

Question 1 (20 points)

(a) (5 points) What is the LIBOR rate and how is it determined? Are there any concerns with using it as a reference rate?

The London Interbank Offered rate is the rate at which large banks in London lend money among themselves

It is obtained based on a survey of estimated borrowing rates of selected banks

Banks have been charged for collusion and manipulating submissions i.e. they understate the rates they think they can borrow to look stronger and therefore there are some concerns in using it as a reference rate.

(b) (5 points) What is one main drawback of a price weighted average approach in computing the stock index versus the value weighted index? Give an example of an index that is computed using the price weighted approach.

The Dow Jones Industrial Average uses the price weighted approach

In the price weighted average approach, all stocks, whether it has large or small market capitalization, get the same weight whereas in the value weighted index, larger stocks would get more weight. Therefore the index constructed from the price weighted approach may not be a very good representation of the market economy.

(c) (5 points) In what situation would VaR or expected shortfall be a better measure of risk than standard deviation?

VaR and expected shortfall are measures that are used to measure a portfolio's exposure to adverse events eg. downside risk.

However, if the distribution of returns are symmetrical, the VaR and expected shortfall is proportional to the variance of returns and using standard deviation is a good enough measure of risk.

(d) (5 points) Historically, has there been a positive relationship between volatility and average returns for large portfolios? What about for individual stocks? If there is no positive relationship, explain why.

For large portfolios, there has been a positive relationship between volatility and average returns but not so for individual stocks. This is because for large portfolios independent risks are averaged out and diversified away. For this reason, there is a clear cut relationship between the remaining systematic risk and returns of the portfolio.

Question 2 (40 points)

Your employer offers two funds for your pension plan, a money market fund and an S&P 500 index fund. The money market fund holds 3-month Treasury bills, which currently offer a 3% safe return per year. The S&P 500 index fund offers an expected return of 10% per year with a standard deviation of 20%.

(a) (7 points) Consider portfolio A, which according to your level of risk aversion, requires an expected return of 8% per year. What should be the composition of Treasury bills and the S&P 500 index fund in this portfolio? What is the standard deviation of its returns? What is your level of risk aversion?

..... Linearity of expected returns require

$$w * 10\% + (1-w) * 3\% = 8\%$$

$$0.07w + 0.03 = 0.08$$

$$w = 0.714$$

meaning 71.4% should be invested in the S&P 500 index and 28.6% in the risk free asset.

the standard deviation of returns is

$$0.714 * 0.20 = 0.1428 \text{ or } 14.28\%$$

to solve for risk aversion level A

$$y^* = \frac{E(r_p) - r_f}{A \sigma_p^2} = \frac{0.1 - 0.03}{A (0.2)^2} = 0.714$$

$$\Rightarrow 0.07 = \frac{0.03 + 0.07}{A} \quad (\text{not very risk averse})$$

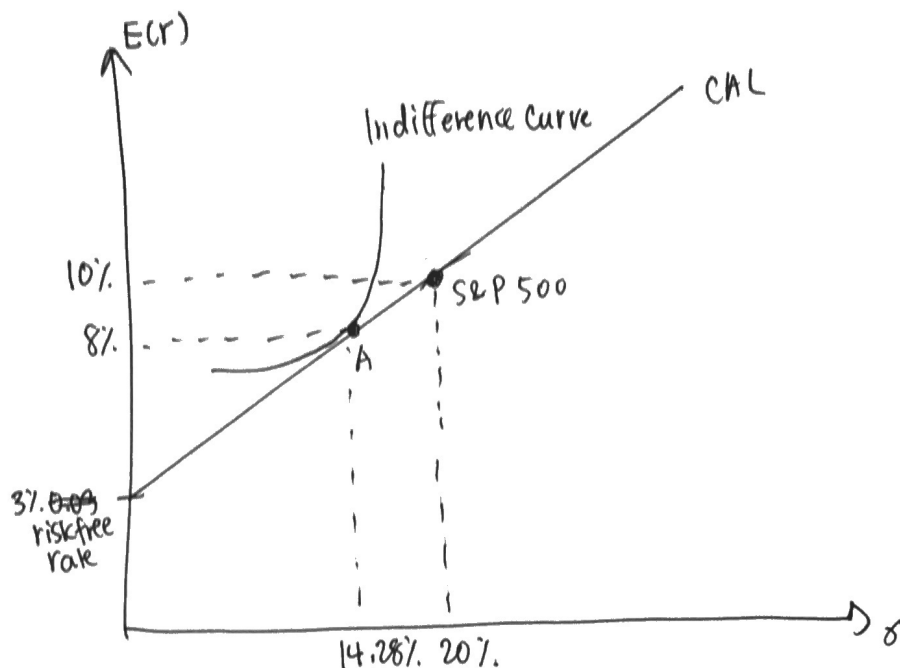
$$A = \frac{0.02856}{0.07} = 2.45$$

(b) (6 points) What is the Sharpe ratio of Portfolio A? What does the Sharpe ratio represent? How is it related to the capital allocation line?

the slope of the CAL is the Sharpe ratio which measures the excess return being earned per unit of volatility

$$\text{Sharpe ratio} = \frac{E(r_A) - r_f}{\sigma_A} = \frac{0.08 - 0.03}{0.1428} = 0.35$$

(c) (7 points) Illustrate Portfolio A on a graph where you also plot the risk-free asset, the S&P 500 index fund, the capital allocation line and your indifference curve. Are you a relatively risk averse investor? Explain.



you are not a relatively risk averse investor since a larger proportion of your wealth is invested in the risky portfolio

(d) (6 points) Now your employer adds an emerging-market fund to the two existing funds. The emerging-market fund offers an expected return of 10% per year, the same as the S&P 500 index fund, but with a standard deviation of 30%, higher than the S&P 500 index fund. Its correlation with the S&P500 index fund is 0. Would you consider including the emerging-market fund as part of your portfolio? Explain.

