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EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

*** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work.***

1. Find the answers following questions (please also show your calculation)

a. $\sum_{i=1}^{5}(a+bx_i)$ $= \sum_{i=1}^{5} a + \sum_{i=1}^{5} bx_i$

$= 5a + (bx_1 + bx_2 + bx_3 + bx_4 + bx_5)$

$= 5a + b(x_1 + x_2 + x_3 + x_4 + x_5)$

b. $\sum_{y=0}^{5} f(x+y)$

$= f(x) + f(x+1) + f(x+2) + f(x+3) + f(x+4) + f(x+5)$

c. $\sum_{i=1}^{10} i^2$

$= 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2$

$= 385$

d. $\sum_{x=1}^{2}\sum_{y=2}^{3}(2x+y)$ $= \sum_{x=1}^{2}\left[(2x+2)+(2x+3)\right]$

$= \sum_{x=1}^{2}(2x+2) + \sum_{x=1}^{2}(2x+3)$

$= \left[(2(1)+2)+(2(2)+2)\right] + \left[(2(1)+3)+(2(2)+3)\right]$

$= 22$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

** when b is constant number

a. Find the value of b

$0.5b + b + 2.25b + 2b + 1.5b + 0.5b + 0.25b = 1$

$b = 0.125$

b. Find the answer for $P(X \le 2)$

$P(X \le 2) = 1 - P(X > 2)$

$= 1 - [P(X=3) + P(X=4)]$

$= 1 - [(0.5(0.125)) + (0.25(0.125))]$

$= 1 - [0.0625 + 0.03125]$

$= 1 - 0.09375$

$= 0.90625$

c. Find the answer for $P(-2 \le X \le 3)$

$P(-2 \le X \le 3) = 1 - P(X=4)$

$= 1 - (0.25 \times 0.125)$

$= 1 - 0.03125$

$= 0.96875$

d. Find the answer for $P(X \ge 1)$

$P(X \ge 1) = 1 - P(X < 1)$

$= 1 - [P(X=0) + P(X=-1) + P(X=-2)]$

$= 1 - [(2.25(0.125)) + 0.125 + (0.5(0.125))]$

$= 1 - [0.28 + 0.125 + 0.06]$

$= 1 - 0.47$

$= 0.53125$

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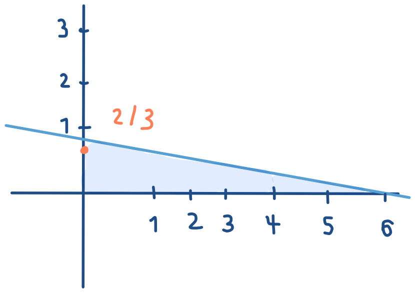
3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \le x \le 3$$

a. Plot graph for $f(x)$



b. Find the answer for $P(1 \le X \le 3)$

$$P(1 \leqslant x \leqslant 3) = \int_1^3 f(x)\,dx$$

$$= -\frac{1}{18}x^2 + \frac{6}{9}x \Big|_1^3$$

$$= \left[\frac{-(3)^2}{18} + \frac{6(3)}{9}\right] - \left[\frac{-(1)^2}{18} + \frac{6(1)}{9}\right]$$

$$= \frac{16}{18}$$

c. Find the answer for $P(X \ge 2)$

$$P(x \geqslant 2) = \int_2^3 f(x)\,dx$$

$$= -\frac{1}{18}x^2 + \frac{6}{9}x \Big|_2^3$$

$$= \left[\frac{-(3)^2}{18} + \frac{6(3)}{9}\right] - \left[\frac{-(2)}{18} + \frac{6(2)}{9}\right]$$

$$= \frac{7}{18}$$

d. Find the expected value of X

$$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_0^3 -\frac{1}{9}x^2 + \frac{6}{9}x\,dx$$

$$= \frac{-x^3}{27} + \frac{6x^2}{18} \Big|_0^3$$

$$= \frac{-27}{27} + \frac{6 \cdot 9}{18}$$

$$= -1 + 3$$

$$= 2$$

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4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

   a. Construct the joint probability distribution function (PDF) table of X and Y

a dice

a coin

$y \backslash x$	1	2	3	4	5	6	marginal (y)
1	1/12	1/12	1/12	1/12	1/12	1/12	1/2
0	1/12	1/12	1/12	1/12	1/12	1/12	1/2
marginal (x)	1/6	1/6	1/6	1/6	1/6	1/6	1

   b. Find the marginal probability distribution function (PDF) of X

The marginal prob of x

$P(X=x) = \frac{1}{6}$

   c. Find the marginal probability distribution function (PDF) of Y

The marginal prob of

$P(Y=y) = \frac{1}{2}$

   d. Find the conditional probability distribution function (PDF) of X given Y is equal to 1 $P(x \mid y=1)$

x	1	2	3	4	5	6
$P(x \mid y=1)$	1/6	1/6	1/6	1/6	1/6	1/6

   e. Find the expected value of X given Y is equal to 1 * 1/2 → 0.5

$$E(X \mid Y=1) = \sum x_i P(X=x_i \mid Y=1)$$
$$= \frac{\sum x_i P(X=x_i, Y=1)}{P(Y=1)}$$
$$= \frac{1}{P(Y=1)} \sum x_i P(X=x_i, Y=1)$$

$$= \frac{1}{0.5}\left[(1\cdot 1/12)+(2\cdot 1/12)+(3\cdot 1/12)+(4\cdot 1/12)+(5\cdot 1/12)+(6\cdot 1/12)\right]$$
$$= \frac{7}{2}$$

