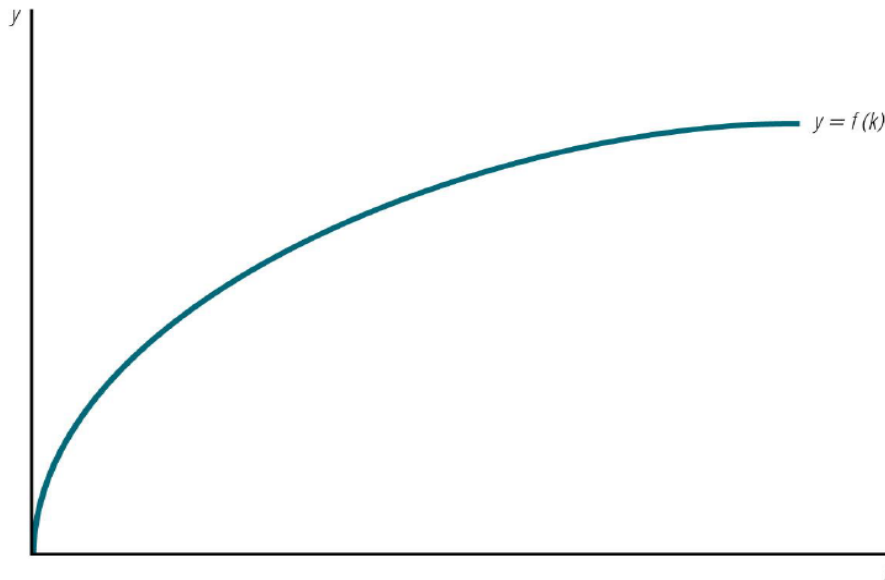
# Basic Equations of the Solow Model (1)

- All key variables are expressed in terms of per-worker:

$$Y/L = F(K/L, 1)$$

→  $y = f(k)$  -- (1) , where  $y = Y/L$  and  $k = K/L$ .

- Assume that  $y = f(k)$  exhibits **diminishing returns to capital**.



## Basic Equations of the Solow Model (2)

- From  $\Delta K = sY - dK$ , capital accumulation can be written as:

$$\Delta k = sy - (n+d)k \quad \text{-- (2)}$$

- Derivation of equation (2):

