**Past Exam Questions**

**Midterm 2009**

3. A company sells 2 products *A* and *B* with the demand functions as follows:

$$q_A = 50 - \frac{1}{2} p_A$$
$$q_B = 76 - p_B$$

The <u>total cost</u> is $c = 3q_A^{\ 2} + 2q_A q_B + 2q_B^{\ 2} + 55$

(a) Calculate the optimum profit and use the **second derivative test** to show that this optimum maximises profit. (10 marks)

(b) If the accountant made a mistake in the calculation of the total cost, the **new total cost** becomes $c = 3q_A^{\ 2} + 2q_A q_B + 1.8q_B^{\ 2} + 55$,

apply the **envelop theorem** to **approximate** the new maximum profit. (3 marks)

**Ans:** P(8,10) = 729, relative max, f*new = 768

4. For a three variable function

$$f(x, y, z) = 2\ln\left(\frac{1}{x}\right) + 3\ln\left(\frac{1}{y}\right) + \ln\left(\frac{1}{z}\right)$$

subject to the constraint

$$3x + 2y - \frac{1}{z} = 1$$

(a) se **the Lagrange multiplier method** to find all critical point(s) of the function and the corresponding minimum value(s) of the function $f(x, y, z)$. (10 marks)

(b) **Estimate** the new optimum value(s) of the function $f(x, y, z)$ if the constraint is changed to $3x + 2y - \frac{1}{z} = 1.1$. (2 marks)

**Ans:** f(1/6, 3/8, 4) = 5.14, f*new = 5.10

6. Find the maximum value of the utility function $U = x^{\frac{4}{5}} y^{\frac{1}{5}}$

subject to the budget constraint $5x + 3y = 75$ and additional constraint $x - y^2 \le 7.8$. (15 marks)

Ans: U(12, 5) = 10.07

7. Find the maximum value of

$$f(x, y) = -5x^2 + 80x - y^2 + 32y \quad \text{subject to } -x - y \le -30,\ -x^2 \le -4. \quad \text{(19 marks)}$$

**Ans:** f(8, 16) = 576, other critical points (14, 16), (-2, 32)

**Midterm 2010**

4. Use Lagrange Multiplier method to find the absolute *minimum* and absolute *maximum* values of $f(x, y, z) = 3xyz$ subjected to a constraint $x^2 + y^2 + z^2 = 12$

   Estimate the new absolute minimum and absolute maximum values of $f(x, y, z)$ if the constraint is changed to $x^2 + y^2 + z^2 = 11.5$ **(7 marks)**

**Ans:** Absolute max f(2,2,2)=f(2,-2,-2) = 24, absolute min f(-2,-2,-2) =f(-2,2,2)=-24, f* new = 22.5 and -22.5

5. Use Lagrange Multiplier method to find the *optimum* value of

$$f(x, y) = \frac{1}{2}x - y + 6 \quad \text{subject to } x + e^{-x} - y \le 0 \text{ and } x \le 2. \quad \textbf{(8 marks)}$$

**Ans:** f(0.693, 0.193) = 6.1535

6. Use Lagrange Multiplier method to solve

$$\max \quad f(x, y) = x + 2y \quad \text{subject to } x^2 + y^2 = 1 \text{ and } -x - y \le 0$$

Use the envelope theorem to estimate the maximum value of $f(x, y) = x + 4y$ subject to the same constraints. **(8 marks)**

**Ans:** f(1/√5, 2/√5) = √5