Even though the emerging markets fund has the same return and higher standard deviation its correlation is < 1 and may lead to achieve lower levels of risk for the portfolio as a whole. This is because we will be able to diversify away some risk. Therefore we should add the emerging fund.

(e) (7 points) Now consider portfolio B, which consists of 80% in the S&P index fund and 20% in the emerging-market fund. Calculate portfolio B's expected return and standard deviation. If you mix portfolio B with the money-market fund to achieve an expected return of 8%, is it better than Portfolio A? Explain.

$$E(r_B) = 0.8 * 10\% + 0.2 * 10\% = 10\%$$

$$\sigma_B^2 = 0.8^2 * (20\%)^2 + 0.2^2 * (30\%)^2 = 0.0292$$

$$\sigma_B = \sqrt{\sigma_B^2} = 17.09\%$$

The weights will be the same as in part a) i.e. 28.6% in the risk-free asset and 71.4% in portfolio B. The standard deviation of this new portfolio will be $0.7143 * 17.09\% = 12.2\%$ which is lower than for the portfolio in part a. We are now better off as we can achieve the same return for lower risk.

(f) (7 points) Is portfolio B the global minimum variance portfolio? If not, solve for the global minimum variance portfolio. Explain whether it is possible for this to be the perfect hedge portfolio.

The correlation between the two risky portfolios $\rho = 0$, in which case

$$w = \frac{\sigma_B^2}{\sigma_A^2 + \sigma_B^2} \quad \text{for the global minimum variance portfolio}$$

$$= \frac{0.30^2}{0.20^2 + 0.30^2}$$

$$= \frac{0.09}{0.04 + 0.09} = 0.6923 \quad \text{i.e. } 69.2\% \text{ in the S\&P 500 fund.}$$

and therefore portfolio B is not the global minimum variance portfolio since it puts 80% weight in the S&P 500 fund and 20% in the emerging market fund.

It is not possible to get a perfect hedge portfolio since $\rho \neq -1$

Question 3 (30 points)

In 1997, the rate of return on short-term government securities (perceived to be risk-free) was about 5%. Suppose the expected rate of return required by the market for a portfolio with a beta measure of 1 is 12%. According to the capital asset pricing model (CAPM):

- (a) (7 points) What is the expected rate of return on the tangency portfolio? What about the expected rate of return on a stock with $\beta = 0$? If a stock has $\beta = 0$ does this mean that it has no risk?

The tangency portfolio is the market portfolio with $\beta = 1$ therefore its expected rate of return is 12%.

$\beta = 0$ means no systematic risk and therefore the return is the risk free rate ie 5%.

If $\beta = 0$ it does not mean there is no risk, it just means the systematic risk is zero but could still have positive total risk.

(b) (7 points) Suppose you consider buying a share of stock at \$40. The stock is expected to pay \$3 dividends next year and you expect it to sell then for \$41. The stock risk has been evaluated by $\beta = -0.5$. Is the stock overpriced or underpriced and would you consider buying or selling the stock?

Using SML $E(r) = 5 + (-0.5)(12 - 5) = 1.5\%$
 which is the fair return for the stock.

The actual expected return is $\frac{44}{40} - 1 = 0.1$ or 10%

The actual expected return exceeds the fair return (by a lot) so the stock is underpriced and you should consider buying the stock.

(c) (8 points) Write out the equilibrium relation underlying CAPM. Describe in terms of the reward to risk ratio the intuition behind this equilibrium relationship. Why is it considered to be an equilibrium relationship?

$$\frac{E(r_i) - r_f}{\text{Cov}(r_i, r_m)} = \frac{E(r_m) - r_f}{\sigma_m^2}$$

This is the equilibrium condition underlying CAPM saying that the reward to risk ratio for all investments should be equal.

If one was better the other, investors would rearrange portfolios by putting more in the one with better tradeoff. They will keep rebalancing portfolios until all reward to

the risk ratios are equalized. When no further changes are made, this becomes the equilibrium situation.

(d) (8 points) What is the two fund separation theorem? Does it imply that passive or active investing is best? Explain why the two fund separation may not hold in the real world.

A theory stating that under the condition in which investors can borrow and lend at the risk free rate and process information in the same way, all investors will choose to hold the same risky portfolio. This is the tangency portfolio or the market portfolio. However, how they allocate funds between this risky portfolio and the risk free asset depends on their level of risk aversion. This implies that passive investing is best.

The two fund separation may not hold in the real world as the borrowing rate is higher than the risk free rate due to default risk. Also, not all agents have access to the same information and process it in the same way.