From  $\Delta K = sY - dK$ , we have:  $\Delta K/K = sY/K - d$

Then,  $\Delta k/k =$

→  $\Delta k =$

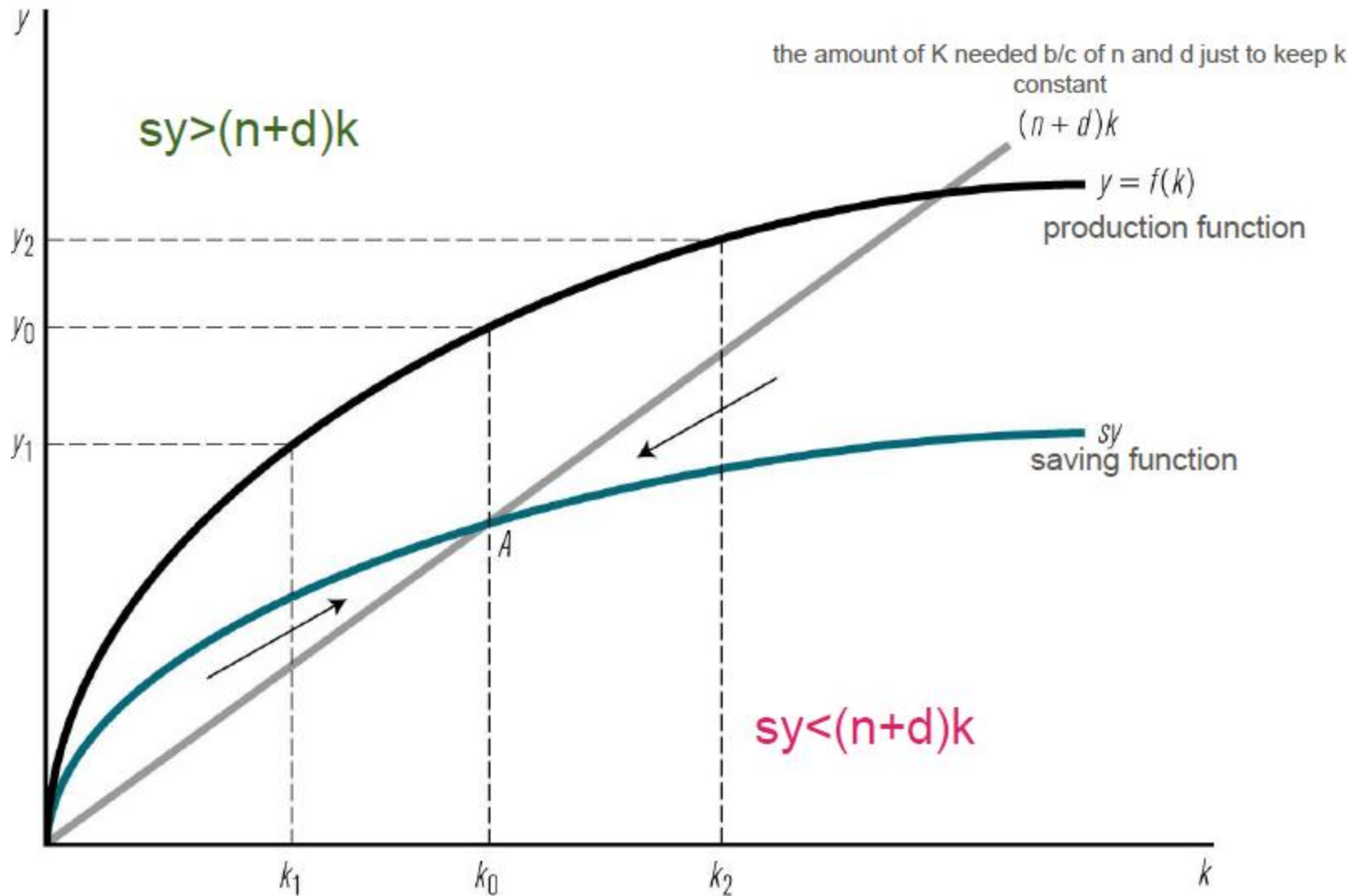
- Interpretations of equation (2):
  - $\Delta k$  is positively related to saving per worker.
  - $\Delta k$  is negatively related to population growth.
  - Depreciation erodes the capital stock.

Thus, saving (and investment) adds to capital per worker, whereas labor force growth and depreciation reduce capital per worker.

# Basic Equations of the Solow Model (3)

- *Capital deepening*: the process through which the economy increases the amount of capital per worker (i.e.  $\Delta k$ )
- *Capital widening*: a widening of both the total amount of capital and the size of the workforce
  - This occurs when  $sy = (n+d)k$  (i.e.  $\Delta k = 0$ ).
- Thus,  $\Delta k = sy - (n+d)k$  means “capital deepening ( $\Delta k$ ) is equal to saving per worker ( $sy$ ) minus the amount needed for capital widening  $[(n+d)k]$ .”
  - If  $sy > (n+d)k$ , then  $\Delta k \_ 0$  (i.e. ).
  - If  $sy < (n+d)k$ , then  $\Delta k \_ 0$  (i.e. ).
  - If  $sy = (n+d)k$ , then  $\Delta k = 0$ .

# The Solow Diagram



# Steady State of the Solow Model

- Point A is where **new savings ( $sy$ )** equals the amount of **new capital needed for growth in the labor force and depreciation  $[(n+d)k]$** . → the **steady state** of the Solow model.
- The output level at the steady state ( $y_0$ ) is referred to as the **steady state (or long run or potential)** level of output per worker.
- Note: at the steady state, **total output** continues to grow at the same rate the **population** and **labor force** grows ( $n$ ), but output per worker ( $y$ ) is constant (i.e. average income remains unchanged).
  - Why?
  - What about the rate of change in total capital and total saving.

# An Increase in the Saving Rate in the Solow Model

- An increase in the saving rate results in an upward shift in the capital deepening curve. → \_\_\_\_\_
- Higher saving rate leads to a *temporary* increase in the economic growth, but the long-run output growth rate remains at  $n$ . Why?

# An Increase in in Population Growth in the Solow Model

- An increase in the pop growth rate causes the capital widening curve to rotate to the left. → \_\_\_\_\_.
- Higher population growth rate ( $n' > n$ ) leads to higher growth rate of total income ( $Y$ ), whereas the per capita income decreases ( $y_0$  to  $y_4$ ).

