

Quiz#5 EE320 Semester 1/2019: Answer the question in the area provided.

A firm produces two different kinds of commodity, namely A and B. The daily cost of producing x units of A and y units of B is

$$C(x, y) = 2x^2 - 4xy + 4y^2 - 40x - 20y + 14$$

Suppose that the firm sells all its output at a price per unit of \$24 for A and \$12 for B. Find the daily production levels x and y that maximize profit. Confirm your result with second-order conditions test.

$$\pi = (24x + 12y) - (2x^2 - 4xy + 4y^2 - 40x - 20y + 14)$$

FOC $\pi_x = \pi_y = 0$

$$\pi_x = 24 - (4x - 4y - 40) = 0$$

$$\pi_y = 12 - (-4x + 8y - 20) = 0$$

$$4x - 4y = 64 \quad \text{--- (1)}$$

$$-4x + 8y = 32 \quad \text{--- (2)}$$

(1) + (2) $\Rightarrow 4y = 96 \Rightarrow y^* = 24$ units.

$$4x^* = 64 + 4(24)$$

$$= 64 + 96 = 160$$

$$x^* = 40 \text{ units}$$

SOC $H_2 = \begin{bmatrix} \pi_{xx} & \pi_{xy} \\ \pi_{yx} & \pi_{yy} \end{bmatrix}$

$$= \begin{bmatrix} -4 & +4 \\ +4 & -8 \end{bmatrix}$$

$$|H_1| = -4 < 0$$

$$|H_2| = 32 - 16 = 16 > 0$$

(i) H is negative definite
at $(x^*, y^*) \rightarrow d^2\pi < 0$

(ii) since H is always
negative definite $\rightarrow d^2\pi < 0$
 $\forall x, y$

(x^*, y^*) is also
global maximum
solution.