Solution Assign. 1 EE320 # Any errors or questions regarding solution, please contact : jak.a@st.econ.tu.ac.th

0.1) $\frac{\partial z}{\partial x} = \frac{3x^2(x^2y^2)-(x^3-y^3)2xy^2}{x^4y^4} = \frac{3x^4y^2-2x^4y^2+2xy^5}{x^4y^4} = \frac{x^4y^2+2xy^5}{(x^2y^2)^2} \longrightarrow x\frac{\partial z}{\partial x} = \frac{x^5y^2+2x^2y^5}{(x^2y^2)^2} = \frac{x^3+2y^3}{x^2y^2}$

$\frac{\partial z}{\partial y} = \frac{x^2y^2(-3y^2)-(x^3-y^3)2x^2y}{x^4y^4} = \frac{-3x^2y^4-2x^5y+2x^2y^4}{(x^2y^2)^2} = \frac{-x^2y^4-2x^5y}{(x^2y^2)^2} \longrightarrow y\frac{\partial z}{\partial y} = \frac{-x^2y^5-2x^5y^2}{(x^2y^2)^2} = \frac{-y^3-2x^3}{x^2y^2}$

$\therefore\ x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{y^3-x^3}{x^2y^2} = -z$ #

0.2) $dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$

$= \frac{(x+y)-(x-y)}{(x+y^2)}dx + \frac{(x+y)(-1)-(x-y)}{(x+y)^2}dy$

$= \frac{2y}{(x+y)^2}dx + \frac{-2y}{(x+y)^2}dy \quad -①$

At $x=1, y=1$ while $dx=2$, $dy=-2$ → plug in ①

$dz = \left(\frac{2}{(2)^2}\right)(2) - \left(\frac{2}{2^2}\right)(-2) = 1+1 = 2$ # z increases by 2

0.3)

$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial r} = (4xy+3y)(2r) + (2x^2+3x+2y)(-4)$

$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s} = (4xy+3y)(2r+2s) + (2x^2+3x+2y)(2)$

At $r=1, s=0$ → $x = (1)^2+0 = 1$

$y = 2(1)+0 = 2$

$\left.\frac{\partial z}{\partial r}\right|_{r=1,s=0} = \big(4(1)(2)+3(2)\big)(2) + \big(2(1)^2+3+2(2)\big)(-4)$

$= (14\times 2) + (9\times(-4)) = 28-36 = -8$

if r increases by 1 unit, z will decrease by 8

$\left.\frac{\partial z}{\partial s}\right|_{r=1,s=0} = (14)(2)+(9)(2) = 46$ ∴ if s increase by 1 unit, z will increase by 46 units

0.4) $f(x,y,z) = 16$, By implicit f^n theorem

$\frac{\partial f}{\partial z} = 4z$ at $x=1, y=2, z=-1$

$= -4 \neq 0$

Hence, there exists $z = \phi(x,y)$ around $x=1, y=2, z=$

$\frac{\partial z}{\partial y} = -\frac{\partial f/\partial y}{\partial f/\partial z} = -\frac{6y}{4z}$

$\therefore\ \left.\frac{\partial z}{\partial y}\right|_{x=1,y=2,z=1} = -\frac{6(2)}{-4} = 3$ # if y increases by 1 unit, z will increase 3 units

0.5) $f(x,y,z) = \ln(x+y+z) + xyz - ze^{x+y+z}$

$\left.\frac{\partial f}{\partial z}\right|_{x=0,y=1,z=0} = \frac{1}{x+y+z} + xy - (ze^{x+y+x} + e^{x+y+z})\Big|_{\substack{x=0\\y=1\\z=0}} = 1+0-(0+e) = 1-e \neq 0$ → Implicit f^n theorem

$\left.\frac{\partial f}{\partial x}\right|_{x=0,y=1,z=0} = \frac{1}{x+y+z} + yz - ze^{x+y+z}\Big|_{\substack{x=0\\y=1\\z=0}} = 1$

$\therefore\ \left.\frac{\partial z}{\partial x}\right|_{\substack{x=0\\y=1\\z=0}} = -\frac{\partial f/\partial x}{\partial f/\partial z} = -\frac{1}{1-e}$ #

