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BACHELOR OF ECONOMICS
THAMMASAT UNIVERSITY

EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. $X_i$is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3.0	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

1.2 Find $\hat{Y}_i$ and $\hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$

Student	Y_i	X_i	$X_i - \bar{X}$	$(X_i - \bar{X})^2$	$Y_i - \bar{Y}$	$(X_i - \bar{X})(Y_i - \bar{Y})$
1	2.8	63	-14.625	213.890625	-0.4125	6.0328125
2	3.4	72	-5.625	31.640625	0.1875	-1.0546875
3	3.0	78	0.375	0.140625	-0.2125	-0.0796875
4	3.5	81	3.375	11.390625	0.2875	0.9703125
5	3.6	87	9.375	87.890625	0.3875	3.6328125
6	3.0	75	-2.625	6.890625	-0.2125	0.5578125
7	2.7	75	-2.625	6.890625	-0.5125	1.3453125
8	3.7	90	12.375	153.140625	0.4875	6.0328125
	$\Sigma Y_i = 25.7$	$\Sigma X_i = 621$	$\Sigma(X_i - \bar{X}) = 0$	$\Sigma(X_i - \bar{X})^2 = 511.875$	$\Sigma(Y_i - \bar{Y}) = 0$	$\Sigma(X_i - \bar{X})(Y_i - \bar{Y}) =$

0 เสมอ 0 เสมอ 17.4375

(1.1) $\bar{X} = \dfrac{\Sigma X_i}{n} = \dfrac{621}{8} = 77.625$

$\bar{Y} = \dfrac{\Sigma Y_i}{n} = \dfrac{25.7}{8} = 3.2125$

$$\hat{\beta}_2 = \frac{\Sigma(X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma(X_i - \bar{X})^2}$$

$$= \frac{17.4375}{511.875}$$

$$= 0.0341$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$$

$$= 3.2125 - (0.0341)(77.625)$$

$$= 0.5655$$

$$\hat{Y}_i = 0.5655 + 0.0341 X_i$$

$\hat{\beta}_1 = 0.5655$ means that if total econometrics exam point is equal to zero, BE student's GPA is 0.5655 ※

$\hat{\beta}_2$: 0.0341 means that if total econometrics exam point increased by 1 point, on average, BE student's GPA increased by 0.0341

$\hat{Y}_i = 0.5155 + 0.0341X_i$

(1.2)

Student	Y_i	X_i	$\hat{Y}_i$	$\hat{u}_i = Y_i - \hat{Y}_i$	$\hat{u}_i^2$	X_i^2
1	2.8	63	2.7138	0.0862	0.00743044	3969
2	3.4	72	3.0207	0.3793	0.14386849	5184
3	3.0	78	3.2253	-0.2253	0.05076009	6084
4	3.5	81	3.3276	0.1724	0.02972176	6561
5	3.6	87	3.5322	0.0678	0.00459684	7569
6	3.0	75	3.123	-0.123	0.015129	5625
7	2.7	75	3.123	-0.423	0.178929	5625
8	3.7	90	3.6345	0.0655	0.00429025	8100
	$\sum Y_i = 25.7$	$\sum X_i = 621$		$\sum \hat{u}_i = -0.0001$	$\sum \hat{u}_i^2 =$ 0.43472587	$\sum X_i^2 = 48717$

$\sum \hat{u}_i \approx 0$

(1.3) $Var(\hat{u}_i) = \hat{\sigma}^2 = \dfrac{\sum \hat{u}_i^2}{n-2} = \dfrac{0.43472587}{8-2} = 0.0725$

$$Var(\hat{\beta}_1) = \frac{\hat{\sigma}^2 \sum X_i^2}{n \sum (X_i - \bar{X})^2} = \frac{(0.0725)(48717)}{8(511.875)} = 0.8625$$

$$Var(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\sum (X_i - \bar{X})^2} = \frac{(0.0725)}{(511.875)} = 0.000142$$ ※

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y? $\rightarrow \hat{Y}_i$

2.5 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1), var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$$\boldsymbol{Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)}$$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

"Practice makes Perfect."

