Assignment 2
Simultaneous Equations Model

Due: 1/9/2020

Demand and Supply Equations

$$\ln S_t = \beta_{10} + \beta_{11}\ln P_{Dt} + \beta_{12}\ln P_{X2t} + \beta_{13}\ln P_{X3t} + \beta_{14}\ln P_{X4t} + \varepsilon_{1t} \quad (1)$$

$$\ln D_t = \beta_{20} + \beta_{21}\ln P_{Dt} + \beta_{22}\ln GDP_t + \varepsilon_{2t} \quad (2)$$

where: S_t = Domestic Supply at time t

D_t = Domestic Demand at time t

P_{Dt} = Domestic Price at time t = $P_{Mt} + T_t$

T_t = Tariff at time t

P_{X2t} = Price of Input 2 at time t

P_{X3t} = Price of Input 3 at time t

P_{X4t} = Price of Input 4 at time t

GDP_t = Gross Domestic Product (Representing Income) at time t

Endogenous variables in this system include S_t, D_t, and P_{Dt}
Exogenous variables in this system include P_{X2t}, P_{X3t}, P_{X4t}, and GDP_t

From Data `Assignment 2.dta`:

1. State reduce form model of these system models.
2. Estimate reduce form model using OLS and prediction of the endogenous variables.
3. Estimate structural form using predicted endogenous variables as independent variables in the structural form model.
4. Estimate the structural models of these system equations using OLS, 2SLS, 3SLS, and I3SLS. Concerning on the asymptotic property, which model is the most appropriated model? Why?
5. What do β_{21} and β_{22} mean?

1.)

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$$\ln S_t = \beta_{10} + \beta_{11}\ln P_{Dt} + \beta_{12}\ln P_{X2t} + \beta_{13}\ln P_{X3t} + \beta_{14}\ln P_{X4t} + \varepsilon_{1t}$$

$$\ln D_t = \beta_{20} + \beta_{21}\ln P_{Dt} + \beta_{22}\ln GDP_t + \varepsilon_{2t}$$

① If D = S

$$\beta_{10} + \beta_{11}\ln P_{Dt} + \beta_{12}\ln P_{x2t} + \beta_{13}\ln P_{x3t} + \beta_{14}\ln P_{x4t} + \varepsilon_{1t} = \beta_{20} + \beta_{21}\ln P_{Dt} + \beta_{22}\ln GDP_t + \varepsilon_{2t}$$

$$\beta_{11}\ln P_{Dt} - \beta_{21}\ln P_{Dt} = \beta_{20} - \beta_{10} + \beta_{22}\ln GDP_t - \beta_{12}\ln P_{x2t} - \beta_{13}\ln P_{x3t} - \beta_{14}\ln P_{x4t} + \varepsilon_{2t} - \varepsilon_{1t}$$

$$(\beta_{11} - \beta_{21})\ln P_{Dt} = \beta_{20} - \beta_{10} + \beta_{22}\ln GDP_t - \beta_{12}\ln P_{x2t} - \beta_{13}\ln P_{x3t} - \beta_{14}\ln P_{x4t} + \varepsilon_{2t} - \varepsilon_{1t}$$

$$\ln P_{Dt} = \frac{\beta_{20} - \beta_{10} + \beta_{22}\ln GDP_t - \beta_{12}\ln P_{x2t} - \beta_{13}\ln P_{x3t} - \beta_{14}\ln P_{x4t} + \varepsilon_{2t} - \varepsilon_{1t}}{\beta_{11} - \beta_{21}}$$

$$\ln P_{Dt} = \frac{(\beta_{20} - \beta_{10})}{(\beta_{11} - \beta_{21})} + \frac{\beta_{22}\ln GDP_t}{(\beta_{11} - \beta_{21})} - \frac{\beta_{12}\ln P_{x2t}}{(\beta_{11} - \beta_{21})} - \frac{\beta_{13}\ln P_{x3t}}{(\beta_{11} - \beta_{21})} - \frac{\beta_{14}\ln P_{x4t}}{(\beta_{11} - \beta_{21})} + \frac{\varepsilon_{21} - \varepsilon_{1t}}{(\beta_{11} - \beta_{21})}$$

