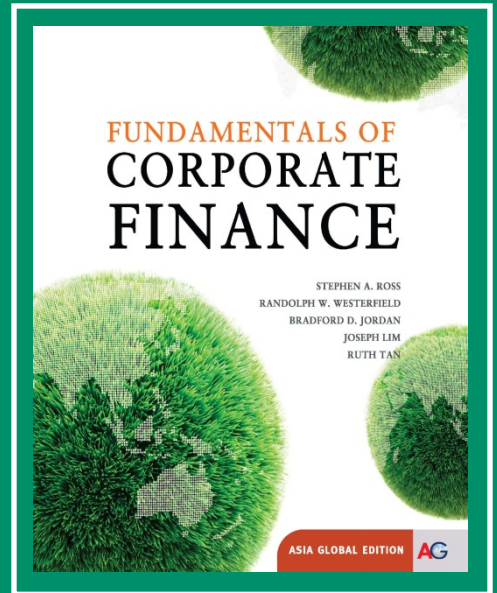


Chapter 6



Discounted Cash Flow Valuation



Key Concepts and Skills

- Be able to compute the future value of multiple cash flows
- Be able to compute the present value of multiple cash flows
- Be able to compute loan payments
- Be able to find the interest rate on a loan
- Understand how interest rates are quoted
- Understand how loans are amortized or paid off

Chapter Outline

- Future and Present Values of Multiple Cash Flows
- Valuing Level Cash Flows: Annuities and Perpetuities
- Comparing Rates: The Effect of Compounding
- Loan Types and Loan Amortization

Multiple Cash Flows – FV

Example 1

- Suppose you invest \$500 in a mutual fund today and \$600 in one year. If the fund pays 9% annually, how much will you have in two years?
 - $FV_2 = 500(1.09) + 600(1.09)$
 $= 594.05 + 654.00$
 $= 1,248.05$
 - Using financial calculator for:
Year 0 CF : 2N; -500 PV; 9 I/Y; CPT FV = 594.05
Year 1 CF : 1N; -600 PV; 9 I/Y; CPT FV = 654.00
Total FV₂ = 594.05 + 654.00 = 1,248.05

Multiple Cash Flows – FV

Example 1 Continued

How much will you have in 5 years if you make no further deposits?

- First way:
 - $FV_5 = 500(1.09) + 600(1.09) = 769.31 + 846.95 = 1,616.26$
 - Using financial calculator for:
Year 0 CF : 5N; -500 PV; 9 I/Y; CPT FV = 769.31
Year 1 CF : 4N; -600 PV; 9 I/Y; CPT FV = 846.95
Total $FV_5 = 769.31 + 846.95 = 1,616.26$
- Second way – use value at year 2:
 - $FV_5 = FV_2 (1.09) = 1,248.05(1.09) = 1,616.26$
 - Using financial calculator: 3 N; -1,248.05 PV; 9 I/Y; CPT FV = 1,616.26

Multiple Cash Flows – FV

Example 2

- Suppose you plan to deposit \$100 into an account in one year and \$300 into the account in three years. How much will be in the account in five years if the interest rate is 8%?

- $FV_5 = 100(1.08) + 300(1.08)$
 $= 136.05 + 349.92$
 $= 485.97$

- Using financial calculator for:

Year 1 CF: 4 N; -100 PV; 8 I/Y; CPT FV = 136.05

Year 3 CF: 2 N; -300 PV; 8 I/Y; CPT FV = 349.92

Total FV = 136.05 + 349.92 = 485.97

Multiple Cash Flows – PV Example

- You are considering an investment that will pay you \$1,000 in one year, \$2,000 in two years and \$3,000 in three years. If you want to earn 10% on your money, how much would you be willing to pay?