(2.1)

X_i	Y_i	$X_i - \bar{X}$	$(X_i - \bar{X})^2$	$Y_i - \bar{Y}$	$(X_i - \bar{X})(Y_i - \bar{Y})$
10	0	-10	100	-9.1	91
12	2	-8	64	-7.1	56.8
14	5	-6	36	-4.1	24.6
16	6	-4	16	-3.1	12.4
18	7	-2	4	-2.1	4.2
22	10	2	4	0.9	1.8
24	10	4	16	0.9	3.6
26	15	6	36	5.9	35.4
28	16	8	64	6.9	55.2
30	20	10	100	10.9	109
$\sum X_i = 200$	$\sum Y_i = 91$	$\sum (X_i - \bar{X}) = 0$	$\sum (X_i - \bar{X})^2 = 440$	$\sum (Y_i - \bar{Y}) = 0$	$\sum (X_i - \bar{X})(Y_i - \bar{Y}) = 394$

$$\bar{X} = \frac{\sum X_i}{n} = \frac{200}{10} = 20$$

$$\bar{Y} = \frac{\sum Y_i}{n} = \frac{91}{10} = 9.1$$

$$\hat{\beta}_2 = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2} = \frac{394}{440} = 0.8955 \text{ ※}$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} = 9.1 - (0.8955)(20) = 9.1 - 17.91 = -8.81 \text{ ※}$$

$$\hat{Y}_i = -8.81 + 0.8955 X_i$$

$\hat{\beta}_1 = -8.81$ means that if total econometrics exam point is equal to zero, BE student's GPA is -8.81 ※

$\hat{\beta}_2 = 0.8955$ means that if total econometrics exam point increased by 1 point, on average, BE student's GPA increased by 0.8955