# Technological Change in the Solow Model

- Introduce the **labor-augmenting technological change (T)**.
- The new production function with technological progress is:

$$Y = F(K, T \times L)$$

*T x L = amount of effective units of labor*

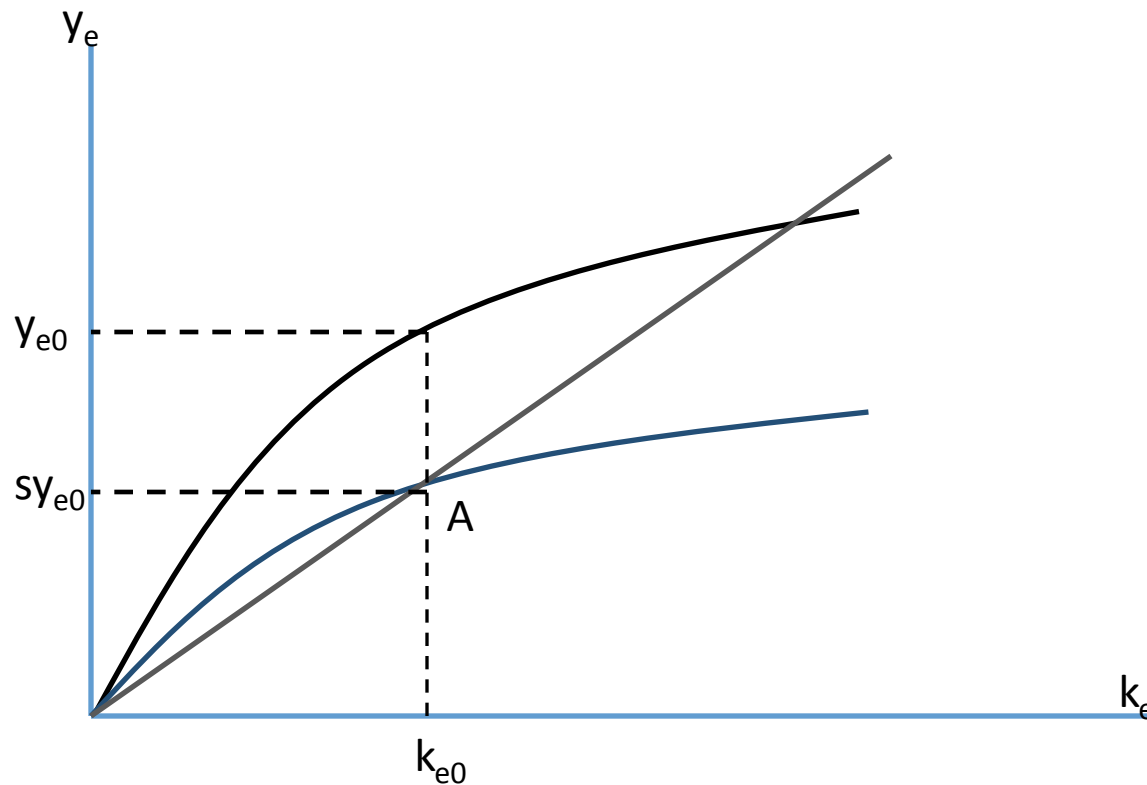
➔ The growth rate of effective supply of labor =  $n + \theta$  (why?)

- **Output per effective worker** is defined as:  $y_e = Y/(T \times L)$
- **Capital per effective worker** is defined as:  $k_e = K/(T \times L)$
- Then, the capital accumulation equation changes to:

$$\Delta k_e =$$

- To keep constant, *saving per effective worker* must be equal to the amount of *new capital needed to compensate for changes in the size of labor force, depreciation, and technological change*.

# The Solow Model with Technical Change



- At the steady state, output per effective worker ( $y_e$ ) is constant.
- Total output grows at the rate  $n+\theta$ , so that the income per capita increases at rate  $\theta$ .