- $$PV = 1,000 / (1.10) + 2,000 / (1.10) + 3,000 / (1.10)$$
$$= 4,815.93$$

- Using financial calculator for:

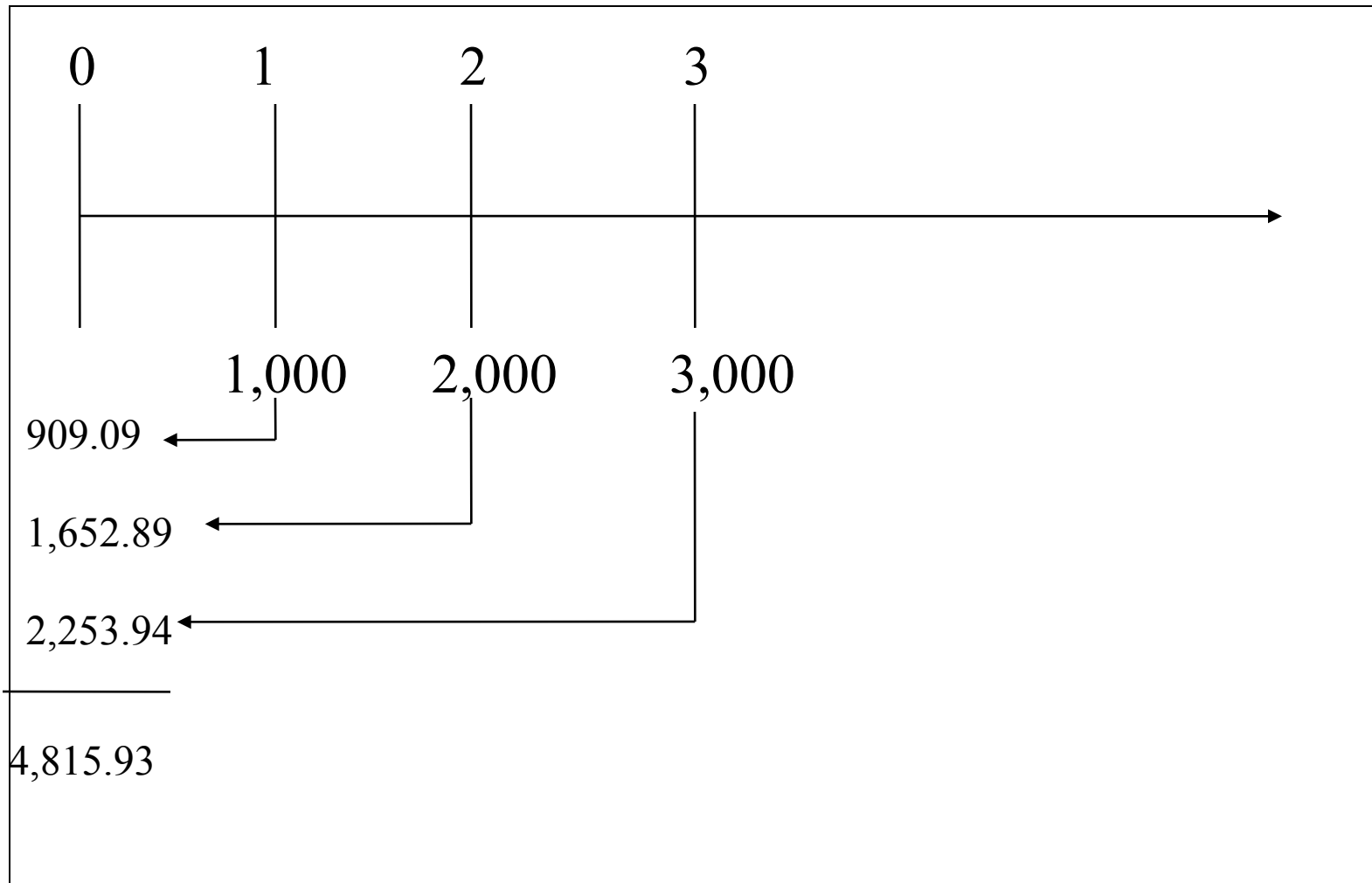
Year 1 CF: 1 N; 1,000 FV; 10 I/Y; CPT PV = -909.09

Year 2 CF: 2 N; 2,000 FV; 10 I/Y; CPT PV = -1,652.89

Year 3 CF: 3 N; 3,000 FV; 10 I/Y; CPT PV = -2,253.94

Total PV = 909.09 + 1,652.89 + 2,253.94 = 4,815.93

Timeline



Multiple Cash Flows Using a Spreadsheet

- You can use the PV or FV functions in Excel to find the present value or future value of a set of cash flows
- Setting the data up is half the battle – if it is set up properly, then you can just copy the formulas
- Click on the Excel icon for an example



