B.E. International Program

Faculty of Economics, Thammasat University

EE 320 Introductory Mathematical Economics (Section 046402)

Semester 1/2013

Homework 3

Due 26 September 2013

There are four questions in total. Each of them is worth 5 points.

1. *(1 point each)* Differentiate the following functions:

a) $y = \frac{x^2-9x+20}{x-5}$

Ans. $\frac{dy}{dx} = 1, x \neq 5$

b) $y = \sqrt{x+\sqrt{x}}$

Ans. $\frac{dy}{dx} = \frac{1}{2(x+\sqrt{x})}\left(1+\frac{1}{2\sqrt{x}}\right) = \frac{\sqrt{x}+1}{2(x\sqrt{x}+x)}$

c) $y = \ln(e^x + x)$

Ans. $\frac{dy}{dx} = \frac{e^x+1}{e^x+x}$

d) $y = x^4 e^x$

Ans. $\frac{dy}{dx} = x^3 e^x (x+4)$

2. Given the total-product function:

$$Q = 2L + 5L^2 - L^3$$

a) *(1 point)* Find the average product (AP) function.

Ans. $AP(L) = 2 + 5L - L^2$

b) *(1 point)* Find the marginal product (MP) function.

Ans. $MP(L) = 2 + 10L - 3L^2$

c) *(2 points)* Determine the slopes of the AP and MP functions. What can you conclude about their relative slopes?

Ans. $\frac{d(AP)}{dL} = 5 - 2L$; $\frac{d(MP)}{dL} = 10 - 6L$

The MP curve is twice as steep as the AP curve.

3. Given the following function

$$f(x) = 2x^3 + 8x^2 - 32x - 50,$$

a) *(2 points)* Find the critical value(s) of *x* and the corresponding stationary value(s) of *f(x)*.

Ans. $f'(x) = 6x^2 + 16x - 32 = 0 \quad \Leftrightarrow \; x^* = -4, \frac{4}{3}$

b) *(2 points)* Evaluate whether the stationary value(s) found in part a) are relative maxima or minima or inflection points by using the first-derivative test.

Ans. f(-4) is a maximum because $f'(-5) = 38 > 0$; $f'(-3) = -26 < 0$.

f(3/4) is a minimum because $f'(1) = -10 < 0$; $f'(2) = 24 > 0$.

4. Let the total cost function be:

$$TC(Q) = 2Q^2 - 8Q + 10.$$

a) *(1 point)* Determine whether *TC(Q)* is a convex or concave function.

Ans. $TC'(Q) = 4Q - 8 \;\Rightarrow\; TC''(Q) = 4 > 0$. Thus, TC(Q) is a convex function.

b) *(2 points)* Find the quantity *Q** that minimizes the total cost.

Ans. $TC'(Q) = 4Q - 8 = 0 \;\Rightarrow Q^* = 2$

c) *(2 points)*Verify that *TC(Q*)* is the lowest cost by using the second derivative test.

Ans. $TC''(2) = 4 > 0$. Thus, TC(2) = 2 is the minimum.

5. Suppose that a firm produces a single commodity. Its demand and total-cost functions are:

$$Q = 70 - P$$

$$C(Q) = Q^2 - 10Q + 500.$$

a) *(1 point)* Set up a profit function, $\pi(Q)$.

Ans. $\pi(Q) = (70 - Q)Q - (Q^2 - 10Q + 500) = -2Q^2 + 80Q - 500$

b) *(2 points)* Find the value of Q* that maximizes profit.

Ans. $\pi'(Q) = -4Q + 80 = 0 \Rightarrow Q^* = 20$

c) *(2 points)* Verify that the quantity Q* found in part (b) gives a maximum profit.

Ans. $\pi''(Q) = -4 < 0$. Thus, $\pi(20) = 300$ is the maximum profit.