Example 3.J: Excess burden *formula under linear model & Tax-Revenue-maxizing tax rate*

Demand: $p^d = a - bQ^d \quad ; \; a \geq 0, \quad b \leq 0.$

Supply : $p^s = c + dQ^s \quad ; \; d \geq 0.$

- Solve for quantity and prices equilibrium when the unit tax is imposed. Analyze the result

Before tax

Slove for p^* and Q^*

$p^* = f(a, b, c, d)$

$Q^* = f(a, b, c, d)$

find Q^* : $p^d = p^s$

$$a - bQ = c + dQ$$

$$a - c = dQ + bQ$$

$$a - c = Q(d + b)$$

$$Q^* = \frac{a - c}{d + b}$$

find p^* :

$$p = a - b\left(\frac{a - c}{d + b}\right)$$

$$p^* = \frac{(c + d)(a - c)}{d + b}$$

After tax

Assume tax per unit $= t$

new s : $p = c + dQ + t$

$p^*_{tax} = f(a, b, c, d, t)$

$Q^*_{tax} = f(a, b, c, d, t)$

find Q^* :

$$a - bQ = c + dQ + t$$

$$a - c - t = dQ + bQ$$

$$a - c - t = Q(d + b)$$

$$Q^* = \frac{a - c - t}{d + b}$$

find p^* :

$$p^* = c + t + d\left(\frac{a - c - t}{d + b}\right)$$

- Derive the excess burden formula for buyers and sellers

Consumers' burden $= (P_B - P^*) \times Q_{tax}$

$$= \left[\left(c + t + d\left(\frac{a-c-t}{d+b}\right)\right) - \left(c + d\left(\frac{a-c}{d+b}\right)\right)\right]\left(\frac{a-c-t}{d+b}\right)$$

$$= c + d\left(Q^*_{tax}\right)$$

$$= c + d\left(\frac{a-c-t}{d+b}\right)$$

producers' burden $= (P^* - P_S) \times Q_{tax}$

$$= \left[\left(c + d\left(\frac{a-c}{d+b}\right)\right) - \left(c + d\left(\frac{a-c-t}{d+b}\right)\right)\right]\left(\frac{a-c-t}{d+b}\right)$$

- Calculate the tax rate that maximizes the tax revenue of government.

$$\text{tax revenue} = t \times Q_{tax}$$

$$\frac{\partial \text{ tax revenue}}{\partial t} = 0$$

$$\Rightarrow t^*$$

$$\text{Total tax} = t \times Q_{tax}$$

$$= t \times \left(\frac{a-c-t}{d+b}\right)$$

$$= \frac{at - ct - t^2}{d+b}$$

$$\frac{d \text{ Total tax}}{dt} = \frac{a-c-2t}{d+b} = 0$$

$$2t = c - a$$

$$t = \frac{c-a}{2}$$

Example 3.K ***Price control and Welfare***

Consider the market for apartment rentals in Chicago. The price of rent is determined by the following system of equations.

Demand: $p = -2q_d + 160$ → p^d

Supply: $p = q_s + 10$ → p^s

- What is the equilibrium price and quantity in the market for apartment rentals?

$p^d = p^s$

$-2q + 160 = q + 10$

$160 - 10 = q + 2q$

$3q = 150$

$Q^* = 50$

$p = q + 10$

$p = 50 + 10$

$p^* = 60$

- Suppose the government tries to control the rent prices through a price ceiling of \$40. Discuss the implication of this policy. Is there any deadweight loss?