Multiple Uneven Cash Flows

– Using the Calculator

- Another way to use the financial calculator for uneven cash flows is to use the cash flow keys
 - Press CF and enter the cash flows beginning with year 0.
 - You have to press the “Enter” key for each cash flow
 - Use the down arrow key to move to the next cash flow
 - The “F” is the number of times a given cash flow occurs in consecutive periods
 - Use the NPV key to compute the present value by entering the interest rate for I, pressing the down arrow, and then computing the answer

Decisions Based on Discounted Cash Flows

- Your broker calls you and tells you that he has this great investment opportunity. If you invest \$100 today, you will receive \$40 in one year and \$75 in two years. If you require a 15% return on investments of this risk, should you take the investment?
 - $PV = -100 + 40 / (1.15) + 75 / (1.15) = -8.51$
 - Use the CF keys to compute the value of the investment
-100 CF₀; 40 CF₁; 75 CF₂; 15 I/Y; CPT NPV = -8.51
 - No, the investment costs more than what you would be willing to pay.

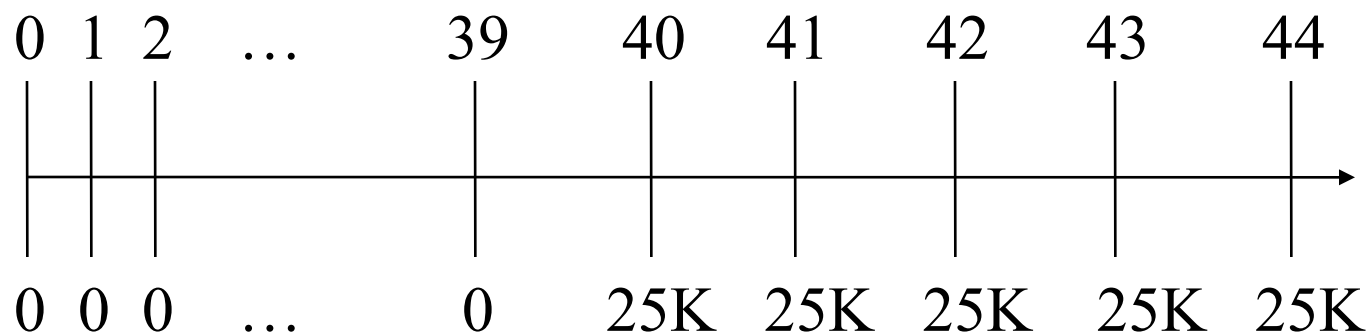
Saving For Retirement

- You are offered the opportunity to put some money away for retirement. You will receive five annual payments of \$25,000 each beginning in 40 years. How much would you be willing to invest today if you desire an interest rate of 12%?

$$\begin{aligned} - \text{PV} &= 25,000 / (1.12) + 25,000 / (1.12) + 25,000 / (1.12) + 25,000 / (1.12) + 25,000 / (1.12) \\ &= 1,084.71 \end{aligned}$$

- Use the CF keys to compute the value of the investment
0 CF₀; 0 CF₁; 39 N; 25,000 CF₂; 5 N; 12 I/Y; CPT NPV = 1,084.71

Saving For Retirement Timeline



Notice that the year 0 cash flow = 0 (0 CF_0)

The cash flows in years 1 – 39 are 0 (0 CF_1 ; 39 N)

The cash flows in years 40 – 44 are 25,000 (25,000 CF_2 ; 5 N)

Quick Quiz – Part I

- Suppose you are looking at the following possible cash flows: Year 1 CF = \$100; Years 2 and 3 CFs = \$200; Years 4 and 5 CFs = \$300. The required discount rate is 7%.
- What is the value of the cash flows at year 5?
- What is the value of the cash flows today?
- What is the value of the cash flows at year 3?

