Assignment 4

1. Let a and b be real numbers with $0 < b < a < 1$. Determine whether each of the following inequalities is true or not. Explain your answer.

   (a) $\frac{(b+1)^3}{|a|} < \frac{(a+1)^3}{b}$

   (b) $\frac{b}{a} < \frac{a^3-1}{b^3-1}$

2. Let x and y be real numbers with $y > x > 1$. Show that

$$y(y-2) > |x-1|^2 - 1.$$

3. Find the solution set for each of following inequalities.

   (a)
$$\frac{x^2-1}{x} < \frac{x+1}{2x}$$

   (b)
$$2 \le \left|\frac{x-1}{x}\right| \le 7$$

   (c)
$$\frac{x^2+|x|+1}{x^7+x^5+x^2+1} \le 0$$

   (d)
$$\frac{|x+3|-2}{5} + \frac{1}{|x-1|+1} \le 1$$

   (e)
$$\frac{|x-1|-x^2-1}{5-|x+3|} \ge 0$$

4. (Optional) Find the solution set for each of the following inequities.

   (a) $\frac{x-1}{x+1} \le 0$

   (b) $\frac{-x^2-x}{2-x} \ge 0$

   (c) $\frac{2x}{x-2} - \frac{3x}{x-4} \le 1$

   (d) $\frac{(2x^2+4x+7)(x-1)}{x^3-5x^2+x-5} \le 0$

   (e) $\frac{x^2+8}{x^2+x+6} \ge 0$

   (f) $\frac{x^2-2x-4}{x^2-x-6} \ge 0$

   (g) $\frac{2x-x^2-3}{x-2x^2-1} \le 1$

   (h) $\frac{(-x^2+5x-7)(2x+1)}{x^4-1} \ge 0$

(i) $\frac{x^2-1}{2-3x+x^2} > \frac{1}{x}$

(j) $\frac{x^4+6x^2}{1-2x} \leq x^2$

(k) $x^5 + 3x^4 - 23x^3 - 51x^2 + 94x + 120 \geq 0$

(l) $\frac{\ln(x^2+x+4)}{|\sin(x)-\cos(x)|-3} < 0$

5. (Optional) Find the solution set for each of the following inequities.

(a) $|x - 1| < 7$

(b) $3 < |x + 2| < 4$

(c) $|2x + 3| \leq 2$

(d) $|3x - 1| \geq 1$

(e) $\frac{|x+2|}{2} > |x|$

(f) $|x + 1| - |2x - 1| < 0$

(g) $|2 - x| - x \leq 0$

(h) $|x + 1| > 3 - |x|$

(i) $x^2 - 2 \geq \frac{1}{2}|x - 1|$

(j) $|x + 3| \geq 2 - x$

(k) $|2x^2 + x - 1| \geq 2$

(l) $\frac{|x|}{2} + 3 > x^2$

(m) $|x| > \frac{2}{|x+1|}$

(n) $\frac{x^2}{x+1} < |x|$

(o) $\frac{4}{|x|} \leq \frac{1}{|x|}$

(p) $|x^2 + 1| < |x + 1|$

(q) $(x^2 + 1)(|x + 2| - |x|) \geq 0$

(r) $\frac{|1-x|}{x^3+2x^2+5x+4} \leq 0$