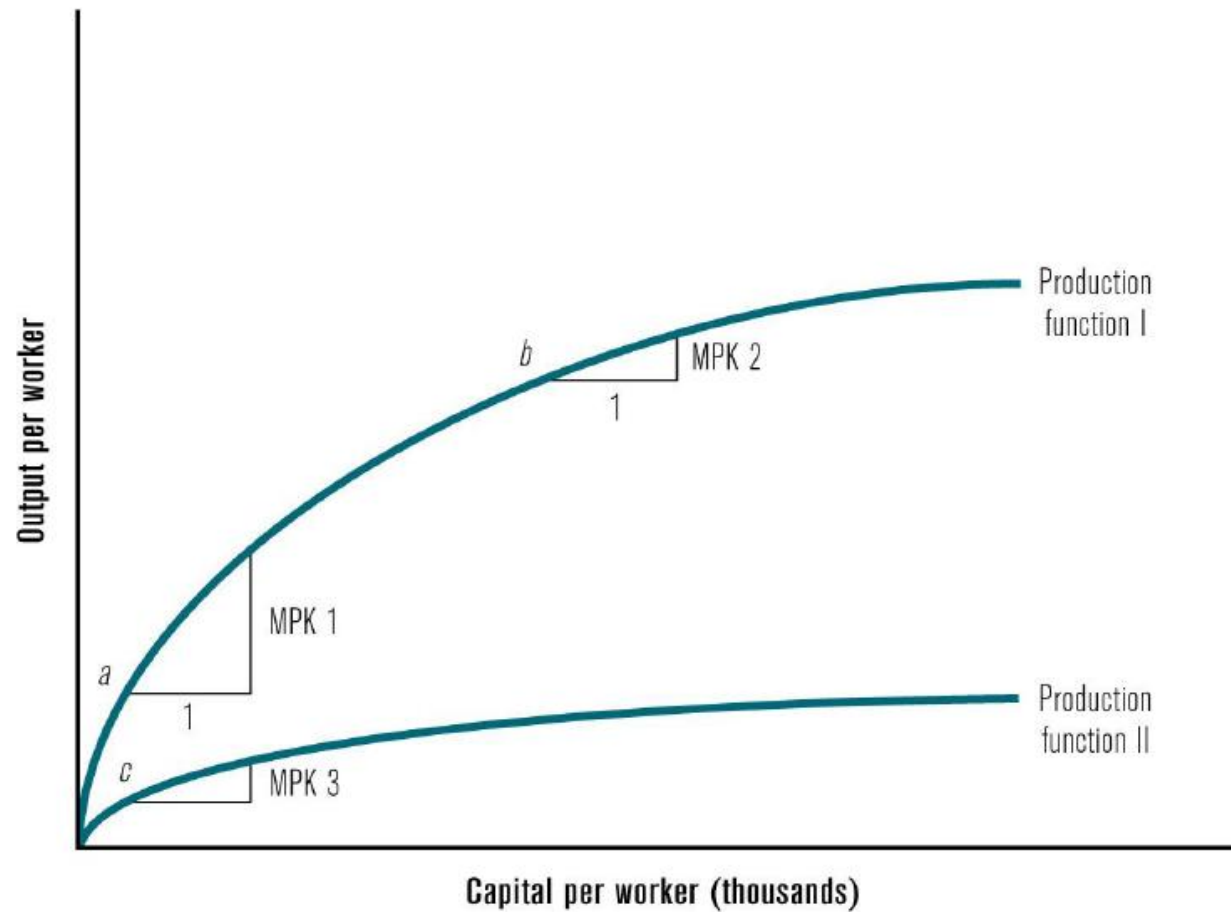
# Strengths and Weaknesses of Solow Model

- *Strength:*
  - It allows for substitution between inputs (need not be fixed).
  - Provides good insights about the **role of technology** and **productivity growth** in the growth process.
- *Weaknesses:*
  - It specifies **productivity growth** as *exogenous*.
    - Didn't specify how it takes place, or how the growth process itself might affect productivity.
  - One sector approach, factors that drive steady state, and assumes saving rate, population growth, and technical change as given.
    - It does not explain how these parameters change over time

# Diminishing Returns and the Production Function

- Three implications of diminishing MP of capital:
  1. Poor countries have a *potential* to grow more rapidly than do rich countries (b/c they face capital scarcity).
  2. Richer countries with capital abundance grow slowly.
  3. Since poor countries have more *potential* to grow faster than do rich countries, they can *catch up* and close the gap in relative income.
- However, the above implications rest on the assumption that all else is equal between the two countries (i.e. same  $s$ ,  $n$ ,  $d$ , etc.).
- But, if the two countries do not have the same technology then the predictions for rich and poor countries might not hold.
  - Ex. If the production function is flatter, then poor countries might not grow faster than rich countries and may never catch up.