Annuities and Perpetuities Defined

- Annuity – finite series of equal payments that occur at regular intervals
 - If the first payment occurs at the end of the period, it is called an ordinary annuity
 - If the first payment occurs at the beginning of the period, it is called an annuity due
- Perpetuity – infinite series of equal payments

Annuities and Perpetuities – Basic Formulas

Perpetuity with 1 payment at end of the period

$$PV = C / r$$

Annuities with 1 payment at end of the period

$$PV = C \left[\frac{1 - \frac{1}{(1+r)^t}}{r} \right]$$

$$FV = C \left[\frac{(1+r)^t - 1}{r} \right]$$

Annuities and the Calculator

- You can use the PMT key on the calculator for the equal payment
- The sign convention still holds
- Ordinary annuity versus annuity due
 - You can switch your calculator between the two types by using the 2nd BGN 2nd Set on the TI BA-II Plus
 - If you see “BGN” or “Begin” in the display of your calculator, it is set for an annuity due
 - Most problems are ordinary annuities

Annuity – Sweepstakes Example

- Suppose you win the \$10 million sweepstakes. The money is paid in equal annual end-of-year installments of \$333,333.33 over 30 years. If the appropriate discount rate is 5%, how much is the sweepstakes actually worth today?

$$PV = C \left[\frac{1 - \frac{1}{(1+r)^t}}{r} \right] = 333,333.33 \left[\frac{1 - \frac{1}{(1+0.05)^{30}}}{0.05} \right] = 5,124.150.29$$

Using financial calculator:

30 N; 5 I/Y; 333,333.33 PMT; CPT PV = 5,124,150.29

Buying a House

- You are ready to buy a house, and you have \$20,000 to cover both the down payment and commission costs. Commission costs are estimated to be 4% of the loan value. You have an annual salary of \$36,000, and the bank is willing to allow your monthly mortgage payment to be equal to 28% of your monthly income. The interest rate on the loan is 6% per year with monthly compounding (.5% per month) for a 30-year fixed rate loan. How much money will the bank lend you? How much can you offer for the house?

Buying a House - Continued

- Bank loan
 - Monthly income = $36,000 / 12 = 3,000$
 - Maximum mortgage payment = $.28(3,000) = 840$

$$PV = C \left[\frac{1 - \frac{1}{(1+r)^t}}{r} \right] = 840 \left[\frac{1 - \frac{1}{(1+0.06/12)^{12(30)}}}{0.06/12} \right] = 140,105$$

Using financial calculator: 360 N; 0.5 I/Y; -840 PMT; CPT PV = 140,105

- Total Price
 - Commission costs = $.04(140,105) = 5,604$
 - Down payment = $20,000 - 5,604 = 14,396$
 - Total Price = $140,105 + 14,396 = 154,501$

Annuities on the Spreadsheet - Example

- The annuity formula has 4 variables namely:
 - discount rate (rate),
 - number of payments (nper),
 - payment amount per period (PMT),
 - present value (pv) / future value (fv).
- Knowing any 3 allows the fourth variable to be solved
- Click on the Excel icon to see an example



Quick Quiz – Part II

- You know the payment amount for a loan, and you want to know how much was borrowed. Do you compute a present value or a future value?
- You want to receive 5,000 per month in retirement. If you can earn 0.75% per month and you expect to need the income for 25 years, how much do you need to have in your account at retirement?

Finding the Payment

- Suppose you want to borrow \$20,000 for a new car. You can borrow at 8% per year, compounded monthly ($8/12 = .66667\%$ per month). If you wish to make monthly installment payments over a 4-year period, what is your monthly payment?

$$PV = C \left[\frac{1 - (1 + r)^{-N}}{r} \right]$$

$$20,000 = C \left[\frac{1 - (1 + 8\%/12)^{-48}}{8\%/12} \right]$$

$$C = 488.26$$

- Using financial calculator: 48 N; 20,000 PV; 0.66667 I/Y; CPT PMT = 488.26

Finding the Number of Payments – Another Example

- Suppose you borrow \$2,000 at 5%, and you are going to make annual payments of \$734.42. How long will you take to pay off the loan?

$$PV = C \left[\frac{1 - \frac{1}{(1+r)^t}}{r} \right] = 734.42 \left[\frac{1 - \frac{1}{(1+0.05)^t}}{0.05} \right] = 2,000 \Rightarrow t = 3$$

- Using financial calculator: 5 I/Y; 2,000 PV; -734.42 PMT; CPT N = 3 years

Finding the Rate

- Suppose you borrow \$10,000 from your parents to buy a car. You agree to pay \$207.58 per month for 60 months. What is the monthly interest rate?

$$PV = C \left[\frac{1 - \frac{1}{(1 + r/12)^{12t}}}{r/12} \right] = 207.58 \left[\frac{1 - \frac{1}{(1 + r/12)^{60}}}{r/12} \right] = 10,000 \Rightarrow r = 9\%$$

- Using financial calculator: 60 N; 10,000 PV; -207.58 PMT; CPT I/Y = .75%

This is the rate for 1 month.

