HW 2 EE 325 due September 16th, 2011 ☺ (submit in class) ☺

1. If X_1, X_2, X_3 is a random sample from a population with mean μ and variance σ^2.

   X_1, X_2, X_3 are not independent $Cov(X_1, X_2) = Cov(X_1, X_3) = Cov(X_2, X_3) = \frac{1}{2}\sigma^2$

   $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

   Find $E(\bar{X})$ and $\mathrm{var}(\bar{X})$

2. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean μ and variance σ^2. $\bar{X}$ is estimator used to estimate mean value.

   $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3 + X_4)$

   2.1 Find $E(\bar{X})$ and $\mathrm{var}(\bar{X})$ in term of μ and σ

   2.2 Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ. Show that $\tilde{X}$ is an unbiased estimator of μ

   2.3 Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ? Why?

3. Data is listed in the table

X_i	Y_i
10	0
12	2
14	4
16	6
18	8
22	12
24	14
26	16
28	18
30	20

   3.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$ Find estimators of β_1 and β_2 from the OLS method and interpret the meaning

   3.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

   3.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

   3.4 If $X_i = 25$, what is the predicted Y?

   3.5 Find $\mathrm{var}(\hat{u}_i)$, $\mathrm{var}(\hat{\beta}_1), \mathrm{var}(\hat{\beta}_2)$