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EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

*** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work. ***

1. Find the answers following questions (please also show your calculation)

a. $\sum_{i=1}^5 (a + bx_i) = 5a + b \sum_{i=1}^5 x_i$
 $= 5a + b(x_1 + x_2 + x_3 + x_4 + x_5)$

b. $\sum_{y=0}^5 f(x+y) = f(x+0) + f(x+1) + f(x+2) + f(x+3) + f(x+4) + f(x+5)$

c. $\sum_{i=1}^{10} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{10(10+1)(2(10)+1)}{6} = 385$

d. $\sum_{x=1}^2 \sum_{y=2}^3 (2x+y) = \sum_{x=1}^2 [2x+2+3] = \sum_{x=1}^2 2x+5 = 2(1)+2(2)+5 = 11$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

** when b is constant number

- a. Find the value of b

$$f(x) = P(x, x) = 0.5b + b + 2.25b + 2b + 1.5b + 0.5b + 0.25b = 1$$

- b. Find the answer for $P(X \leq 2)$

$$\begin{aligned} P(X \leq 2) &= 1 - P(X > 2) \\ &= 1 - P(X=3) - P(X=4) \\ &= 1 - (0.5)(0.125) - (0.25)(0.125) = 0.96875 \end{aligned}$$

- c. Find the answer for $P(-2 \leq X \leq 3)$

$$\begin{aligned} P(-2 \leq X \leq 3) &= 1 - P(X < -2) \\ &= 1 - (0.25)(0.125) \\ &= 0.96875 \end{aligned}$$

- d. Find the answer for $P(X \geq 1)$

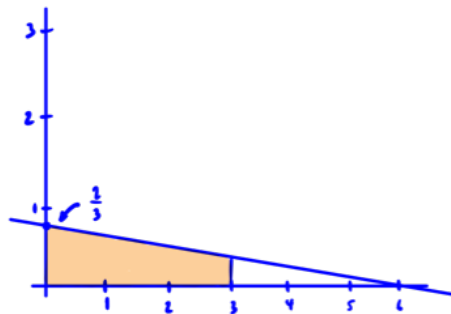
$$\begin{aligned} P(X \geq 1) &= 1 - P(X < 1) \\ &= 1 - P(X=-2) - P(X=-1) - P(X=0) \\ &= 1 - (2.25)(0.125) - (1)(0.125) - (0.5)(0.125) \\ &= 0.515625 \end{aligned}$$

Student Name Pattarit Yamploy ID 610440393

3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \leq x \leq 3 \quad (x, y) = (6, \frac{2}{3})$$

- a. Plot graph for $f(x)$



- b. Find the answer for $P(1 \leq X \leq 3)$

$$\begin{aligned} P(1 \leq X \leq 3) &= \int_1^3 f(x) dx \\ &= \left[-\frac{1}{18}x^2 + \frac{6}{9}x \right]_1^3 \\ &= \left[-\frac{(3)^2}{18} + \frac{6(3)}{9} \right] - \left[-\frac{(1)^2}{18} + \frac{6(1)}{9} \right] = -\frac{9}{18} + \frac{18}{9} - \left(-\frac{1}{18} + \frac{6}{9} \right) = -\frac{3}{18} + \frac{11}{9} = \frac{16}{18} \end{aligned}$$

- c. Find the answer for $P(X \geq 2)$

$$\begin{aligned} P(X \geq 2) &= \int_2^3 f(x) dx \\ &= \left[-\frac{1}{18}x^2 + \frac{6}{9}x \right]_2^3 \\ &= \left[-\frac{(3)^2}{18} + \frac{6(3)}{9} \right] - \left[-\frac{(2)^2}{18} + \frac{6(2)}{9} \right] = -\frac{9}{18} + \frac{18}{9} - \left(-\frac{4}{18} + \frac{12}{9} \right) = \frac{3}{18} \end{aligned}$$

- d. Find the expected value of X

$$\begin{aligned} E(X) &= \int_{-\infty}^{\infty} x f(x) dx \\ &= \int_0^3 x \left(-\frac{1}{9}x + \frac{6}{9} \right) dx \\ &= \int_0^3 \left(-\frac{1}{9}x^2 + \frac{6}{9}x \right) dx \\ &= \left[-\frac{x^3}{27} + \frac{6x^2}{18} \right]_0^3 \end{aligned} \quad \left| \quad \begin{aligned} &= -\frac{(3)^3}{27} + \frac{6(3)^2}{18} \\ &= -1 + 6 \\ &= 5 \end{aligned} \right.$$

Student Name Pattarit Yamploy ID 610460393

4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

- a. Construct the joint probability distribution function (PDF) table of X and Y

x/y	1	2	3	4	5	6	
0	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{2}$
1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{2}$
	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	1

- b. Find the marginal probability distribution function (PDF) of X

the marginal probability of x $P(X=x)$ is $\frac{1}{6}$

- c. Find the marginal probability distribution function (PDF) of Y

the PDF, $P(Y=y)$ is $\frac{1}{2}$

- d. Find the conditional probability distribution function (PDF) of X given Y is equal to

x	1	2	3	4	5	6
$P(X=x Y=1)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

- e. Find the expected value of X given Y is equal to 1

$$E(X|Y=1) = \sum x_i P(X=x_i|Y=1) = \frac{1}{P(Y=1)} \sum x_i P(X=x_i, Y=1)$$

$$= \frac{1}{0.5} \left[(1 \cdot \frac{1}{12}) + (2 \cdot \frac{1}{12}) + (3 \cdot \frac{1}{12}) + (4 \cdot \frac{1}{12}) + (5 \cdot \frac{1}{12}) + (6 \cdot \frac{1}{12}) \right] = \frac{7}{2}$$

