

EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. X_i is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3.0	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

$Y_i - \bar{Y}$	$X_i - \bar{X}$
-0.4125	-14.625
0.1875	-8.625
-0.2125	0.375
0.2875	3.375
0.3875	9.375
-0.2125	-2.625
-0.5125	-2.625
0.4875	12.375

$$\bar{Y} = 3.2125$$

$$\bar{X} = 77.625$$

$$\beta_2 = \frac{\sum (Y_i - \bar{Y})(X_i - \bar{X})}{\sum (X_i - \bar{X})^2}$$

$$\beta_1 = \bar{Y} - \beta_2 \bar{X}$$

- 1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of β_1 and β_2 . Interpret the regression.

- 1.2 Find $\hat{Y}_i$ and $\hat{u}_i$ and show that $\sum_{i=1}^n \hat{u}_i \approx 0$

- 1.3 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1)$, and $var(\hat{\beta}_2)$

$$1.1) \beta_2 = \frac{\sum (y_i)(x_i)}{\sum (x_i)^2}$$

$$= \frac{6.0328 - 1.0524 - 0.0797 + 0.9703 + 3.6328 + 0.5578 + 1.3453 + 6.0328}{511.875}$$

$$= \frac{17.4347}{511.875} = 0.0341 \quad \#$$

$$\beta_1 = \bar{Y} - \beta_2 \bar{X}$$

$$= 3.2125 - (0.0341) 77.625$$

$$= 3.2125 - 2.6470$$

$$= 0.5655 \quad \#$$

$$Y_i = 0.5655 + 0.0341 X_i + u_i \quad \#$$

$$1.2) \quad \hat{Y}_1 = 0.5655 + 0.0341(63) \\ = 2.7138$$

$$\hat{Y}_2 = 0.5655 + 0.0341(72) \\ = 3.0207$$

$$\hat{Y}_3 = 3.2253$$

$$\hat{Y}_4 = 3.3276$$

$$\hat{Y}_5 = 3.5322$$

$$\hat{Y}_6 = 3.1230$$

$$\hat{Y}_7 = 3.1230$$

$$\hat{Y}_8 = 3.6345$$

$$\hat{U} = Y_i - \hat{Y}_i$$

$$\hat{U}_1 = 0.0862$$

$$\hat{U}_2 = 0.3793$$

$$\hat{U}_3 = -0.2253$$

$$\hat{U}_4 = 0.1724$$

$$\hat{U}_5 = 0.0678$$

$$\hat{U}_6 = -0.123$$

$$\hat{U}_7 = -0.423$$

$$\hat{U}_8 = 0.0655$$

$$\sum_{i=1}^n \hat{U}_i = -0.0001 \\ \approx 0 \quad \#$$

$$1.3) \text{var}(\hat{u}_i), \text{var}(\hat{\beta}_1), \text{var}(\hat{\beta}_2)$$

$$\text{var}(\hat{u}_i) = \frac{5512}{n-2} = \frac{\sum_{i=1}^8 \hat{u}_i^2}{n-2} = \frac{\sum_{i=1}^8 \hat{u}_i^2}{6}$$

$$= \frac{0.4347}{6} = 0.0725 \quad \#$$

$$\text{var}(\hat{\beta}_1) = \frac{\sum (x_i)^2 b^2}{n \sum x_i^2}, \quad x_i = x_i - \bar{x}$$

$$= \frac{(48717)(0.0725)}{8(511.875)} = 0.8625 \quad \#$$

$$\text{var}(\hat{\beta}_2) = \frac{b_0^2}{\sum x_i^2} = \frac{0.0725}{511.875} = 0.0001 \quad \#$$

2. Data is listed in the table

$y_i - \bar{y}$	$x_i - \bar{x}$
-9.1	-10
-7.1	-8
-4.1	-6
-3.1	-4
-2.1	-2
0.9	2
0.9	4
5.9	6
6.9	8
10.9	10

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

$$\bar{x} = 20$$

$$\bar{y} = 9.1$$

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i), var(\hat{\beta}_1), var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$$Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$$

Find an OLS estimator of β_1 . Then, provide a proof that this is an unbiased estimator.

Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

"Practice makes Perfect."

$$2.1) \beta_2 = \frac{\sum (x_i)(y_i)}{\sum (x_i)^2}$$

$$= (9.1)(-10) + (-7.1)(-8) + (-4.1)(-6)$$

$$+ (-3.1)(-4) + (-2.1)(-2) + (0.9)(2)$$

$$+ (0.9)(4) + (5.9)(6) + (6.9)(8) + (10.9)(10)$$

$$(-10)^2 + (-8)^2 + (-6)^2 + (-4)^2 + (-2)^2$$

$$+ (2)^2 + (4)^2 + (6)^2 + (8)^2 + (10)^2$$

$$= \frac{91 + 56.8 + 24.6 + 12.4 + 4.2 + 1.8 + 3.6 + 35.4 + 55.2}{440} \quad \times 10^9$$

$$= \frac{394}{440} = 0.8955 \#$$

$$\beta_1 = \bar{y} - \beta_2 \bar{x}$$

$$= 9.1 - 0.8955(20)$$

$$= 9.1 - 17.91$$

$$= -8.81$$

$$y_i = -8.81 + 0.8955x_i + u_i$$

$$2.2) \hat{y}_1 = -8.81 + 0.8955(10) = 0.145$$

$$\hat{y}_2 = -8.81 + 0.8955(12) = 1.936$$

$$\hat{y}_3 = -8.81 + 0.8955(14) = 3.727$$

$$\hat{y}_4 = -8.81 + 0.8955(16) = 5.518$$

$$\hat{y}_5 = -8.81 + 0.8955(18) = 7.309$$

$$\hat{y}_6 = -8.81 + 0.8955(22) = 10.891$$

$$\hat{y}_7 = -8.81 + 0.8955(24) = 12.682$$

$$\hat{y}_8 = -8.81 + 0.8955(26) = 14.473$$

$$\hat{y}_9 = -8.81 + 0.8955(28) = 16.264$$

$$= 16.264$$

$$\hat{y}_{10} = -8.81 + 0.8955(30) = 18.055$$

$$= 18.055$$

$$\hat{u}_i = y_i - \hat{y}_i$$

$$\hat{u}_1 = 0 - 0.145 = -0.145$$

$$\hat{u}_2 = 2 - 1.936 = 0.064$$

$$\hat{u}_3 = 5 - 3.727 = 1.273$$

$$\hat{u}_4 = 6 - 5.518 = 0.482$$

$$\hat{u}_5 = 7 - 7.309 = -0.309$$

$$\hat{u}_6 = 10 - 10.891 = -0.891$$

$$\hat{u}_7 = 10 - 12.682 = -2.682$$

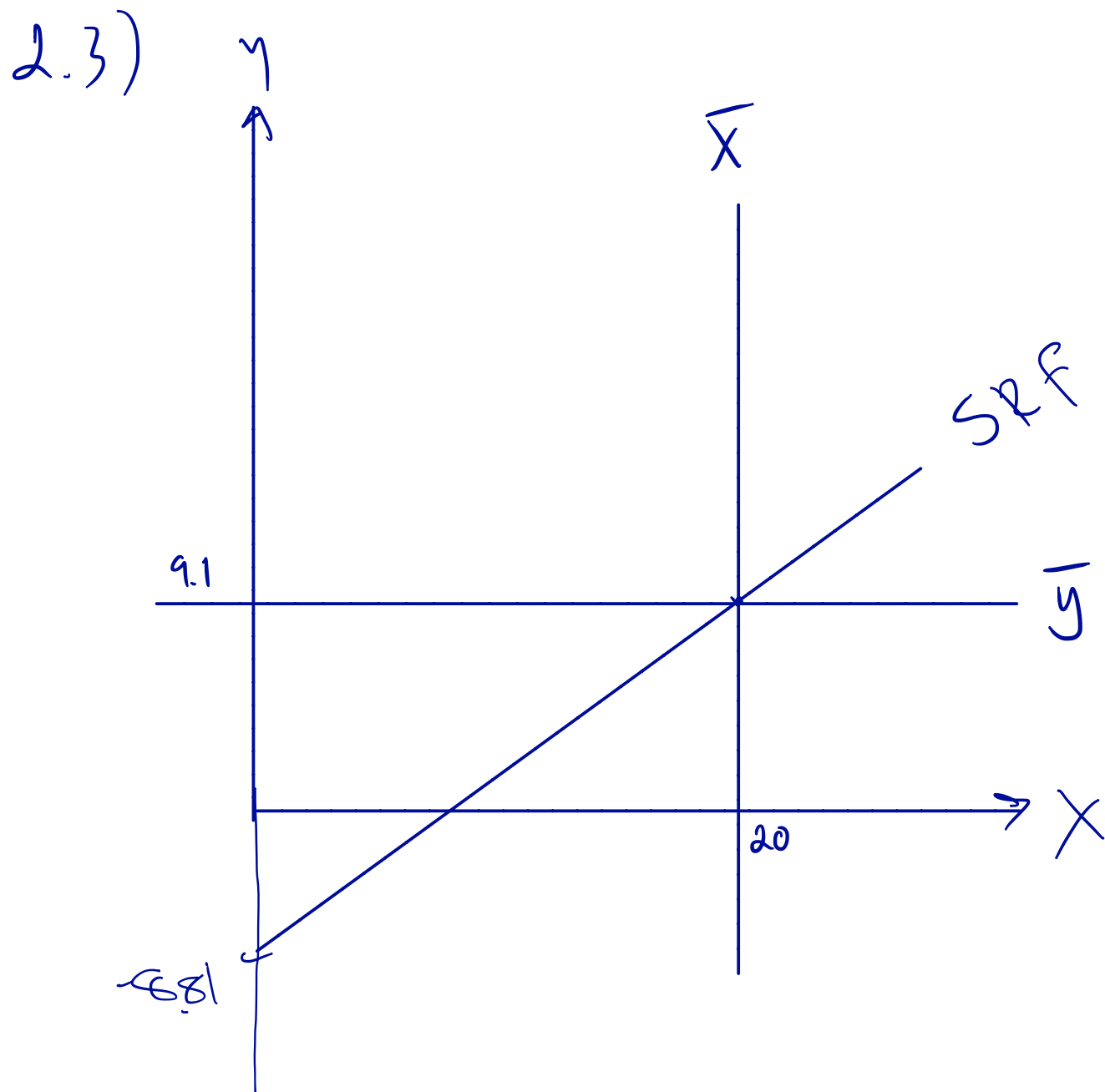
$$\hat{u}_8 = 15 - 14.473 = 0.527$$

$$\hat{u}_9 = 16 - 16.264 = -0.264$$

$$\hat{u}_{10} = 20 - 18.055 = 1.945$$

$$\begin{aligned}\sum u_i &= -0.145 + 0.064 + 1.273 + 0.482 \\ &\quad - 0.309 - 0.891 - 2.682 + 0.527 \\ &\quad - 0.264 + 1.945 = 0\end{aligned}$$