1.1) $\frac{\partial Q_x}{\partial P_y} = -(-\frac{1}{2}) P_y^{-3/2}(50)$

$= 25 P_y^{-3/2} > 0 \quad \therefore$ They are substitute product

1.2) $\frac{\partial Q_x}{\partial I} = I > 0$

$\therefore$ X is <u>not</u> inferior good

1.3) Q_x at $P_x = 10$, $P_y = 25$, $I = 10$

$$Q_x = 100 - 4(10) - 50(25)^{-\frac{1}{2}} + 0.5(10)^2$$
$$= 100 - 40 - 50\left(\frac{1}{5}\right) + 50$$
$$= 100 - 40 - 10 + 50 = 100 \text{ units } \#$$

1.4) $\varepsilon_{Q,P_x} = \frac{\partial \ln Q}{\partial \ln P_x} = \frac{\partial Q}{Q} \cdot \frac{P}{\partial P} = \frac{\partial Q_x}{\partial P_x} \cdot \frac{P_x}{Q_x}$; At $P_x = 10$ $I = 10$ $P_y = 25 \rightarrow Q_x = 100$

$$= (-4)\left(\frac{P_x}{Q_x}\right)$$
$$= (-4)\left(\frac{10}{100}\right) = -0.4$$

$\therefore |\varepsilon_{Q,P_x}| = |-0.4| < 1$ is inelastic

1% change in P_x leads to -0.4% change in Q_x

1.5) $\varepsilon_{Q_x,P_y} = \frac{\partial \ln Q_x}{\partial \ln P_y} = \frac{\partial Q}{Q} \cdot \frac{P_y}{\partial P_y} = \frac{\partial Q_x}{\partial P_y} \cdot \frac{P_y}{Q_x}$

$$= 25(P_y)^{-3/2}\left(\frac{P_y}{Q_y}\right) = 25(25)^{-3/2}\left(\frac{25}{100}\right)$$
$$= \left(\frac{1}{5}\right)\left(\frac{1}{4}\right)$$
$$= 0.05$$

$\therefore \varepsilon_{Q_x,P_y} = 0.05 < 1$ is inelastic

1.6) $\varepsilon_{Q_x,I} = \frac{\partial Q_x}{\partial I} \cdot \frac{I}{Q_x}$

$$= (I)\left(\frac{I}{Q_x}\right) = (10)\left(\frac{10}{100}\right) = 1$$

$\therefore \varepsilon_{Q_x,I} = 1$ meaning that product is necessary

2.1) DRTS : $f(tK,tL) < tf(K,L)$

$$f(tK,tL) = A[t^nK^n + t^nL^n]$$
$$= t^n A[K^n + L^n]$$
$$= t^n f(K,L)$$

$\therefore f(tK,tL) < tf(K,L)$ if <u>$n < 1$</u> #

2.2) $\frac{\partial Q}{\partial K} = nAK^{n-1} = MP_K$ | $\frac{\partial Q}{\partial L} = nAL^{n-1} = MP_L$

$\frac{\partial MP_K}{\partial K} = \underbrace{n(n-1)}_{<0} AK^{n-2} < 0$ | $\frac{\partial MP_L}{\partial L} = \underbrace{n(n-1)}_{<0} AL^{n-2} < 0$

since $n < 1$

$$\therefore \left.\begin{array}{l} MP_K \text{ decreases as } K \text{ increases} \\ MP_L \text{ decreases as } L \text{ increases} \end{array}\right\} \text{Diminishing Return}$$

2.3) $MRTS_{L,K}$; $dQ = MP_K \cdot dK + MP_L \, dL$

$$0 = MP_K \cdot dK + MP_L \, dL$$
$$-\frac{dK}{dL} = \frac{MP_L}{MP_K} = MRTS_{L,K}$$
$$MRTS_{L,K} = \left(\frac{L}{K}\right)^{n-1} ; \quad n < 1$$

2.4) $\frac{\partial MRTS}{\partial L} = (n-1) L^{n-2} K^{1-n} < 0$ from $(n-1) < 0$