price selling = 40\$

$40 = -2q_d + 160$

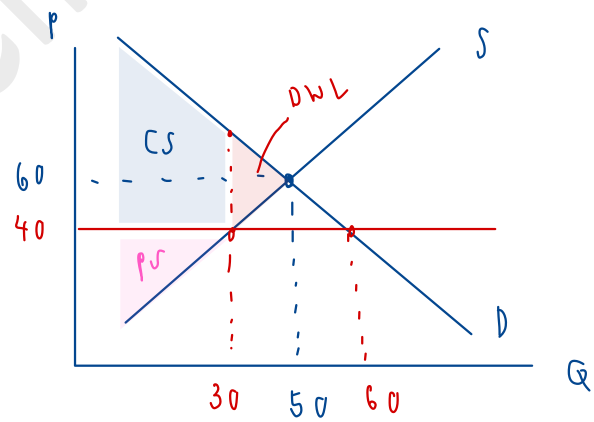
$2q_d = 160 - 40$

$q_d = \frac{120}{2}$

$q_d = 60$

$40 = q_s + 10$

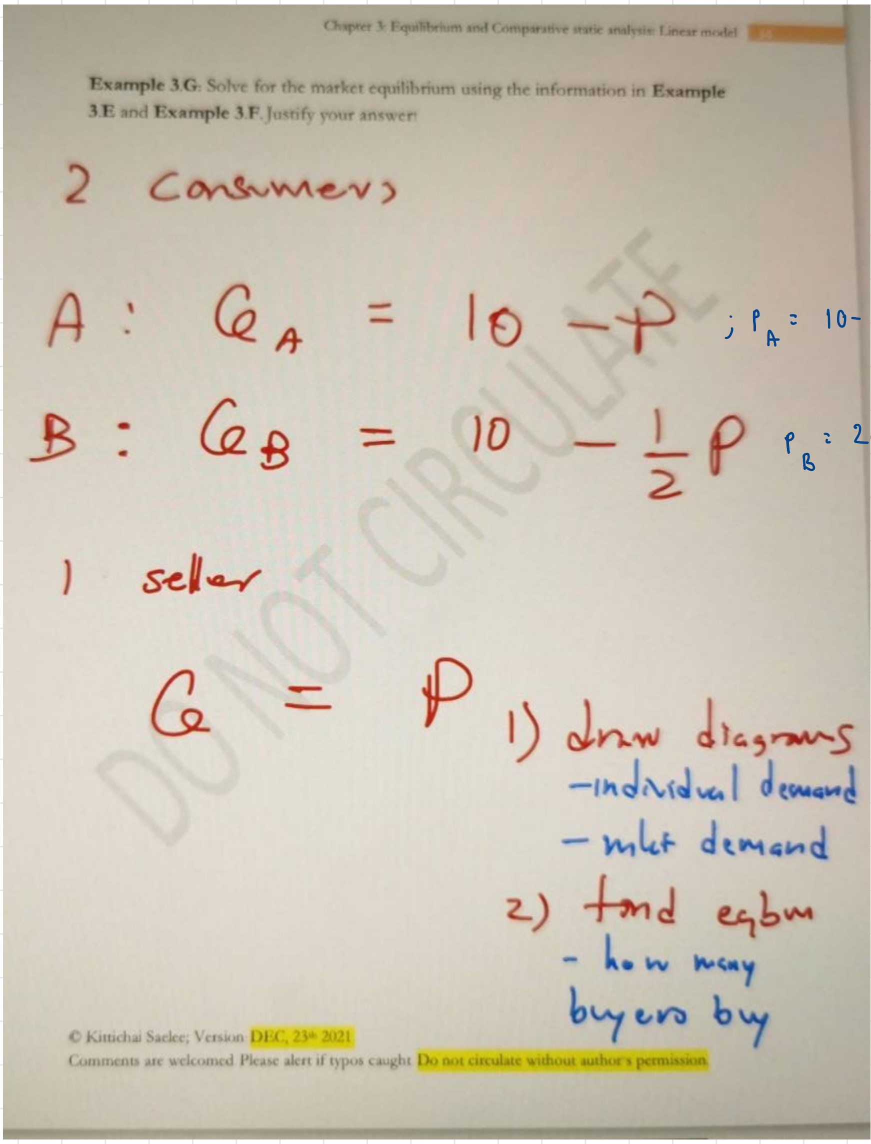
$q_s = 30$



① when price decrease to 40, output also decrease

② CS increase but PS decrease

③ So, there is deadweight loss



Example 3.G: Solve for the market equilibrium using the information in **Example 3.E** and **Example 3.F**. Justify your answer.

2 Consumers

A : $Q_A = 10 - P$; $P_A = 10 - Q_A$

B : $Q_B = 10 - \frac{1}{2}P$; $P_B = 20 - 2Q_B$

1 seller

$Q = P$

1) draw diagrams
- individual demand
- mkt demand

2) find eqbm
- how many buyers buy

$P \geq 10$; $Q_A = 0$, $Q_B = 10 - \frac{1}{2}P$

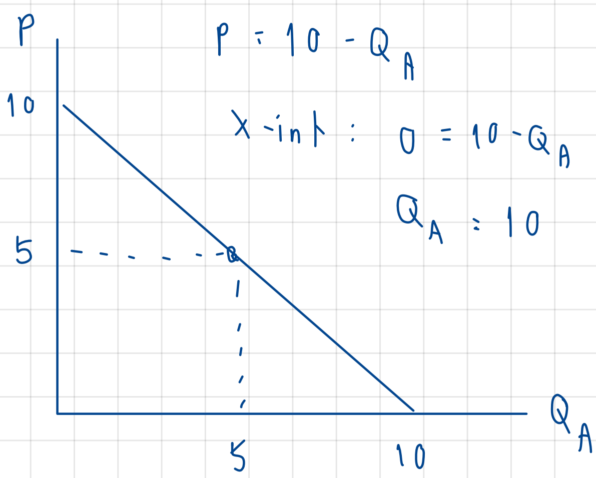
$\rightarrow Q_{market} = 10 - \frac{1}{2}P$