② sub $\ln P_{Dt}$ to $\ln S_t$

$$\ln S_t = \beta_{10} + \beta_{11}\left(\frac{(\beta_{20} - \beta_{10})}{(\beta_{11} - \beta_{21})} + \frac{\beta_{22}\ln GDP_t}{(\beta_{11} - \beta_{21})} - \frac{\beta_{12}\ln P_{x2t}}{(\beta_{11} - \beta_{21})} - \frac{\beta_{13}\ln P_{x3t}}{(\beta_{11} - \beta_{21})} - \frac{\beta_{14}\ln P_{x4t}}{(\beta_{11} - \beta_{21})} + \frac{\varepsilon_{21} - \varepsilon_{1t}}{(\beta_{11} - \beta_{21})}\right) + \beta_{12}\ln P_{x2t}$$
$$+ \beta_{13}\ln P_{x3t} + \beta_{14}\ln P_{x4t} + \varepsilon_{1t}$$

$$\ln S_t = \beta_{10} + \beta_{11}\left(\frac{(\beta_{20} - \beta_{10})}{(\beta_{11} - \beta_{21})}\right) + \beta_{11}\left(\frac{\beta_{22}\ln GDP_t}{(\beta_{11} - \beta_{21})}\right) + \beta_{11}\left(-\frac{\beta_{12}\ln P_{x2t}}{(\beta_{11} - \beta_{21})}\right) + \beta_{11}\left(-\frac{\beta_{13}\ln P_{x3t}}{(\beta_{11} - \beta_{21})}\right)$$
$$+ \beta_{11}\left(-\frac{\beta_{14}\ln P_{x4t}}{(\beta_{11} - \beta_{21})}\right) + \beta_{11}\left(\frac{\varepsilon_{21} - \varepsilon_{1t}}{(\beta_{11} - \beta_{21})}\right) + \beta_{21}\ln P_{x2t} + \beta_{13}\ln P_{x3t} + \beta_{14}\ln P_{x4t} + \varepsilon_{1t}$$

where $\pi_{10} = \beta_{10} + \dfrac{\beta_{11}(\beta_{20} - \beta_{10})}{(\beta_{11} - \beta_{21})}$

$$\pi_{11} = \frac{\beta_{11}\beta_{22}}{(\beta_{11} - \beta_{21})}$$

$$\pi_{12} = -\frac{\beta_{11}\beta_{12}}{\beta_{11} - \beta_{12}}$$

$$\pi_{13} = -\frac{\beta_{11}\beta_{13}}{\beta_{11} - \beta_{12}}$$

$$\pi_{14} = -\frac{\beta_{11}\beta_{14}}{\beta_{11} - \beta_{12}}$$

$$W_{1t} = \frac{\beta_{11}(\varepsilon_{2t} - \varepsilon_{1t})}{\beta_{11} - \beta_{21}} + \varepsilon_{1t}$$

③ sub $\ln P_{Dt}$ to $\ln D_t$

$$\ln D_t = \beta_{20} + \beta_{21}\left[\frac{(\beta_{20}-\beta_{10})}{(\beta_{11}-\beta_{21})} + \frac{\beta_{22}\ln GDP_t}{(\beta_{11}-\beta_{21})} - \frac{\beta_{12}\ln P_{x2t}}{(\beta_{11}-\beta_{21})} - \frac{\beta_{13}\ln P_{x3t}}{(\beta_{11}-\beta_{21})} - \frac{\beta_{14}\ln P_{x4t}}{(\beta_{11}-\beta_{21})} + \frac{\varepsilon_{21}-\varepsilon_{1t}}{(\beta_{11}-\beta_{21})}\right]$$
$$+ \beta_{22}\ln GDP_t + \varepsilon_{2t}$$

$$\ln D_t = \beta_{20} + \beta_{21}\frac{(\beta_{20}-\beta_{10})}{(\beta_{11}-\beta_{21})} + \frac{\beta_{21}\beta_{22}\ln GDP_t}{(\beta_{11}-\beta_{21})} - \frac{\beta_{21}\beta_{12}\ln P_{x2t}}{(\beta_{11}-\beta_{21})} - \frac{\beta_{21}\beta_{13}\ln P_{x3t}}{(\beta_{11}-\beta_{21})} - \frac{\beta_{21}\beta_{14}\ln P_{x4t}}{(\beta_{11}-\beta_{21})}$$
$$+ \frac{\beta_{21}(\varepsilon_{21}-\varepsilon_{1t})}{(\beta_{11}-\beta_{21})} + \beta_{22}\ln GDP_t + \varepsilon_{2t}$$