The annual rate is $0.75\% \times 12 = 9\%$

Annuity – Finding the Rate Without a Financial Calculator

- Trial and Error Process
 - Choose an interest rate and compute the PV of the payments based on this rate
 - Compare the computed PV with the actual loan amount
 - If the computed PV $>$ loan amount, then the interest rate is too low
 - If the computed PV $<$ loan amount, then the interest rate is too high
 - Adjust the rate and repeat the process until the computed PV and the loan amount are equal

Quick Quiz – Part III

You want to receive \$5,000 per month for the next 5 years.

- How much would you need to deposit today if you can earn 0.75% per month?
- What monthly rate would you need to earn if you only have \$200,000 to deposit?
- Suppose you have \$200,000 to deposit and can earn 0.75% per month.
 - How many months could you receive the \$5,000 payment?
 - How much could you receive every month for 5 years?

Future Values for Annuities

- Suppose you begin saving for your retirement by depositing \$2,000 per year in a savings account. If the interest rate is 7.5%, how much will you have in 40 years?

$$FV = C \left[\frac{(1 + r)^t - 1}{r} \right] = 2,000 \left[\frac{(1 + 0.075)^{40} - 1}{0.075} \right] = 454,513.04$$

- Using financial calculator: 40 N; 7.5 I/Y; -2,000 PMT;
CPT FV = 454,513.04

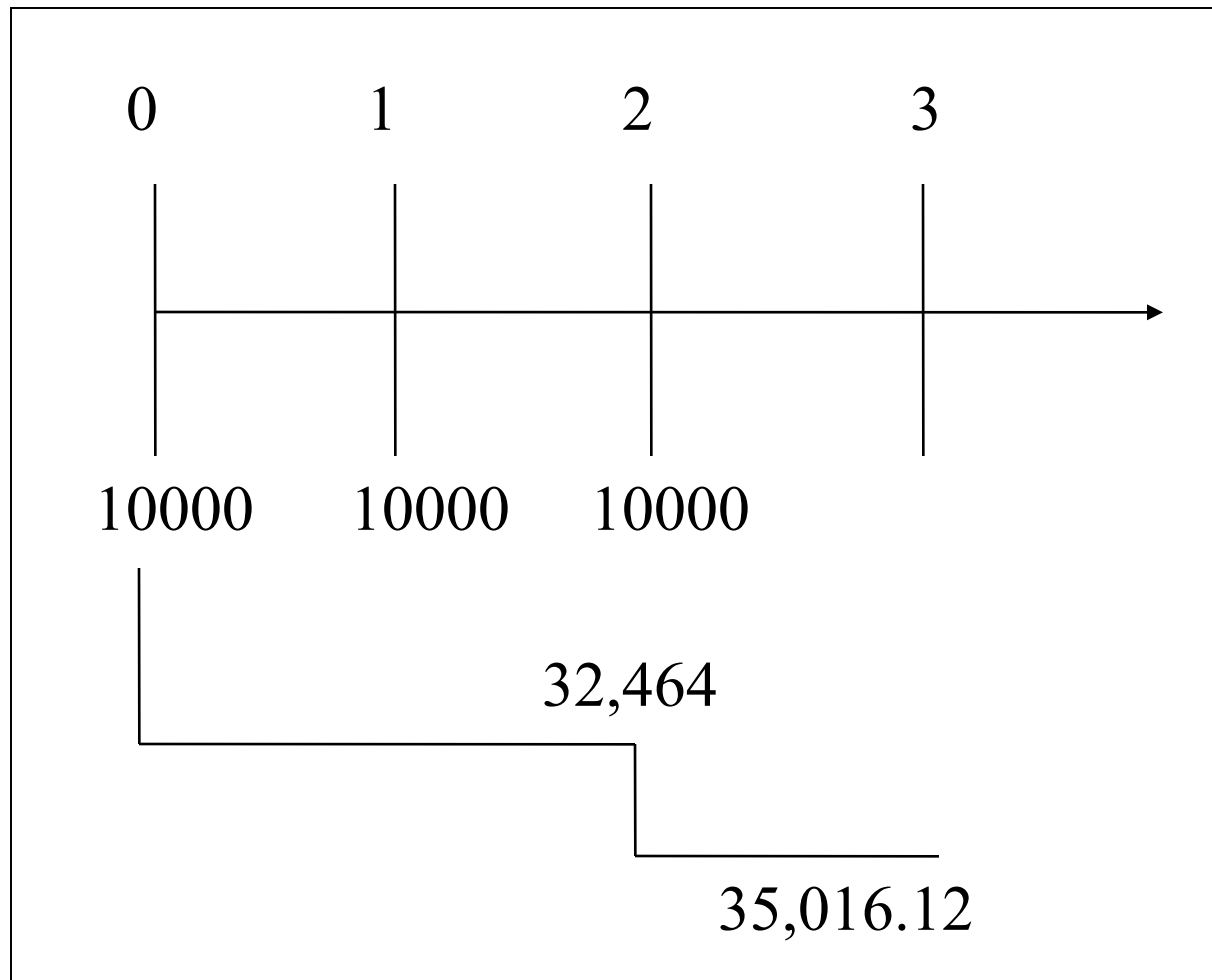
Annuity Due

- You are saving for a new house and you put \$10,000 per year in an account paying 8%. The first payment is made today. How much will you have at the end of 3 years?

$$FV = C \left[\frac{(1+r)^t - 1}{r} \right] (1+r) = 10,000 \left[\frac{(1+0.08)^3 - 1}{0.08} \right] (1.08) = 35,061.12$$

- Using financial calculator: 2nd BGN 2nd Set (you should see BGN in the display)
- 3 N; -10,000 PMT; 8 I/Y; CPT FV = 35,061.12
- 2nd BGN 2nd Set (be sure to change it back to an ordinary annuity).
- Note that the answer is the value at the end of year 3.

Annuity Due Timeline



Quick Quiz – Part IV

- You want to have \$1 million to use for retirement in 35 years. If you can earn 1% per month, how much do you need to deposit on a monthly basis if the first payment is made in one month?
