

$$\begin{aligned}\textcircled{1} \text{ firm 1 ; } \pi_1 &= TR_1 - TC_1 \\ \pi_1 &= (a - bq_1 - bq_2 - bq_3)q_1 - C_1 \\ \frac{\partial \pi_1}{\partial q_1} &= a - 2bq_1 - bq_2 - bq_3 = 0 \\ a - bq_2 - bq_3 &= 2bq_1 \\ q_1 &= \frac{a - bq_2 - bq_3}{2b}\end{aligned}$$

$$\begin{aligned}\text{substitute } \textcircled{2} \text{ into } q_1 \\ q_1 &= \frac{a - \left(\frac{a - bq_3}{3b}\right)b - bq_3}{2b} \\ q_1 &= \frac{3a - a + bq_3 - 3bq_3}{6b} \\ q_1 &= \frac{a - bq_3}{3b} \quad \text{--- } \textcircled{1}\end{aligned}$$

$$\begin{aligned}\text{substitute } \textcircled{3} \text{ into } q_1 \\ q_1 &= \frac{a - \left(\frac{a}{4b}\right)b}{3b} \\ q_1 &= \frac{4a - a}{12b} \\ q_1 &= \frac{a}{4b}\end{aligned}$$

$$\begin{aligned}\text{firm 3 ; } \pi_3 &= TR_3 - TC_3 \\ \pi_3 &= (a - bq_1 - bq_2 - bq_3)q_3 - C_3 \\ \frac{\partial \pi_3}{\partial q_3} &= a - bq_1 - bq_2 - 2bq_3 = 0 \\ a - bq_1 - bq_2 &= 2bq_3 \\ q_3 &= \frac{a - bq_1 - bq_2}{2b}\end{aligned}$$

$$\begin{aligned}\text{substitute } \textcircled{1} \&\textcircled{2} \text{ into } q_3 \\ q_3 &= \frac{a - b\left(\frac{a - bq_3}{3b}\right) - b\left(\frac{a - bq_3}{3b}\right)}{2b} \\ q_3 &= \frac{3a - a + bq_3 - a + bq_3}{6b} \\ q_3 &= \frac{a + 2bq_3}{6b} \\ 6bq_3 &= a + 2bq_3 \\ 4bq_3 &= a \\ q_3 &= \frac{a}{4b}\end{aligned}$$

$$\begin{aligned}\text{firm 2 ; } \pi_2 &= TR_2 - TC_2 \\ \pi_2 &= (a - bq_1 - bq_2 - bq_3)q_2 - C_2 \\ \frac{\partial \pi_2}{\partial q_2} &= a - bq_1 - 2bq_2 - bq_3 = 0 \\ a - bq_1 - bq_3 &= 2bq_2\end{aligned}$$

$$\begin{aligned}\text{substitute } \textcircled{1} \text{ into } 2bq_2 \\ 2bq_2 &= a - \left(\frac{a - bq_2 - bq_3}{2b}\right)b - bq_3 \\ 2bq_2 &= \frac{2a - a + bq_2 + bq_3 - 2bq_3}{2} \\ 4bq_2 &= a + bq_2 - 2bq_3 \\ 3bq_2 &= a - bq_3 \\ q_2 &= \frac{a - bq_3}{3b} \quad \text{--- } \textcircled{2}\end{aligned}$$

$$\begin{aligned}\text{substitute } \textcircled{3} \text{ into } q_2 \\ q_2 &= \frac{a - \left(\frac{a}{4b}\right)b}{3b} \\ q_2 &= \frac{4a - a}{12b} \\ q_2 &= \frac{a}{4b}\end{aligned}$$

$$\begin{aligned}\text{Equilibrium price ; } P &= a - bQ \\ P &= a - b(q_1 + q_2 + q_3) \\ P &= a - b\left(\frac{a}{4b} + \frac{a}{4b} + \frac{a}{4b}\right) \\ P &= a - \frac{3a}{4} \\ P &= \frac{a}{4} = 0.25a\end{aligned}$$

$$\begin{aligned}\text{firm 1 ; } \pi_1 &= P \cdot q_1 - C_1 \\ &= 0.25a \cdot \frac{a}{4b} - C_1 \\ &= \frac{a^2}{16b} - C_1\end{aligned}$$

$$\begin{aligned}\text{firm 2 ; } \pi_2 &= P \cdot q_2 - C_2 \\ &= 0.25a \cdot \frac{a}{4b} - C_2 \\ &= \frac{a^2}{16b} - C_2\end{aligned}$$

$$\begin{aligned}\text{firm 3 ; } \pi_3 &= P \cdot q_3 - C_3 \\ &= 0.25a \cdot \frac{a}{4b} - C_3 \\ &= \frac{a^2}{16b} - C_3\end{aligned}$$

