

Properties of Absolute-Value Inequalities

Theorem 2: Properties of Absolute-Value Inequalities

Let x and y be real numbers.

- (1) If $x \neq 0$, then $|x| > 0$.
- (2) $|x| = 0$ if and only if $x = 0$
- (3) $|-x| = |x|$, $|x| \geq |x|$, $|x| \leq |x|$
- (4) $-|x| \leq x \leq |x|$
- (5) $|x + y| \leq |x| + |y|$
- (6) $|x - y| \leq |x| + |y|$
- (7) $||x| - |y|| \leq |x - y|$
- (8) $|xy| = |x||y|$
- (9) $\left|\frac{x}{y}\right| \leq \frac{|x|}{|y|}$
- (10) $|x| < |y|$ if and only if $x^2 < y^2$, $|x| \leq |y|$ if and only if $x^2 \leq y^2$
- (11) $|x|^2 = x^2$