where
$$\pi_{20} = \beta_{20} + \beta_{21}\frac{(\beta_{20}-\beta_{10})}{(\beta_{11}-\beta_{21})}$$
$$\pi_{21} = \frac{\beta_{21}\beta_{22}}{(\beta_{11}-\beta_{21})}$$
$$\pi_{22} = -\frac{\beta_{21}\beta_{12}}{(\beta_{11}-\beta_{21})}$$
$$\pi_{23} = -\frac{\beta_{21}\beta_{13}}{(\beta_{11}-\beta_{21})}$$
$$\pi_{24} = -\frac{\beta_{21}\beta_{14}}{(\beta_{11}-\beta_{21})}$$
$$W_{2t} = \frac{\beta_{21}(\varepsilon_{21}-\varepsilon_{1})}{(\beta_{11}-\beta_{21})} + \varepsilon_{2t}$$

$$\pi_{30} = \frac{\beta_{20}-\beta_{10}}{(\beta_{11}-\beta_{21})}$$
$$\pi_{31} = \frac{-\beta_{12}}{(\beta_{11}-\beta_{21})}$$
$$\pi_{32} = \frac{-\beta_{13}}{(\beta_{11}-\beta_{21})}$$
$$\pi_{33} = \frac{-\beta_{14}}{(\beta_{11}-\beta_{21})}$$
$$\pi_{34} = \frac{\beta_{22}}{(\beta_{11}-\beta_{21})}$$
$$W_{3t} = \frac{\varepsilon_{2t}-\varepsilon_{1t}}{(\beta_{11}-\beta_{21})}$$

$$\ln S_t = \pi_{10} + \pi_{11}(\ln P_{x2t}) + \pi_{12}(\ln P_{x3t}) + \pi_{13}(\ln P_{x4t}) + \pi_{14}(\ln GDP_t) + W_{1T}$$
$$\ln D_t = \pi_0 + \pi_{21}(\ln P_{x2t}) + \pi_{22}(\ln P_{x3t}) + \pi_{23}(\ln P_{x4t}) + \pi_{24}(\ln GDP_t) + W_{2T}$$
$$\ln P_{Dt} = \pi_0 + \pi_{31}(\ln P_{x2t}) + \pi_{32}(\ln P_{x3t}) + \pi_{33}(\ln P_{x4t}) + \pi_{34}(\ln GDP_t) + W_{3T}$$

2.)

```
. tsset obs
        time variable:  obs, 1986 to 2007
                delta:  1 unit

. g lnst=ln(st)

. g lndt=ln(dt)

. g pd=pm+t

. g lnpx2=ln(px2)

. g lnpx3=ln(px3)

. g lnpx4=ln(px4)

. g lngdp=ln(gdp)

. g lnpd=ln(pd)

. reg lnst lnpx2 lnpx3 lnpx4 lngdp

      Source |       SS           df       MS      Number of obs   =        22
-------------+----------------------------------   F(4, 17)        =     37.32
       Model |  4.64569724         4  1.16142431   Prob > F        =    0.0000
    Residual |  .529104674        17  .031123804   R-squared       =    0.8978
-------------+----------------------------------   Adj R-squared   =    0.8737
       Total |  5.17480192        21  .246419139   Root MSE        =    .17642

------------------------------------------------------------------------------
        lnst |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       lnpx2 |  -.4503744   .1515961    -2.97   0.009    -.7702142   -.1305347
       lnpx3 |  -.9242052   .2783356    -3.32   0.004    -1.511442   -.3369685
       lnpx4 |  -.3883793   .4222332    -0.92   0.371    -1.279214    .5024549
       lngdp |   .3438812   .1913463     1.80   0.090    -.0598242    .7475865
       _cons |   24.65741   5.309757     4.64   0.000      13.4548    35.86002
------------------------------------------------------------------------------

. predict lnsthat
. reg lndt lnpx2 lnpx3 lnpx4 lngdp

      Source |       SS           df       MS      Number of obs   =        22
-------------+----------------------------------   F(4, 17)        =     26.43
       Model |   3.4026552         4  .850663799   Prob > F        =    0.0000
    Residual |   .54721789        17  .032189288   R-squared       =    0.8615
-------------+----------------------------------   Adj R-squared   =    0.8289
       Total |  3.94987309        21  .188089195   Root MSE        =    .17941

------------------------------------------------------------------------------
        lndt |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       lnpx2 |  -.4887365   .1541691    -3.17   0.006    -.8140049   -.1634682
       lnpx3 |  -.7243134   .2830597    -2.56   0.020    -1.321517   -.1271097
       lnpx4 |   -.577921   .4293997    -1.35   0.196    -1.483875    .3280333
       lngdp |   .1265855    .194594     0.65   0.524    -.2839719    .5371429
       _cons |   27.18614   5.399879     5.03   0.000     15.79339    38.57889
------------------------------------------------------------------------------

. predict lndthat
(option xb assumed; fitted values)
```