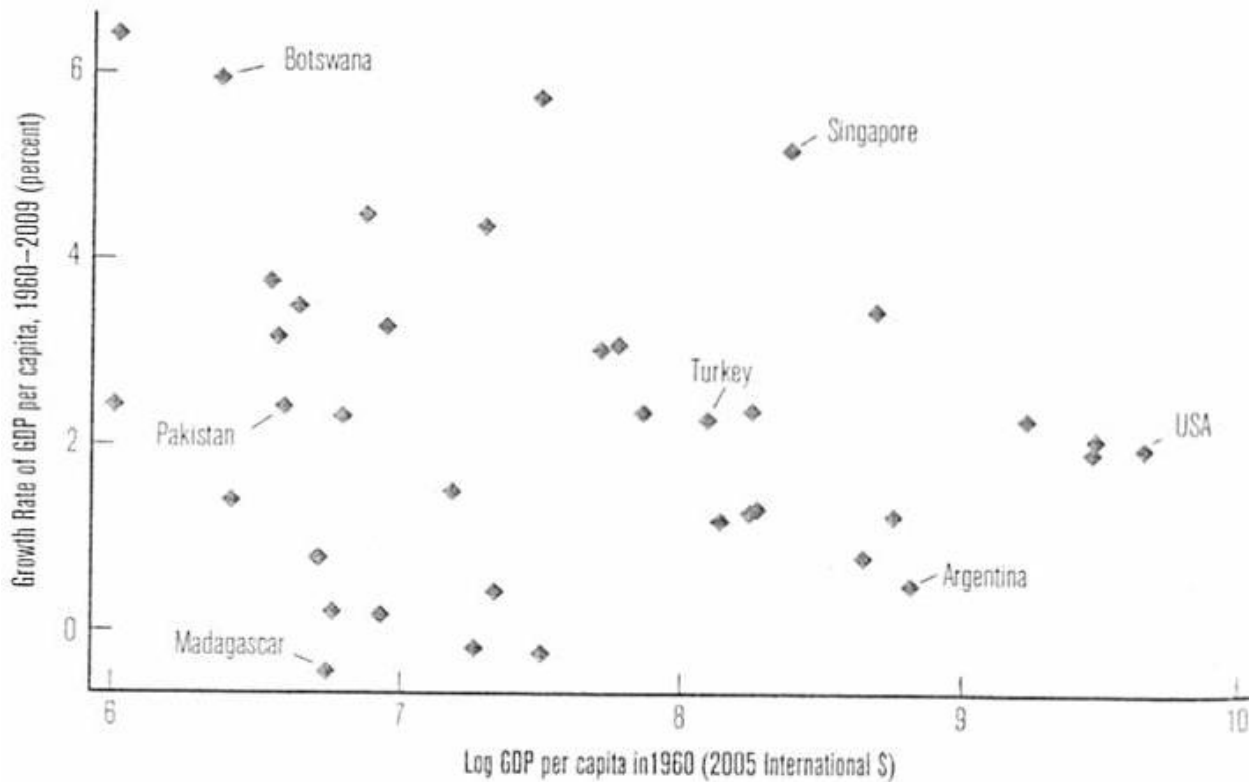
# Diminishing Marginal Product of Capital