   f. Find the variance of X given Y is equal to 1

$$Var(X \mid Y=1) = \sum (X - E(X \mid Y=1))^2 \cdot P(X \mid Y=1)$$
$$= \left[\left((1-\tfrac{7}{2})^2\cdot\tfrac{1}{6}\right) + \left((2-\tfrac{7}{2})^2\cdot\tfrac{1}{6}\right) + \left((3-\tfrac{7}{2})^2\cdot\tfrac{1}{6}\right) + \left((4-\tfrac{7}{2})^2\cdot\tfrac{1}{6}\right) + \left((5-\tfrac{7}{2})^2\cdot\tfrac{1}{6}\right) + \left((6-\tfrac{7}{2})^2\cdot\tfrac{1}{6}\right)\right]$$
$$= \left(\tfrac{25}{4}\cdot\tfrac{1}{6}\right)+\left(\tfrac{9}{4}\cdot\tfrac{1}{6}\right)+\left(\tfrac{1}{4}\cdot\tfrac{1}{6}\right)+\left(\tfrac{1}{4}\cdot\tfrac{1}{6}\right)+\left(\tfrac{9}{4}\cdot\tfrac{1}{6}\right)+\left(\tfrac{25}{4}\cdot\tfrac{1}{6}\right)$$
$$= \frac{10}{3}$$

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5. If X_1, X_2, X_3 is a random sample from a population with mean u and variance σ^2. X_1, X_2, X_3 are not independent
$$Cov(X_1, X_2) = Cov(X_1, X_3) = Cov(X_2, X_3) = \frac{1}{4}\sigma^2$$
$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$
Find $E(\bar{X})$ and $var(\bar{X})$

$$E(\bar{x}) = E\left(\frac{1}{N}\sum_{i=1}^{3} x_i\right)$$
$$= \frac{1}{N} E(x_1 + x_2 + x_3)$$
$$= \frac{1}{N}\left(E(x_1) + E(x_2) + E(x_3)\right)$$
$$= \frac{1}{3}\left[\mu_x + \mu_x + \mu_x\right]$$
$$= \frac{1}{3} \times 3\mu_x$$
$$= \mu_x$$

$$var(\bar{x}) = var\left(\frac{1}{N}\sum_{i=1}^{3} x_i\right)$$
$$= \frac{1}{N^2} Var(x_1 + x_2 + x_3)$$
$$= \frac{1}{N^2}\left[Var(x_1) + Var(x_2) + Var(x_3) + 2cov(x_1, x_2) + 2cov(x_1, x_3) + 2cov(x_2, x_3)\right]$$
$$= \frac{1}{3^2}\left[\frac{1}{4}\sigma_x^2 + \frac{1}{4}\sigma_x^2 + \frac{1}{4}\sigma_x^2 \frac{1}{2}\sigma_x^2 + \frac{1}{2}\sigma_x^2 + \frac{1}{2}\sigma_x^2\right]$$
$$= \frac{1}{9} \cdot \frac{9}{4}\sigma_x^2$$
$$= \frac{\sigma_x^2}{4}$$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean u and variance σ^2. $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

a. Find $E(\bar{X})$ and $var(\bar{X})$ in term of μ and σ

$$E(\bar{x}) = E\left(\frac{1}{N}\sum_{i=1}^{4} x_i\right)$$
$$= \frac{1}{N} E(x_1 + x_2 + x_3 + x_4)$$
$$= \frac{1}{N}\left[E(x_1) + E(x_2) + E(x_3) + E(x_4)\right]$$
$$= \frac{1}{4}\left[\mu_x + \mu_x + \mu_x + \mu_x\right]$$
$$= \mu_x$$

$$var(\bar{x}) = var\left(\frac{1}{N}\sum_{i=1}^{4} x_i\right)$$
$$= \frac{1}{N^2} Var(x_1 + x_2 + x_3 + x_4)$$
$$= \frac{1}{N^2}\left[Var(x_1) + Var(x_2) + Var(x_3) + Var(x_4)\right]$$
$$= \frac{1}{4^2}\left[\sigma_x^2 + \sigma_x^2 + \sigma_x^2 + \sigma_x^2\right]$$
$$= \frac{1}{16} \cdot 4\sigma_x^2$$
$$= 0.25\sigma_x^2$$

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b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ. Show that $\tilde{X}$ is an unbiased estimator of u

$\tilde{x} = \frac{1}{4}(0.5x_1 + x_2 + 0.5x_3 + 2x_4)$

$E(\tilde{x}) = E(\frac{1}{N}\sum_{i=1}^{4} x_i)$

$= \frac{1}{4}E(0.5x_1 + x_2 + 0.5x_3 + 2x_4)$

$= \frac{1}{4}[E(0.5x_1) + E(0.5x_3) + E(x_2) + 2E(x_4)]$

$= \frac{1}{4}(4\mu_x)$

$= \mu_x$

$\tilde{x}$ is unbiased estimator for μ

$var(\tilde{x}) = Var(\frac{1}{N}\sum_{i=1}^{4} x_i)$

$= \frac{1}{4} Var(0.5x_1 + x_2 + 0.5x_3 + 2x_4)$

$= \frac{1}{4^2}[Var(0.5x_1) + Var(x_2) + Var(0.5x_3) + Var(2x_4)]$

$= \frac{1}{4^2}[0.25\sigma^2 + \sigma^2_x + 0.25\sigma^2_x + 4\sigma^2_x]$

$= \frac{5.5\sigma^2_x}{16}$

$= 0.34\sigma^2_x$

c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ? Why?

$\bar{x}$ is more efficient estimator of μ_x than $\tilde{x}$ due to the smaller variance.