

EE435 Assignment 2

1. State reduce form model of these system models.

$$\ln S_t = \beta_{10} + \beta_{11} \ln P_{Dt} + \beta_{12} \ln P_{X2t} + \beta_{13} \ln P_{X3t} + \beta_{14} \ln P_{X4t} + \varepsilon_{1t} \quad - (1) \quad \ln S_t : \ln D_t \quad - (3)$$

$$\ln D_t = \beta_{20} + \beta_{21} \ln P_{Dt} + \beta_{22} \ln GDP_t + \varepsilon_{2t} \quad - (2)$$

$$\beta_{10} + \beta_{11} \ln P_{Dt} + \beta_{12} \ln P_{X2t} + \beta_{13} \ln P_{X3t} + \beta_{14} \ln P_{X4t} + \varepsilon_{1t} : \beta_{20} + \beta_{21} \ln P_{Dt} + \beta_{22} \ln GDP_t + \varepsilon_{2t}$$

$$(\beta_{11} - \beta_{21}) \ln P_{Dt} = \beta_{20} + \beta_{22} \ln GDP_t + \varepsilon_{2t} - \beta_{10} - \beta_{12} \ln P_{X2t} - \beta_{13} \ln P_{X3t} - \beta_{14} \ln P_{X4t} - \varepsilon_{1t}$$

$$\ln P_{Dt} = \frac{\beta_{20}}{\beta_{11} - \beta_{21}} + \frac{\beta_{22} \ln GDP_t}{\beta_{11} - \beta_{21}} + \frac{\varepsilon_{2t}}{\beta_{11} - \beta_{21}} - \frac{\beta_{10}}{\beta_{11} - \beta_{21}} - \frac{\beta_{12} \ln P_{X2t}}{\beta_{11} - \beta_{21}} - \frac{\beta_{13} \ln P_{X3t}}{\beta_{11} - \beta_{21}} - \frac{\beta_{14} \ln P_{X4t}}{\beta_{11} - \beta_{21}} - \frac{\varepsilon_{1t}}{\beta_{11} - \beta_{21}} \quad - (4)$$

$$\textcircled{4} \text{ in } \textcircled{2} \quad \ln D_t : \beta_{20} + \beta_{21} \left(\frac{\beta_{20}}{\beta_{11} - \beta_{21}} + \frac{\beta_{22} \ln GDP_t}{\beta_{11} - \beta_{21}} + \frac{\varepsilon_{2t}}{\beta_{11} - \beta_{21}} - \frac{\beta_{10}}{\beta_{11} - \beta_{21}} - \frac{\beta_{12} \ln P_{X2t}}{\beta_{11} - \beta_{21}} - \frac{\beta_{13} \ln P_{X3t}}{\beta_{11} - \beta_{21}} - \frac{\beta_{14} \ln P_{X4t}}{\beta_{11} - \beta_{21}} - \frac{\varepsilon_{1t}}{\beta_{11} - \beta_{21}} \right) + \beta_{22} \ln GDP_t + \varepsilon_{2t}$$

$$\text{Let } \pi_{20} = \beta_{20} + \frac{\beta_{21} (\beta_{20} - \beta_{10})}{\beta_{11} - \beta_{21}} \quad \pi_{21} = \beta_{22} + \frac{\beta_{21} \beta_{22}}{\beta_{11} - \beta_{21}} \quad \pi_{22} = - \frac{\beta_{21} \beta_{12}}{\beta_{11} - \beta_{21}} \quad \pi_{23} = - \frac{\beta_{21} \beta_{13}}{\beta_{11} - \beta_{21}} \quad \pi_{24} = - \frac{\beta_{21} \beta_{14}}{\beta_{11} - \beta_{21}} \quad W_{2t} = \frac{\beta_{21} (\varepsilon_{2t} - \varepsilon_{1t})}{\beta_{11} - \beta_{21}} + \varepsilon_{2t}$$

$$\ln D_t = \pi_{20} + \pi_{21} \ln GDP_t + \pi_{22} \ln P_{X2t} + \pi_{23} \ln P_{X3t} + \pi_{24} \ln P_{X4t} + W_{2t}$$