② Assume $q_1 + q_2 + q_3 + \dots + q_n = A$

$$P = a - b(q_1 + q_2 + q_3 + \dots + q_n)$$

$$P = a - bq_1 - bq_2 - \dots - bq_n$$

$$\pi_1 = (a - bq_1 - bq_2 - bq_3 - \dots - bq_n)q_1 - C_1$$

⋮

$$\pi_n = (a - bq_1 - bq_2 - bq_3 - \dots - bq_n)q_n - C_n$$

$$\frac{\partial \pi_1}{\partial q_1} = a - bq_1 - bq_2 - bq_3 - \dots - bq_n = 0$$

$$q_1 = \frac{a}{2b} - 0.5(q_2 + q_3 + \dots + q_n)$$

$$q_n = \frac{a}{2b} - 0.5(q_2 + q_3 + \dots + q_{n-1})$$

$$\therefore q_1 - 0.5q_1 = \frac{a}{2b} - 0.5(q_2 + q_3 + \dots + q_n)$$

$$0.5q_1 = \frac{a}{2b} - 0.5A$$

$$q_1 = \frac{a}{b} - A \quad \text{————— ①}$$

$$q_2 = \frac{a}{b} - A$$

$$q_3 = \frac{a}{b} - A$$

⋮

$$q_n = \frac{a}{b} - A$$

$$\text{Since } A = q_1 + q_2 + q_3 + \dots + q_n$$

$$A = n\left(\frac{a}{b} - A\right)$$

$$A = n\left(\frac{a}{b}\right) - nA$$

$$A + nA = n\left(\frac{a}{b}\right)$$

$$(1+n)A = n\left(\frac{a}{b}\right)$$

$$A = \frac{na}{(1+n)b}$$

substitute A into ①

$$q_1 = \frac{a}{(1+n)b}$$

$$\therefore q_i = \frac{a}{(1+n)b}$$

Equilibrium price ; $P = a - b(A)$

$$P = a - b\left(\frac{na}{(1+n)b}\right)$$

$$P = a - \left(\frac{n}{1+n}\right)a$$

$$P = \frac{a(1+n) - na}{1+n}$$

$$P = \frac{a}{1+n}$$

$$\pi_i = P \cdot q_i - C_i$$

$$\pi_i = \frac{a}{1+n} \cdot \frac{a}{(1+n)b} - C_i$$

$$\pi_i = \frac{a^2}{(1+n)^2 b} - C_i$$

- ③ If $n \rightarrow \infty$; it will make $q_i = \frac{a}{(1+n)b} \rightarrow$ nearly zero and each firms will sell at q nearly zero unit
- ; it will make $A = nq_i \rightarrow$ nearly zero Q of every firms combined will be nearly $\rightarrow \infty$ units
- ; it will make $P = \frac{a}{1+n} \rightarrow$ nearly zero When supply increase , price will decrease $\rightarrow$ nearly zero.
- ; it will make $\pi_i = \frac{a^2}{(1+n)^2b} - C_i \rightarrow -C_i$ Each firm will lose its profit

- If $n = 1$; it will make $q_i = \frac{a}{(1+n)b} = \frac{a}{2b}$ Since $Q = \frac{a}{2b} < Q = \frac{na}{(1+n)b}$, monopoly will sell less quantity
- ; it will make $A = nq_i = Q$ Since $n=1$, the firm will be a monopoly.
- ; it will make $P = \frac{a}{1+n} = \frac{a}{2}$ Since $P_M = \frac{a}{2} > P = \frac{a}{1+n}$, monopoly will set a higher price.
- ; it will make $\pi_i = \frac{a^2}{(1+n)^2b} - C_i = \frac{a^2}{4b} - C_i$, which higher than $\pi_i = \frac{a^2}{(1+n)^2b}$
- As a result , a monopoly will get higher profit.