$P < 10$; $Q_A = 10 - P$, $Q_B = 10 - \frac{1}{2}P$

$\rightarrow Q_{market} = Q_A + Q_B$

$= 20 - \frac{3}{2}P$

A

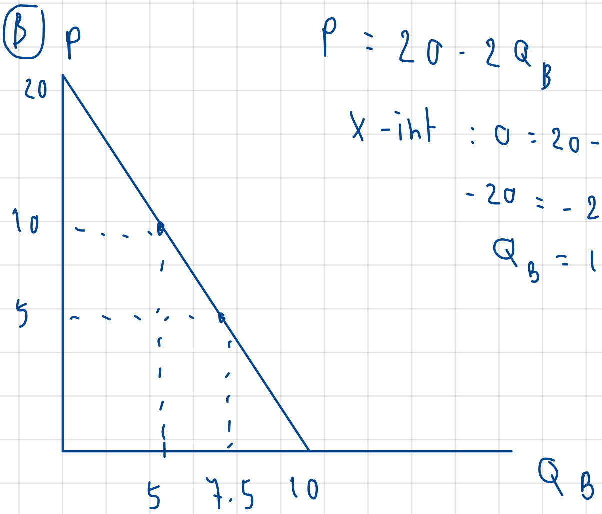


$P = 10 - Q_A$

X-int : $0 = 10 - Q_A$

$Q_A = 10$

B



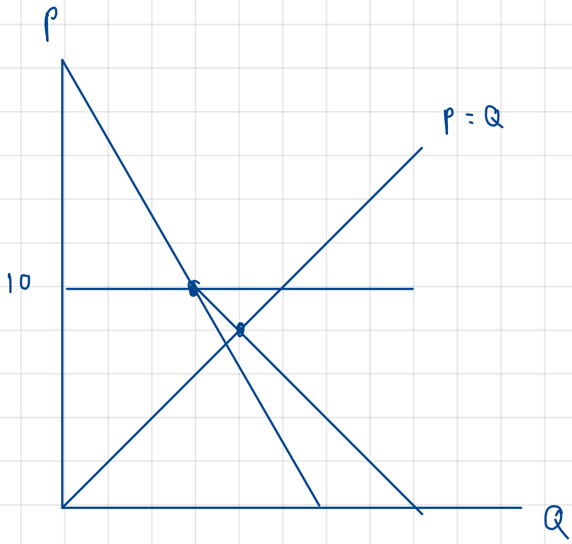
$P = 20 - 2Q_B$

X-int : $0 = 20 - 2Q_B$

$-20 = -2Q_B$

$Q_B = 10$

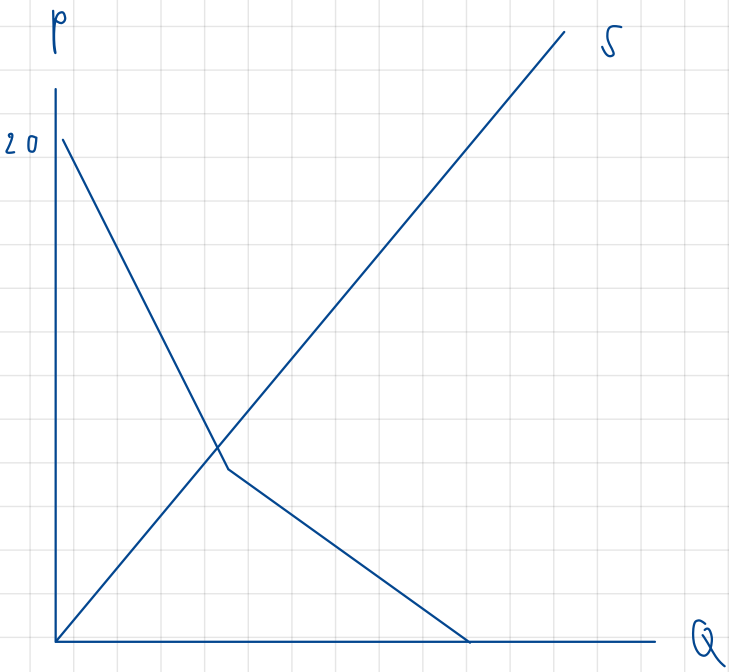
Mkt



$Q^s = Q^d$

$Q = 20 - \frac{3}{2}Q$

$Q = 8$



$$Q^D_{Mkt} \begin{cases} 10 - \frac{1}{2}P & ; P > 10 \\ 20 - \frac{3}{2}P & ; P \le 10 \end{cases}$$

$\therefore$ there are 1 buyer in the market