$$\textcircled{4} \text{ in } \textcircled{1} \quad \ln S_t = \beta_{10} + \beta_{11} \left(\frac{\beta_{20}}{\beta_{11} - \beta_{21}} + \frac{\beta_{22} \ln GDP_t}{\beta_{11} - \beta_{21}} + \frac{\varepsilon_{2t}}{\beta_{11} - \beta_{21}} - \frac{\beta_{10}}{\beta_{11} - \beta_{21}} - \frac{\beta_{12} \ln P_{X2t}}{\beta_{11} - \beta_{21}} - \frac{\beta_{13} \ln P_{X3t}}{\beta_{11} - \beta_{21}} - \frac{\beta_{14} \ln P_{X4t}}{\beta_{11} - \beta_{21}} - \frac{\varepsilon_{1t}}{\beta_{11} - \beta_{21}} \right) + \beta_{12} \ln P_{X2t} + \beta_{13} \ln P_{X3t} + \beta_{14} \ln P_{X4t} + \varepsilon_{1t}$$

$$\text{Let } \pi_{10} = \beta_{10} + \frac{\beta_{11} (\beta_{20} - \beta_{10})}{\beta_{11} - \beta_{21}} \quad \pi_{11} = \frac{\beta_{11} \beta_{22}}{\beta_{11} - \beta_{21}} \quad \pi_{12} = - \frac{\beta_{11} \beta_{12}}{\beta_{11} - \beta_{21}} + \beta_{12} \quad \pi_{13} = - \frac{\beta_{11} \beta_{13}}{\beta_{11} - \beta_{21}} + \beta_{13}$$

$$\pi_{14} = - \frac{\beta_{11} \beta_{14}}{\beta_{11} - \beta_{21}} + \beta_{14} \quad W_{1t} = \frac{\beta_{11} (\varepsilon_{2t} - \varepsilon_{1t})}{\beta_{11} - \beta_{21}} + \varepsilon_{1t}$$

$$\ln S_t = \pi_{10} + \pi_{11} \ln GDP_t + \pi_{12} \ln P_{X2t} + \pi_{13} \ln P_{X3t} + \pi_{14} \ln P_{X4t} + W_{1t}$$

2. Estimate reduce form model using OLS and prediction of the endogenous variables.

```
. tsset obs
      time variable: obs, 1986 to 2007
              delta: 1 unit
```

```
. g lnst=ln(st)
```

```
. g lndt=ln(dt)
```

```
. g pd=pm+t
```

```
. g lnpd=ln(pd)
```

```
. g lnpx2=ln(px2)
```

```
. g lnpx3=ln(px3)
```

```
. g lnpx4=ln(px4)
```

```
. g lngdp=ln(gdp)
```

```
. reg lnst lngdp lnpx2 lnpx3 lnpx4
```

Source	SS	df	MS	Number of obs	=	22
Model	4.64569724	4	1.16142431	F(4, 17)	=	37.32
Residual	.529104674	17	.031123804	Prob > F	=	0.0000
				R-squared	=	0.8978
				Adj R-squared	=	0.8737
Total	5.17480192	21	.246419139	Root MSE	=	.17642

lnst	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lngdp	.3438812	.1913463	1.80	0.090	-.0598242	.7475865
lnpx2	-.4503744	.1515961	-2.97	0.009	-.7702142	-.1305347
lnpx3	-.9242052	.2783356	-3.32	0.004	-1.511442	-.3369685
lnpx4	-.3883793	.4222332	-0.92	0.371	-1.279214	.5024549
_cons	24.65741	5.309757	4.64	0.000	13.4548	35.86002

```
. reg lndt lngdp lnp2 lnp3 lnp4
```

Source	SS	df	MS	Number of obs	=	22
Model	3.4026552	4	.850663799	F(4, 17)	=	26.43
Residual	.54721789	17	.032189288	Prob > F	=	0.0000
				R-squared	=	0.8615
				Adj R-squared	=	0.8289
Total	3.94987309	21	.188089195	Root MSE	=	.17941

lndt	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lngdp	.1265855	.194594	0.65	0.524	-.2839719	.5371429
lnpx2	-.4887365	.1541691	-3.17	0.006	-.8140049	-.1634682
lnpx3	-.7243134	.2830597	-2.56	0.020	-1.321517	-.1271097
lnpx4	-.577921	.4293997	-1.35	0.196	-1.483875	.3280333
_cons	27.18614	5.399879	5.03	0.000	15.79339	38.57889

```
. reg ln2d lngdp lnp2 lnp3 lnp4
```