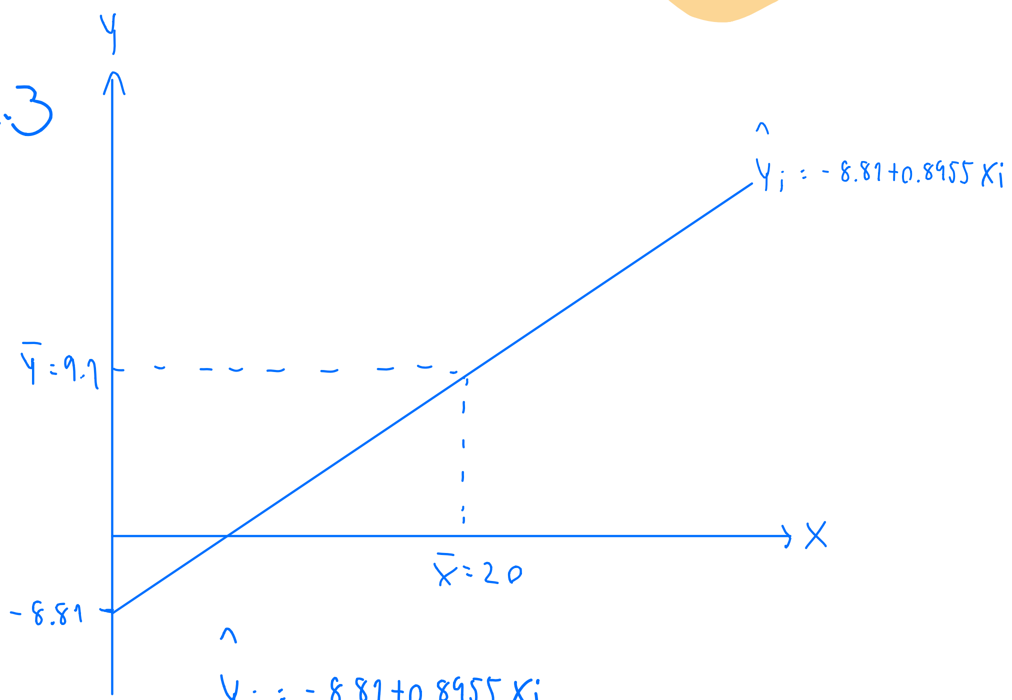
2.2

$\hat{Y}_i = -8.81 + 0.8955 X_i$

X_i	Y_i	$\hat{Y}_i$	$\hat{u}_i = Y_i - \hat{Y}_i$	$\hat{u}_i^2$	X_i^2
10	0	0.145	-0.145	0.021025	100
12	2	1.936	0.064	0.004096	144
14	5	3.727	1.273	1.620529	196
16	6	5.518	0.482	0.232324	256
18	7	7.309	-0.309	0.095481	324
22	10	10.891	-0.891	0.793881	484
24	10	12.682	-2.682	7.193124	576
26	15	14.473	0.527	0.277729	676
28	16	16.264	-0.264	0.069696	784
30	20	18.055	1.945	3.783025	900
$\Sigma X_i = 200$	$\Sigma Y_i = 91$		$\Sigma \hat{u}_i = 0$	$\Sigma \hat{u}_i^2 = 14.09091$	$\Sigma X_i^2 = 4{,}440$

$\Sigma \hat{u}_i \approx 0$

2.3



$\hat{Y}_i = -8.81 + 0.8955 X_i$

$\bar{Y} = -8.81 + 0.8955 \bar{X}$

$\bar{Y} = -8.81 + 0.8955 (20)$

$\bar{Y} = 9.1$

$\therefore$ the regression line pass through the sample means of Y and X ✗

2.4) if $x_i = 18$

$$\hat{Y}_i = -8.81 + 0.8955 X_i$$

$$\hat{Y}_i = -8.81 + 0.8955(18)$$

$$\hat{Y}_i = 7.309\cancel{1}$$

2.5)

$$Var(\hat{u}_i) = \hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{14.09091}{10-2} = 1.7614$$

$$Var(\hat{\beta}_1) = \frac{\hat{\sigma}^2 \sum x_i^2}{n \sum (X_i - \bar{X})^2} = \frac{(1.7614)(4440)}{10\ (440)} = 1.7774$$

$$Var(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\sum (X_i - \bar{X})^2} = \frac{1.7614}{440} \approx 0.0040$$

Note: $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 x_i$

3. Minimize $\sum_{i=1}^{n} \hat{u}_i^2 = \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2$

$= \sum_{i=1}^{n} (Y_i - \hat{\beta}_1 - \hat{\beta}_2 x_i)$ —— 1

Goal: choose $\hat{\beta}_1$ such that for a given sample or set of data, $\sum \hat{u}_i^2$ is 'as smallest as possible'

F.OC: $\dfrac{\partial \sum_{i=1}^{n} \hat{u}_i^2}{\partial \hat{\beta}_1} = 0$ —— 2

$$\frac{\partial \sum_{i=1}^{n} (Y_i - \hat{\beta}_1 - \hat{\beta}_2 x_i)^2}{\partial \hat{\beta}_1} = 0$$

$$2 \sum_{i=1}^{n} (Y_i - \hat{\beta}_1 - \hat{\beta}_2 x_i)(-1) = 0$$

$$-2 \sum \hat{u}_i = 0$$

$$\boxed{\sum_{i=1}^{n} \hat{u}_i = 0}$$ —— 4

$$\sum_{i=0}^{n} \hat{u}_i = 0$$

$$\sum_{i=1}^{n} (Y_i - \hat{\beta}_1 - \hat{\beta}_2 x_i) = 0$$

$$\sum_{i=1}^{n} Y_1 - \sum_{i=1}^{n} \hat{\beta}_1 - \sum_{i=1}^{n} \hat{\beta}_2 x_i = 0$$

$$\sum_{i=1}^{n} Y_i - n \cdot \hat{\beta}_1 - \hat{\beta}_2 \sum_{i=1}^{n} x_i = 0$$

$$n\hat{\beta}_1 = \sum_{i=1}^{n} Y_i - \hat{\beta}_2 \sum_{i=1}^{n} x_i$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} Y_i}{n} - \frac{\hat{\beta}_2 \sum_{i=1}^{n} x_i}{n}$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}, \quad \hat{\beta}_2 = \sum k_i Y_i$$

unbiased: $\boxed{E(\hat{\beta}_1) = \beta_1}$

from $\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$

$$\hat{\beta}_1 = \bar{Y} - \left(\sum_{i=1}^{n} k_i Y_i\right) \cdot \bar{X}$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} Y_i}{n} - \sum_{i=1}^{n} \bar{X} k_i Y_i$$

