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EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. $X_i$is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3.0	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$
Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

1.2 Find $\hat{Y}_i \quad and\ \hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1), var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$\boldsymbol{Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)}$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

"Practice makes Perfect."

(1)

Student	Y_i	X_i	$X_i - \bar{X}$	$(X_i - \bar{X})^2$	$Y_i - \bar{Y}$	$(X_i - \bar{X})(Y_i - \bar{Y})$
1	2.8	63	-14.625	213.890625	-0.4125	6.0328125
2	3.4	72	-5.625	31.640625	0.1875	-1.0546875
3	3	78	0.375	0.140625	-0.2125	-0.0796875
4	3.5	81	3.375	11.390625	0.2875	0.9703125
5	3.6	87	9.375	87.890625	0.3875	3.6328125
6	3.0	75	-2.625	6.890625	-0.2125	0.5578125
7	2.7	75	-2.625	6.890625	-0.5125	1.3453125
8	3.7	90	12.375	153.140625	0.4875	6.0328125
	$\Sigma Y_i = 25.7$	$\Sigma X_i = 621$	$\Sigma(X_i - \bar{X}) = 0$	$\Sigma(X_i - \bar{X})^2 = 511.875$	$\Sigma(Y_i - \bar{Y}) = 0$	$\Sigma(X_i - \bar{X})(Y_i - \bar{Y}) = 17.4375$

(1.1) $\bar{x} = \frac{\Sigma x_i}{n} = \frac{621}{8} = 77.625$ $\qquad \bar{Y} = \frac{\Sigma Y_i}{n} = \frac{25.7}{8} = 3.2125$

$$\hat{\beta}_2 = \frac{\Sigma(x_i - \bar{x})(Y_i - \bar{Y})}{\Sigma(x_i - \bar{x})^2} = \frac{17.4375}{511.875} = 0.0341$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{x} = 3.2125 - 0.0341(77.625) = 0.5655$$

$$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 x_i$$

$$\hat{Y}_i = 0.5655 + 0.0341\, x_i$$

$\hat{\beta}_1 = 0.5655$ mean that if point equal to zero, student's GPA is 0.5655

$\hat{\beta}_2 = 0.0341$ mean that if exam point change by 1 point, on average, student's GPA will change by 0.0341

(1.2)

Y_i	X_i	$\hat{Y}_i$	$\hat{u}_i = Y_i - \hat{Y}_i$	$\hat{u}_i^2$	X_i^2	$(Y_i - \bar{Y})^2$
2.8	63	2.7138	0.0862	0.00743044	3969	0.17015625
3.4	72	3.0207	0.3793	0.14386849	5184	0.03515625
3	78	3.2253	-0.2253	0.05076009	6084	0.04515625
3.5	81	3.3276	0.1724	0.02972176	6561	0.8265625
3.6	87	3.5322	0.0678	0.00459684	7569	0.15015625
3	75	3.123	-0.123	0.015129	5625	0.04515625
2.7	75	3.123	-0.423	0.178929	5625	0.2625625
3.7	90	3.6345	0.0655	0.00429025	8100	0.23765625
			$\Sigma \hat{u}_i = -0.0001$, $\Sigma \hat{u}_i \approx 0$	$\Sigma \hat{u}_i^2 = 0.43472587$	$\Sigma X_i^2 = 48717$	$\Sigma(Y_i - \bar{Y})^2 = 1.02875$

$$\Sigma \hat{u}_i \approx 0$$

(1.3) $VAR(\hat{u}_i) = \hat{\sigma}^2 = \frac{\Sigma \hat{u}_i^2}{n-2} = \frac{0.43472587}{8-2} = 0.0725$

$$VAR(\hat{\beta}_1) = \frac{\hat{\sigma}^2 \Sigma x_i^2}{n \Sigma(x_i - \bar{x})^2} = \frac{0.0725(48717)}{8(511.875)} = 0.8625$$

$$VAR(\hat{\beta}_2) = \frac{\sigma^2}{\Sigma(x_i - \bar{x})^2} = \frac{0.0725}{511.875} = 0.000142$$

(2)

(2.1)

Y_i	X_i	$\hat{Y}_i$	$\hat{u}_i = Y_i - \hat{Y}_i$	$\hat{u}_i^2$	X_i^2	$(Y_i - \bar{Y})^2$
2.8	63	2.7138	0.0862	0.00743044	3969	0.17015625
3.4	72	3.0207	0.3793	0.14386849	5184	0.03515625
3	78	3.2253	-0.2253	0.05076009	6084	0.04515625
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2.7	75	3.123	-0.423	0.178929	5625	0.2625625
3.7	90	3.6345	0.0655	0.00429025	8100	0.23765625
			$\Sigma\hat{u}_i = -0.0002$ $\Sigma\hat{u}_i \approx 0$	$\Sigma\hat{u}_i^2 = 0.43472587$	$\Sigma X_i^2 = 48717$	$\Sigma(Y_i - \bar{Y})^2 = 1.02875$