Source	SS	df	MS	Number of obs	=	22
Model	.17707359	4	.044268398	F(4, 17)	=	6.76
Residual	.111247189	17	.006543952	Prob > F	=	0.0019
				R-squared	=	0.6142
				Adj R-squared	=	0.5234
Total	.288320779	21	.013729561	Root MSE	=	.08089

ln2d	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lngdp	.1632779	.0877392	1.86	0.080	-.0218357	.3483914
lnpx2	.1318015	.0695123	1.90	0.075	-.0148567	.2784596
lnpx3	.0939842	.127627	0.74	0.472	-.1752851	.3632535
lnpx4	.4939641	.1936093	2.55	0.021	.0854842	.9024439
_cons	2.87652	2.434717	1.18	0.254	-2.260283	8.013322

3. Estimate structural form using predicted endogenous variables as independent variables in the structural form model.

```
. predict lnsthat, xb
variable lnsthat already defined
r(110);

. predict lndthat, xb
variable lndthat already defined
r(110);

. predict lnpdhat, xb

. reg lnst lnpdhat lnpx2 lnpx3 lnpx4
```

Source	SS	df	MS	Number of obs	=	22
Model	4.64569773	4	1.16142443	F(4, 17)	=	37.32
Residual	.529104183	17	.031123775	Prob > F	=	0.0000
				R-squared	=	0.8978
				Adj R-squared	=	0.8737
Total	5.17480192	21	.246419139	Root MSE	=	.17642

lnst	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnpdhat	2.106112	1.171903	1.80	0.090	-.3663879	4.578612
lnpx2	-.727963	.1840856	-3.95	0.001	-1.11635	-.3395762
lnpx3	-1.122146	.2824139	-3.97	0.001	-1.717988	-.5263052
lnpx4	-1.428722	.4751381	-3.01	0.008	-2.431176	-.4262679
_cons	18.59912	8.546622	2.18	0.044	.5673274	36.63092

```
. reg lndt lnpdhat lngdp
```

Source	SS	df	MS	Number of obs	=	22
Model	3.26129847	2	1.63064924	F(2, 19)	=	44.99
Residual	.688574614	19	.036240769	Prob > F	=	0.0000
				R-squared	=	0.8257
				Adj R-squared	=	0.8073
Total	3.94987309	21	.188089195	Root MSE	=	.19037

lndt	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnpdhat	-2.574157	.5697943	-4.52	0.000	-3.76675	-1.381563
lngdp	.5212927	.1344816	3.88	0.001	.2398194	.802766
_cons	35.93498	7.189835	5.00	0.000	20.88648	50.98347

4. Estimate the structural models of these system equations using OLS, 2SLS, 3SLS, and I3SLS. Concerning the asymptotic property, which model is the most appropriate model? Why?

```
. . reg3 (lnst lnpx2 lnpx3 lnpx4) (lndt lnpx2 lngdp), ols
```

Multivariate regression

Equation	Obs	Parms	RMSE	"R-sq"	F-Stat	P
lnst	22	4	.1652258	0.9103	43.14	0.0000
lndt	22	2	.1391259	0.9069	92.53	0.0000

	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnst						
lnpx2	-1.111835	.4515147	-2.46	0.019	-2.027549	-.1961207
lnpx3	-.4189546	.1431634	-2.93	0.006	-.7093034	-.1286059
lnpx4	-.9424196	.2585266	-3.65	0.001	-1.466736	-.4181034
_cons	41.4946	3.661911	11.33	0.000	34.0679	48.9213
lndt						
lnpx2	-2.181329	.2946999	-7.40	0.000	-2.779008	-1.58365
lngdp	.5776586	.0887536	6.51	0.000	.397658	.7576593
_cons	31.03578	3.761201	8.25	0.000	23.40771	38.66385

```
. reg3 (lnst lnpx2 lnpx3 lnpx4) (lndt lnpx2 lngdp), 2sls inst (lnpx2 lnpx3 lnpx4 lngdp)
```

Two-stage least-squares regression

Equation	Obs	Parms	RMSE	"R-sq"	F-Stat	P
lnst	22	4	.329951	0.6424	10.67	0.0000
lndt	22	2	.1454858	0.8982	77.04	0.0000