$\sum k_i = 0$

$\sum k_i X_i = 1$

$$\hat{\beta}_1 = \sum_{i=1}^{n} \left(\frac{Y_i}{n} - \bar{X} k_i Y_i \right)$$

$$\hat{\beta}_1 = \sum_{i=1}^{n} \left(\frac{1}{n} - \bar{X} k_i \right) Y_i$$

$$\hat{\beta}_1 = \sum_{i=1}^{n} \left(\frac{1}{n} - \bar{X} k_i \right) \left(\beta_1 + \beta_2 X_i + u_i \right)$$

$$\hat{\beta}_1 = \sum_{i=1}^{n} \left[\frac{\beta_1}{n} + \frac{\beta_2 X_i}{n} + \frac{u_i}{n} - \bar{X} k_i \beta_1 - \bar{X} k_i \beta_2 X_i - \bar{X} k_i u_i \right]$$

$$\hat{\beta}_1 = \frac{1}{n} \sum_{i=1}^{n} \beta_1 + \beta_2 \sum_{i=1}^{n} \frac{X_i}{n} + \frac{1}{n} \sum u_i - \beta_1 \bar{X} \sum_{i=1}^{n} k_i - \beta_2 \bar{X} \sum_{i=1}^{n} k_i X_i - \bar{X} \sum_{i=1}^{n} k_i u_i$$

$$\hat{\beta}_1 = \frac{1}{\cancel{n}} (\cancel{n} \beta_1) + \beta_2 \bar{X} + \frac{1}{n} \sum u_i - \cancel{\beta_1 \bar{X} (0)} - \beta_2 \bar{X} (1) - \bar{X} \sum_{i=1}^{n} k_i u_i$$

$$\hat{\beta}_1 = \beta_1 + \cancel{\beta_2 \bar{X}} + \frac{1}{n} \sum u_i \cancel{- \beta_2 \bar{X}} - \bar{X} \sum_{i=1}^{n} k_i u_i$$

$$\hat{\beta}_1 = \beta_1 + \frac{1}{n} \sum u_i - \bar{X} \sum_{i=1}^{n} k_i u_i$$

$$E(\hat{\beta}_1) = E(\beta_1) + E\left(\frac{1}{n}\sum u_i\right) - E\left(\bar{X}\sum_{i=1}^{n} k_i u_i\right)$$

$$E(\hat{\beta}_1) = \beta_1 + \frac{1}{n} E\left(\sum u_i\right) - \bar{X} E\left(\sum_{i=1}^{n} k_i u_i\right)$$

$$E(\hat{\beta}_1) = \beta_1 + \frac{1}{n} E(u_1 + u_2 + \dots + u_n) - \bar{X} E(k_1 u_1 + k_2 u_2 + \dots + k_n u_n)$$

$$E(\hat{\beta}_1) = \beta_1 + \frac{1}{n}\left[E(u_1) + E(u_2) + \dots + E(u_n)\right] - \bar{X}\left[k_1 E(u_1) + k_2 E(u_2) + \dots + k_n E(u_n)\right]$$

$$E(\hat{\beta}_1) = \beta_1 + \frac{1}{n}\left[0 + 0 + \dots + 0\right] - \bar{X}\left[k_1(0) + k_2(0) + \dots k_n(0)\right]$$

X_i is nonstochastic

k_i " ―――――― "

$E(u_i) = 0$

$$E(\hat{\beta}_1) = \beta_1 + \frac{1}{n}(0) + \bar{X}(0)$$

$$E(\hat{\beta}_1) = \beta_1$$

$\therefore \hat{\beta}_1$ is unbiased estimator