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{200}{10} = 20$$

$$\bar{Y} = \frac{\Sigma Y_i}{n} = \frac{91}{10} = 9.1$$

$$\hat{\beta}_2 = \frac{\Sigma(x_i - \bar{x})(Y_i - \bar{Y})}{\Sigma(x_i - \bar{x})^2} = \frac{394}{440} = 0.8955$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2\bar{x} = 9.1 - (0.8955)(20) = -8.81$$

$$\hat{Y}_i = -8.81 + 0.8955\, x_i$$

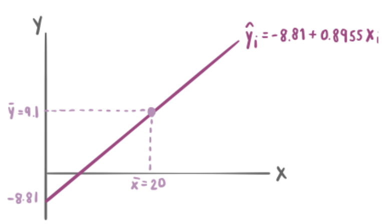
$\hat{\beta}_2 = 0.8955$ means that if x change by 1 unit, Y change 0.8955

(2.2)

X_i	Y_i	$\hat{Y}_i$	$\hat{u}_i = Y_i - \hat{Y}_i$	$\hat{u}_i^2$	X_i^2
10	0	0.145	-0.145	0.021025	100
12	2	1.936	0.064	0.004096	144
14	5	3.727	1.273	1.620529	196
16	6	5.518	0.482	0.232324	256
18	7	7.309	-0.309	0.095481	324
22	10	10.891	-0.891	0.793881	484
24	10	12.682	-2.682	7.193124	576
26	15	14.473	0.527	0.277729	676
28	16	16.264	-0.264	0.069696	784
30	20	18.055	1.945	3.783025	900
$\Sigma X_i = 200$	$\Sigma Y_i = 91$		$\Sigma\hat{u}_i \approx 0$	$\Sigma\hat{u}_i^2 = 14.09091$	$\Sigma X_i^2 = 4{,}440$

$$\Sigma\hat{u}_i \approx 0$$

(2.3)



$$\hat{Y}_i = -8.81 + 0.8955\, x_i$$

$$\bar{Y} = -8.81 + 0.8955\,\bar{x}$$

$$\bar{Y} = -8.81 + 0.8955\,(20)$$

$$\bar{Y} = 9.1$$

$\therefore$ Regression line passes $\bar{x}, \bar{Y}$

(2.4) $x_i = 16$; $\hat{Y}_i = -8.81 + 0.8955(16)$

$\hat{Y}_i = 5.518$

(2.5) $VAR(\hat{u}_i) = \hat{\sigma}^2 = \frac{\Sigma \hat{u}_i^2}{n-2} = \frac{14.09091}{10-2} = 1.7614$

$VAR(\hat{\beta}_1) = \frac{\hat{\sigma}^2 \Sigma x_i^2}{n\Sigma(x_i-\bar{x})^2} = \frac{(1.7614)(4440)}{10(440)} = 1.7774$

$VAR(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\Sigma(x_i-\bar{x})^2} = \frac{1.7614}{440} = 0.004$

(3) $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 \hat{x}_i + \hat{u}_i$

min $\Sigma \hat{u}_i^2$ $\hat{u}_i = \hat{y}_i - \hat{\beta}_1 - \hat{\beta}_2 x_i$

$\Sigma \hat{u}_i^2 = \Sigma(\hat{y}_i - \hat{\beta}_1 - \hat{\beta}_2 \hat{x}_i)^2$

take differential : $-2\Sigma(\hat{y}_i - \hat{\beta}_1 - \hat{\beta}_2 \hat{x}_i) = 0$

$\Sigma \hat{Y}_i - \Sigma \hat{\beta}_1 - \beta_2 \Sigma \hat{x}_i = 0$

$\Sigma \hat{\beta}_1 = \Sigma \hat{Y}_i - \beta_2 \Sigma \hat{x}_i$

$n\hat{\beta}_1 = \Sigma \hat{Y}_i - \beta_2 \Sigma \hat{x}_i$

$\div n$; $\beta_1 = \bar{Y} - \beta_2 \bar{x}$

$E(\hat{\beta}_1) = \beta_1$; unbias

$\hat{\beta}_1 = \bar{Y} - \beta_2 \bar{x}$

$\hat{\beta}_1 = \beta_1$

$E(\hat{\beta}_1) = \beta_1$