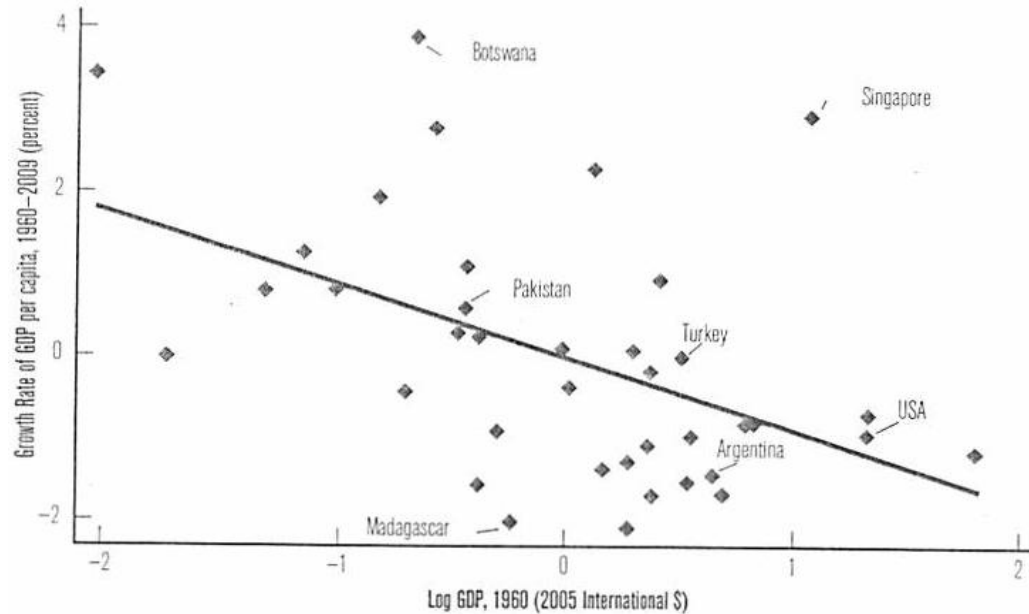
# The Convergence Debate

- Question: Has convergence actually happened?
  - Yes, but only for some countries that share the same features.
  - Ex. Japan Other?
- In general, there is no “**unconditional convergence**.”
  - *Unconditional* – assumption that all countries share the same key parameters (pop growth, saving rate, depreciation).
- But there may be “**conditional convergence**” across rich and poor countries.
  - By allowing countries to have their steady states, which are **conditional upon the countries’ population growth rate (and other characteristics)**, the growth rates tend to be higher for poor countries and smaller for rich countries.

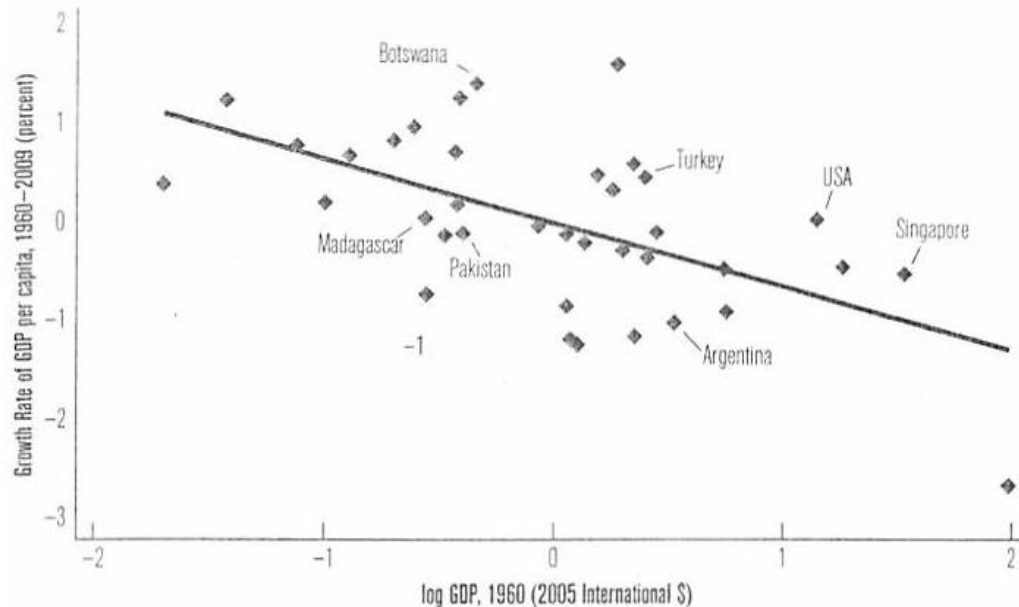
# GDP Growth, Unconditional



GDP growth, conditional on population growth



GDP growth, conditional on opennes, savings, and population growth



# Beyond the Solow Model: New Approaches to Growth

- The Solow model assumes **exogenously** fixed saving rate, growth rate of labor supply, and the pace of technological change.
- Recent works provides models where these variables are **determined within** or **endogenously** in the model.
- These new models allow for **increasing returns to scale** and **positive externalities**.
  - The impact of investment in K or L would be larger than suggested by Solow.
  - New knowledge may have larger contribution to economic growth.
  - Economies do not necessary reach a steady-state level of income.
- They are called ***endogenous growth models***, which we will study in the next topic.