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we plug $\bar{X} = 20$ into the function,

$$\begin{aligned}\bar{Y} &= -8.81 + 0.8955(20) \\ &= 9.1\end{aligned}$$

$\therefore$ this regression function
pass through $(\bar{X}, \bar{Y})$

$$2.4) X_i = 18, \hat{Y}_i = ?$$

$$\begin{aligned}\hat{Y}_i &= -8.81 + 0.8955(18) \\ &= 7.309\end{aligned}$$

$$d.5) \text{var}(\hat{u}), \text{var}(\hat{\beta}_1), \text{var}(\hat{\beta}_2)$$

$$\text{var}(\hat{u}) = \frac{\sum u_i^2}{n-2} = \frac{\sum (y_i - \hat{y})^2}{n-2}$$

$$= \frac{14.0909}{8} = 1.7614$$

$$\text{var}(\hat{\beta}_1) = \frac{\sum x_i^2}{n \sum (x_i - \bar{x})^2} = \frac{4440}{10(440)} = 1.0091$$

$$\text{var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum x_i^2} = \frac{1.7614}{4,440} = 0.004.$$

$$3.) \quad \hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 \hat{x}_i + \hat{u}_i$$

$$\min \sum \hat{u}_i^2 \quad \hat{u}_i = \hat{y}_i - \hat{\beta}_1 - \hat{\beta}_2 \hat{x}_i$$

↓

$$\sum \hat{u}_i^2 = \sum (\hat{y}_i - \hat{\beta}_1 - \hat{\beta}_2 \hat{x}_i)^2$$

diff to find min.

$$-2 \sum (\hat{y}_i - \hat{\beta}_1 - \hat{\beta}_2 \hat{x}_i) = 0$$

$$\sum \hat{y}_i - \sum \hat{\beta}_1 - \beta_2 \sum \hat{x}_i = 0$$

$$\sum \hat{\beta}_1 = \sum \hat{y}_i - \beta_2 \sum \hat{x}_i$$

$$n \hat{\beta}_1 = \sum \hat{y}_i - \beta_2 \sum \hat{x}_i \quad \left(\div n \right)$$

$$\beta_1 = \bar{y} - \beta_2 \bar{x}$$

unbiased when $E(\hat{\beta}_1) = \beta_1$

$$\hat{\beta}_1 = \overline{y} - \beta_2 \overline{x}$$

$$\hat{\beta}_1 = \beta_1 + \cancel{\beta_2 x} - \cancel{\beta_2 x}$$

$$\hat{\beta}_1 = \beta_1$$

$$E(\hat{\beta}_1) = \beta_1$$