3.)

```
. reg lnst lnpdhat lnpx2 lnpx3 lnpx4
```

Source	SS	df	MS
Model	4.64569773	4	1.16142443
Residual	.529104183	17	.031123775
Total	5.17480192	21	.246419139

Number of obs	=	22
F(4, 17)	=	37.32
Prob > F	=	0.0000
R-squared	=	0.8978
Adj R-squared	=	0.8737
Root MSE	=	.17642

lnst	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lnpdhat	2.106112	1.171903	1.80	0.090	-.3663879	4.578612
lnpx2	-.727963	.1840856	-3.95	0.001	-1.11635	-.3395762
lnpx3	-1.122146	.2824139	-3.97	0.001	-1.717988	-.5263052
lnpx4	-1.428722	.4751381	-3.01	0.008	-2.431176	-.4262679
_cons	18.59912	8.546622	2.18	0.044	.5673274	36.63092

```
. reg lndt lnpdhat lngdp
```

Source	SS	df	MS
Model	3.26129847	2	1.63064924
Residual	.688574614	19	.036240769
Total	3.94987309	21	.188089195

Number of obs	=	22
F(2, 19)	=	44.99
Prob > F	=	0.0000
R-squared	=	0.8257
Adj R-squared	=	0.8073
Root MSE	=	.19037

lndt	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lnpdhat	-2.574157	.5697943	-4.52	0.000	-3.76675	-1.381563
lngdp	.5212927	.1344816	3.88	0.001	.2398194	.802766
_cons	35.93498	7.189835	5.00	0.000	20.88648	50.98347

4.)

```
. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), ols
```

Multivariate regression

Equation	Obs	Parms	RMSE	"R-sq"	F-Stat	P
lnst	22	4	.1652258	0.9103	43.14	0.0000
lndt	22	2	.1391259	0.9069	92.53	0.0000

	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lnst						
lnpd	-1.111835	.4515147	-2.46	0.019	-2.027549	-.1961207
lnpx2	-.4189546	.1431634	-2.93	0.006	-.7093034	-.1286059
lnpx3	-.9424196	.2585266	-3.65	0.001	-1.466736	-.4181034
lnpx4	-.521346	.3441643	-1.51	0.139	-1.219344	.1766516
_cons	41.4946	3.661911	11.33	0.000	34.0679	48.9213
lndt						
lnpd	-2.181329	.2946999	-7.40	0.000	-2.779008	-1.58365
lngdp	.5776586	.0887536	6.51	0.000	.397658	.7576593
_cons	31.03578	3.761201	8.25	0.000	23.40771	38.66385

```
. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), ols inst(lnpx2 lnpx3 lnpx4 lngdp)
```

Multivariate regression

Equation	Obs	Parms	RMSE	"R-sq"	F-Stat	P
lnst	22	4	.1652258	0.9103	43.14	0.0000
lndt	22	2	.1391259	0.9069	92.53	0.0000

	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lnst						
lnpd	-1.111835	.4515147	-2.46	0.019	-2.027549	-.1961207
lnpx2	-.4189546	.1431634	-2.93	0.006	-.7093034	-.1286059
lnpx3	-.9424196	.2585266	-3.65	0.001	-1.466736	-.4181034
lnpx4	-.521346	.3441643	-1.51	0.139	-1.219344	.1766516
_cons	41.4946	3.661911	11.33	0.000	34.0679	48.9213
lndt						
lnpd	-2.181329	.2946999	-7.40	0.000	-2.779008	-1.58365
lngdp	.5776586	.0887536	6.51	0.000	.397658	.7576593
_cons	31.03578	3.761201	8.25	0.000	23.40771	38.66385

β_{11} implies price elasticity of quantity supply

```
. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), 2sls inst(lnpx2 lnpx3 lnpx4 lngdp)
```