$\therefore$ the increase in L will decrease $MRTS_{L,K}$

2.5) $\partial Q = \frac{\partial Q}{\partial K} \cdot \frac{\partial K}{\partial t} \cdot dt + \frac{\partial Q}{\partial L} \cdot \frac{\partial L}{\partial t} \cdot dt$

$$= nAK^{n-1}(t+2) \cdot dt + (nAL^{n-1})(e^t)dt$$

$\frac{\partial Q}{\partial t} = nAK^{n-1}(t+2) + (nAL^{n-1})(e^t) > 0$ for period $t > 0$

$\therefore$ Hence, Q increases over time

2.6) $\left.\begin{array}{l} K(t=0) = 3 \\ L(t=0) = 4 \end{array}\right\} Q(t=0) = A[3^n + 4^n]$

$$\frac{dQ}{dt} \cdot \frac{1}{Q} = \frac{nAK^{n-1}(t+2) + (nAL^{n-1})(e^t)}{A[K^n + L^n]}$$
$$= \frac{n(3)^{n-1}(2) + (n)(4)^{n-1}(1)}{3^n + 4^n}$$
$$= \frac{2n(3)^{n-1} + n(4)^{n-1}}{3^n + 4^n}$$

3.1) $MU_x = \frac{\partial u}{\partial x} = \frac{1}{2} x^{-\frac{1}{2}}$

$MU_y = \frac{\partial u}{\partial y} = \frac{1}{2} y^{-\frac{1}{2}}$

3.2) $\left.\begin{aligned} \frac{\partial MU_x}{\partial x} &= -\frac{1}{4} x^{-\frac{3}{2}} < 0 \\ \frac{\partial MU_y}{\partial y} &= -\frac{1}{4} y^{-\frac{3}{2}} < 0 \end{aligned}\right\}$ Diminishing MU

3.3) $MU_x = -\frac{1}{4} x^{-\frac{3}{2}}$ doesn't depend on y ; Δy doesn't affect MU_x

3.4) $U(x=1, y=2) = \sqrt{1} + \sqrt{2} = 1 + \sqrt{2}$

3.5) $du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy$

$du = MU_x dx + MU_y dy$

$= \left(\frac{1}{2}\right) x^{-\frac{1}{2}} dx + \left(\frac{1}{2}\right) y^{-\frac{1}{2}} dy$

At $x = 1$, $y = 2$, $dx = 3$, $dy = -1$

$du = \left(\frac{1}{2}\right)(1)^{-\frac{1}{2}}(3) + \left(\frac{1}{2}\right)(2)^{-\frac{1}{2}}(-1)$

$= \frac{3}{2} + \left(-\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{3}{2} - \frac{1}{2\sqrt{2}}$

$= \frac{6 - \sqrt{2}}{4}$

3.6) $du = MU_x dx + MU_y dy$

$0 = MU_x dx + MU_y dy$

$-\frac{dy}{dx} = \frac{MU_x}{MU_y} = MRS_{xy}$

$MRS_{xy} = \left(\frac{x}{y}\right)^{-\frac{1}{2}}$

$\frac{\partial MRS}{\partial x} = -\frac{1}{2}(x)^{-\frac{3}{2}}(y)^{\frac{1}{2}} < 0$

$\therefore MRS_{xy}$ is decreasing in x

4) $\pi = pq - wL$; $q = L^{\frac{1}{2}}$

$\pi(L) = p(L)^{\frac{1}{2}} - wL$

$\frac{\partial \pi}{\partial L} = \frac{1}{2} p(L)^{-\frac{1}{2}} - w$

FOC : $w = \frac{p}{2\sqrt{L}}$

$\sqrt{L} = \frac{p}{2w}$

$L^* = \frac{p^2}{4w^2}$

$\partial L^* / \partial w = (-2) w^{-3} \left(\frac{p^2}{4}\right) = -\frac{p^2}{2w^3} < 0$

$\partial L^* / \partial p = \frac{1}{2}\left(\frac{p}{w^2}\right) > 0$