- f. Find the variance of X given Y is equal to 1

$$\text{Var}(X|Y=1) = \sum (x - E(X|Y=1))^2 \cdot P(X|Y=1)$$

$$= \left[\left((1 - \frac{7}{2})^2 \cdot \frac{1}{6} \right) + \left((2 - \frac{7}{2})^2 \cdot \frac{1}{6} \right) + \left((3 - \frac{7}{2})^2 \cdot \frac{1}{6} \right) + \left((4 - \frac{7}{2})^2 \cdot \frac{1}{6} \right) + \left((5 - \frac{7}{2})^2 \cdot \frac{1}{6} \right) + \left((6 - \frac{7}{2})^2 \cdot \frac{1}{6} \right) \right]$$

$$= \frac{10}{3}$$

5. If X_1, X_2, X_3 is a random sample from a population with mean μ and variance σ^2 . X_1, X_2, X_3 are not independent

$$\text{Cov}(X_1, X_2) = \text{Cov}(X_1, X_3) = \text{Cov}(X_2, X_3) = \frac{1}{4}\sigma^2$$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$E(\bar{X}) = E\left(\frac{1}{N} \sum_{i=1}^3 X_i\right)$ $= \frac{1}{N} E(X_1 + X_2 + X_3)$ $= \frac{1}{N} [E(X_1) + E(X_2) + E(X_3)]$ $= \frac{1}{3} [\mu_X + \mu_X + \mu_X]$ $= \frac{1}{3} \cdot 3\mu_X = \mu_X$	$\text{var}(\bar{X}) = \text{var}\left(\frac{1}{N} \sum_{i=1}^3 X_i\right)$ $= \frac{1}{N^2} [\text{var}(X_1) + \text{var}(X_2) + \text{var}(X_3) + 2\text{cov}(X_1, X_2) + 2\text{cov}(X_1, X_3) + 2\text{cov}(X_2, X_3)]$ $= \frac{1}{3^2} \left[\frac{1}{4}\sigma_X^2 + \frac{1}{4}\sigma_X^2 + \frac{1}{4}\sigma_X^2 + \frac{1}{4}\sigma_X^2 + \frac{1}{4}\sigma_X^2 + \frac{1}{4}\sigma_X^2 \right]$ $= \frac{1}{3^2} \cdot \frac{9}{4}\sigma_X^2 = \frac{\sigma_X^2}{4}$
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6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean μ and variance σ^2 . $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

- a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$E(\bar{X}) = E\left(\frac{1}{N} \sum_{i=1}^4 X_i\right)$ $= \frac{1}{N} E(X_1 + X_2 + X_3 + X_4)$ $= \frac{1}{N} [E(X_1) + E(X_2) + E(X_3) + E(X_4)]$ $= \frac{1}{4} [\mu_X + \mu_X + \mu_X + \mu_X]$ $= \frac{1}{4} \cdot 4\mu_X = \mu_X$	$\text{var}(\bar{X}) = \text{var}\left(\frac{1}{N} \sum_{i=1}^4 X_i\right)$ $= \frac{1}{N^2} [\text{var}(X_1) + \text{var}(X_2) + \text{var}(X_3) + \text{var}(X_4)]$ $= \frac{1}{4^2} [\sigma_X^2 + \sigma_X^2 + \sigma_X^2 + \sigma_X^2]$ $= \frac{1}{16} \cdot 4\sigma_X^2 = \frac{\sigma_X^2}{4} = 0.25\sigma_X^2$
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Student Name Pattarit Vanphay ID 6104190393

- b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ . Show that $\tilde{X}$ is an unbiased estimator of μ

$$\begin{aligned}\tilde{X} &= \frac{1}{4} (0.5x_1 + x_2 + 0.5x_3 + 2x_4) \\ E(\tilde{X}) &= E\left(\frac{1}{4} \sum_{i=1}^4 x_i\right) \\ &= \frac{1}{4} E(0.5x_1 + x_2 + 0.5x_3 + 2x_4) \\ &= \frac{1}{4} [E(0.5x_1) + E(x_2) + E(0.5x_3) + E(2x_4)] \\ &= \frac{1}{4} [0.5E(x_1) + E(x_2) + 0.5E(x_3) + 2E(x_4)] \\ &= \frac{1}{4} (0.5\mu_x + \mu_x + 0.5\mu_x + 2\mu_x) \\ &= \frac{1}{4} \cdot 4\mu_x = \mu_x\end{aligned}$$

$$\begin{aligned}\text{Var}(\tilde{X}) &= \text{Var}\left(\frac{1}{4} \sum_{i=1}^4 x_i\right) \\ &= \frac{1}{16} \text{Var}(0.5x_1 + x_2 + 0.5x_3 + 2x_4) \\ &= \frac{1}{16} [\text{Var}(0.5x_1) + \text{Var}(x_2) + \text{Var}(0.5x_3) + \text{Var}(2x_4)] \\ &= \frac{1}{16} [0.25 \text{Var}(x_1) + \text{Var}(x_2) + 0.25 \text{Var}(x_3) + 4 \text{Var}(x_4)] \\ &= \frac{1}{16} [0.25\sigma_x^2 + \sigma_x^2 + 0.25\sigma_x^2 + 4\sigma_x^2] \\ &= \frac{5.5}{16} \sigma_x^2 \\ &= 0.34 \sigma_x^2\end{aligned}$$

- c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ ? Why?

$\bar{X}$ is a more efficient estimator of μ than $\tilde{X}$

because it has smaller variance.