- What if the first payment is made today?
- You are considering preferred stock that pays a quarterly dividend of \$1.50. If your desired return is 3% per quarter, how much would you be willing to pay?

Annuity Problem

- An investment will provide you with \$100 at the end of each year for the next 10 years. What is the present value of that annuity if the discount rate is 8% annually?
- What is the present value of the above if the payments are received at the beginning of each year?
- If you deposit the \$100 at the end of each year into an account earning 8%, what will the future value be in 10 years?
- If you open the account with \$1,000 today and then make the \$100 deposits at the end of each year, what will the future value be in 10 years?

Work the Web Example

- Another online financial calculator can be found at http://www.cpf.gov.sg/cpfhsg/Hsg_mnthlyinstal_calc.asp
- Click on the web surfer and work the following example
 - You just inherited \$5 million. If you can earn 6% on your money, how much can you withdraw each month for the next 40 years?
 - The website assumes ordinary annuity.
 - Monthly withdrawal = \$27,511

Table 6.2

I. Symbols:

PV = Present value, what future cash flows are worth today

FV_t = Future value, what cash flows are worth in the future

r = Interest rate, rate of return, or discount rate per period—typically, but not always, one year

t = Number of periods—typically, but not always, the number of years

C = Cash amount

II. Future Value of C per Period for t Periods at r Percent per Period:

$$FV_t = C \times \{[(1 + r)^t - 1]/r\}$$

A series of identical cash flows is called an *annuity*, and the term $[(1 + r)^t - 1]/r$ is called the *annuity future value factor*.

III. Present Value of C per Period for t Periods at r Percent per Period:

$$PV = C \times \{1 - [1/(1 + r)^t]\}/r$$

The term $\{1 - [1/(1 + r)^t]\}/r$ is called the *annuity present value factor*.

IV. Present Value of a Perpetuity of C per Period:

$$PV = C/r$$

A *perpetuity* has the same cash flow every year forever.