Two-stage least-squares regression

Equation	Obs	Parms	RMSE	"R-sq"	F-Stat	P
lnst	22	4	.329951	0.6424	10.67	0.0000
lndt	22	2	.1454858	0.8982	77.04	0.0000

	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lnst						
lnpd	2.10611	2.191774	0.96	0.343	-2.339013	6.551233
lnpx2	-.7279628	.3442892	-2.11	0.041	-1.426214	-.0297119
lnpx3	-1.122146	.528189	-2.12	0.041	-2.193363	-.0509293
lnpx4	-1.428722	.8886357	-1.61	0.117	-3.230959	.3735147
_cons	18.59914	15.98447	1.16	0.252	-13.81886	51.01715
lndt						
lnpd	-2.574157	.4354519	-5.91	0.000	-3.457295	-1.69102
lngdp	.5212921	.1027745	5.07	0.000	.3128558	.7297283
_cons	35.93499	5.494663	6.54	0.000	24.7913	47.07868

Endogenous variables: lnst lnpd lndt
Exogenous variables: lnpx2 lnpx3 lnpx4 lngdp

```
. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), 3sls inst(lnpx2 lnpx3 lnpx4 lngdp)
```

Three-stage least-squares regression

Equation	Obs	Parms	RMSE	"R-sq"	chi2	P
lnst	22	4	.2963642	0.6266	57.47	0.0000
lndt	22	2	.135203	0.8982	178.41	0.0000

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
lnst						
lnpd	2.171576	1.926095	1.13	0.260	-1.603501	5.946652
lnpx2	-.7990055	.2985983	-2.68	0.007	-1.384247	-.2137635
lnpx3	-1.329743	.4560002	-2.92	0.004	-2.223487	-.4359989
lnpx4	-1.171403	.775654	-1.51	0.131	-2.691657	.348851
_cons	17.84948	14.04122	1.27	0.204	-9.670808	45.36976
lndt						
lnpd	-2.574157	.4046743	-6.36	0.000	-3.367304	-1.78101
lngdp	.5212921	.0955104	5.46	0.000	.3340951	.708489
_cons	35.93499	5.106302	7.04	0.000	25.92682	45.94316

Endogenous variables: lnst lnpd lndt
Exogenous variables: lnpx2 lnpx3 lnpx4 lngdp

```
. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), 3sls ireg3 inst(lnpx2 lnpx3 lnpx4 lngdp)

Iteration 1:   tolerance =   .1059484
Iteration 2:   tolerance =  .04569793
Iteration 3:   tolerance =  .01846611
Iteration 4:   tolerance =  .00725496
Iteration 5:   tolerance =  .00281814
Iteration 6:   tolerance =  .00108981
Iteration 7:   tolerance =  .00042072
Iteration 8:   tolerance =  .00016231
Iteration 9:   tolerance =   .0000626
Iteration 10:   tolerance =  .00002414
Iteration 11:   tolerance =  9.310e-06
Iteration 12:   tolerance =  3.590e-06
Iteration 13:   tolerance =  1.384e-06
Iteration 14:   tolerance =  5.339e-07

Three-stage least-squares regression, iterated
```

Equation	Obs	Parms	RMSE	"R-sq"	chi2	P
lnst	22	4	.3022006	0.6117	54.83	0.0000
lndt	22	2	.135203	0.8982	178.41	0.0000

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
lnst						
lnpd	2.212666	2.005956	1.10	0.270	-1.718936	6.144268
lnpx2	-.8435967	.3049354	-2.77	0.006	-1.441259	-.2459342
lnpx3	-1.460044	.4623671	-3.16	0.002	-2.366267	-.5538216
lnpx4	-1.009892	.7998393	-1.26	0.207	-2.577548	.557764
_cons	17.37893	14.61488	1.19	0.234	-11.26571	46.02357
lndt						
lnpd	-2.574157	.4046743	-6.36	0.000	-3.367304	-1.78101
lngdp	.5212921	.0955104	5.46	0.000	.3340951	.708489
_cons	35.93499	5.106302	7.04	0.000	25.92682	45.94316

```
Endogenous variables:  lnst lnpd lndt
Exogenous variables:   lnpx2 lnpx3 lnpx4 lngdp
```

5.) β_{11} implies price elasticity of quantity supply

β_{21} implies price elasticity of quantity demand