	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnst						
lnpx2	2.10611	2.191774	0.96	0.343	-2.339013	6.551233
lnpx3	-.7279628	.3442892	-2.11	0.041	-1.426214	-.0297119
lnpx4	-1.122146	.528189	-2.12	0.041	-2.193363	-.0509293
_cons	-1.428722	.8886357	-1.61	0.117	-3.230959	.3735147
lndt						
lnpx2	18.59914	15.98447	1.16	0.252	-13.81886	51.01715
lngdp	-2.574157	.4354519	-5.91	0.000	-3.457295	-1.69102
_cons	.5212921	.1027745	5.07	0.000	.3128558	.7297283
	35.93499	5.494663	6.54	0.000	24.7913	47.07868

Endogenous variables: lnst lnpx2 lndt

Exogenous variables: lnpx2 lnpx3 lnpx4 lngdp

```
. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), 3sls inst (lnpx2 lnpx3 lnpx4 lngdp)
```

Three-stage least-squares regression

Equation	Obs	Parms	RMSE	"R-sq"	chi2	P
lnst	22	4	.2963642	0.6266	57.47	0.0000
lndt	22	2	.135203	0.8982	178.41	0.0000

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
lnst						
lnpd	2.171576	1.926095	1.13	0.260	-1.603501	5.946652
lnpx2	-.7990055	.2985983	-2.68	0.007	-1.384247	-.2137635
lnpx3	-1.329743	.4560002	-2.92	0.004	-2.223487	-.4359989
lnpx4	-1.171403	.775654	-1.51	0.131	-2.691657	.348851
_cons	17.84948	14.04122	1.27	0.204	-9.670808	45.36976
lndt						
lnpd	-2.574157	.4046743	-6.36	0.000	-3.367304	-1.78101
lngdp	.5212921	.0955104	5.46	0.000	.3340951	.708489
_cons	35.93499	5.106302	7.04	0.000	25.92682	45.94316

Endogenous variables: lnst lnpd lndt

Exogenous variables: lnpx2 lnpx3 lnpx4 lngdp

```
. reg3 (lnst lnpd lnpx2 lnpx3 lnpx4) (lndt lnpd lngdp), 3sls ireg3 inst (lnpx2 lnpx3 lnpx4 lngdp)
```

```
Iteration 1:  tolerance =  .1059484
Iteration 2:  tolerance =  .04569793
Iteration 3:  tolerance =  .01846611
Iteration 4:  tolerance =  .00725496
Iteration 5:  tolerance =  .00281814
Iteration 6:  tolerance =  .00108981
Iteration 7:  tolerance =  .00042072
Iteration 8:  tolerance =  .00016231
Iteration 9:  tolerance =  .0000626
Iteration 10: tolerance =  .00002414
Iteration 11: tolerance =  9.310e-06
Iteration 12: tolerance =  3.590e-06
Iteration 13: tolerance =  1.384e-06
Iteration 14: tolerance =  5.339e-07
```

Three-stage least-squares regression, iterated

Equation	Obs	Parms	RMSE	"R-sq"	chi2	P
lnst	22	4	.3022006	0.6117	54.83	0.0000
lndt	22	2	.135203	0.8982	178.41	0.0000

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
lnst						
lnpd	2.212666	2.005956	1.10	0.270	-1.718936	6.144268
lnpx2	-.8435967	.3049354	-2.77	0.006	-1.441259	-.2459342
lnpx3	-1.460044	.4623671	-3.16	0.002	-2.366267	-.5538216
lnpx4	-1.009892	.7998393	-1.26	0.207	-2.577548	.557764
_cons	17.37893	14.61488	1.19	0.234	-11.26571	46.02357
lndt						
lnpd	-2.574157	.4046743	-6.36	0.000	-3.367304	-1.78101
lngdp	.5212921	.0955104	5.46	0.000	.3340951	.708489
_cons	35.93499	5.106302	7.04	0.000	25.92682	45.94316

Endogenous variables: lnst lnpd lndt

Exogenous variables: lnpx2 lnpx3 lnpx4 lngdp

In case that the asymptotic property is prioritized, the 3SLS is the most appropriated. The OLS result is not asymptotically efficient by its nature. While among ILS, 2SLS and 3SLS, the result is more consistent and efficient. However, 3SLS gives the lowest standard deviations, meaning that it has more asymptotically efficient comparing to other models.

5. What do β_{11} and β_{22} mean?

The relationship between log-log implies that there is a proportional change between independent and dependent variables. This marginal effect shows that one percentage change in independent variables will cause a proportional change on the dependent variable by the percentage of the coefficient.

Therefore, β_{11} represents to price elasticity of supply and β_{21} represents to price elasticity of demand.