

Solution: Quiz 4

1. Find the solution set for

$$5|x| < |3x + 5 - |3x||.$$

Solution: Two approaches are given here.

Approach 1:

$$\begin{aligned} 5|x| &< |3x + 5 - |3x|| \\ (5x)^2 - (3x + 5 - |3x|)^2 &< 0 \\ (5x - [3x + 5 - |3x|])(5x + [3x + 5 - |3x|]) &< 0 \\ (2x - 5 + |3x|)(8x + 5 - |3x|) &< 0 \end{aligned}$$

Consider 2 cases for $|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$. Note: $|3x| = 3|x|$.

Case1: $x < 0$ which implies $|x| = -x$

$$(2x - 5 + |3x|)(8x + 5 - |3x|) = (2x - 5 - 3x)(8x + 5 + 3x) = (-x - 5)(11x + 5) < 0$$

$$\Rightarrow (x + 5)(11x + 5) > 0$$

	$x \in (-\infty, -5)$	$x \in (-5, -\frac{5}{11})$	$x \in (-5/11, \infty)$
$x + 5$	-	+	+
$11x + 5$	-	-	+
$(x + 5)(11x + 5)$	+	-	+

From the table, $x \in (-\infty, -5) \cup (-5/11, \infty)$ and the solution set in this case is

$$\boxed{\{(-\infty, -5) \cup (-5/11, \infty)\} \cap (-\infty, 0) = (-\infty, -5) \cup (-5/11, 0)}.$$

Case2: $x \geq 0$ which implies $|x| = x$

$$(2x - 5 + |3x|)(8x + 5 - |3x|) = (2x - 5 + 3x)(8x + 5 - 3x) = (5x - 5)(5x + 5) < 0 \Rightarrow (x - 1)(x + 1) < 0$$

$$\Rightarrow (x + 1)(x - 1) < 0$$

	$x \in (-\infty, -1)$	$x \in (-1, 1)$	$x \in (1, \infty)$
$x + 1$	-	+	+
$x - 1$	-	-	+
$(x + 1)(x - 1)$	+	-	+

From the table, $x \in (-1, 1)$ and the solution set in this case is $\boxed{(-1, 1) \cap [0, \infty) = [0, 1)}.$

From cases 1 and 2, the solution set is

$$\{(-\infty, -5) \cup (-5/11, 0)\} \cup [0, 1) = (-\infty, -5) \cup (-5/11, 1)$$

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Approach 2:

Consider 2 cases for $|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$.

Case1: $x < 0$ which implies $|x| = -x$. Note: $|3x| = 3|x|$.

$$\begin{aligned} 5|x| &< |3x + 5 - |3x|| \\ -5x &< |3x + 5 + 3x| \\ -5x &< |6x + 5| \end{aligned}$$

Consider 2 sub-cases:

$$1.1 \quad x < -5/6 \text{ or } |6x + 5| = -(6x + 5)$$

$$-5x < -(6x + 5) \quad \Leftrightarrow \quad x < -5 \Rightarrow x \in (-\infty, -5) \cap (-\infty, -5/6) = (-\infty, -5)$$

$$1.2 \quad x \geq -5/6 \text{ or } |6x + 5| = 6x + 5$$

$$-5x < 6x + 5 \quad \Leftrightarrow \quad -5 < 11x \Rightarrow x > -5/11 \Rightarrow x \in (-5/11, \infty) \cap [-5/6, \infty) = (-5/11, \infty)$$

From subcases 1.1 and 1.2, case 1 has solution set:

$$\boxed{\{(-\infty, -5) \cup (-5/11, \infty)\} \cap (-\infty, 0) = (-\infty, -5) \cup (-5/11, 0)}$$

Case 2: $x \geq 0$ which implies $|x| = x$. Note: $|3x| = 3|x|$.

$$\begin{aligned} 5|x| &< |3x + 5 - |3x|| \\ 5x &< |3x + 5 - 3x| \\ 5x &< |5| = 5 \\ x &< 1 \end{aligned}$$

Hence, the solution set for case 2 is: $\boxed{(-\infty, 1) \cap [0, \infty) = [0, 1)}$

From cases 1 and 2, the solution set is:

$$\{(-\infty, -5) \cup (-5/11, 0)\} \cup [0, 1) = (-\infty, -5) \cup (-5/11, 1)$$

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