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EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.),2020

Use 4 decimal places for numerical answers

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

$\bar{x} = \frac{\sum_i x_i}{n}$ | $\bar{x} = \frac{621}{8} = 77.625$ | $\bar{y} = \frac{\sum_i y_i}{n}$ | $\bar{y} = \frac{25.7}{8} = 3.2125$

Table 1.a

Student	Y_i	X_i	$(x_i-\bar{x})(y_i-\bar{y})$
1	2.8	63	$(-14.625)(-0.4125) = 6.0328$
2	3.4	72	$(-5.625)(0.1875) = -1.0547$
3	3	78	$(0.375)(-0.2125) = -0.0797$
4	3.5	81	$(3.375)(0.2875) = 0.9703$
5	3.6	87	$(9.375)(0.3875) = 3.6328$
6	3.0	75	$(-2.625)(-0.2125) = 0.5578$
7	2.7	75	$(-2.625)(-0.5125) = 1.3453$
8	3.7	90	$(12.375)(0.4875) = 6.0328$
			17.4375

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1. (Note: *NIID* = Normally, Identically, and Independently Distributed).

$\hat{\beta}_1 = \frac{\sum_i (x_i-\bar{x})(y_i-\bar{y})}{\sum_i (x_i-\bar{x})^2}$

$\hat{\beta}_1 = \frac{17.4375}{511.875}$

$\hat{\beta}_1 = 0.0340659$

$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$

$\hat{\beta}_0 = 3.2125 - 0.0340659(77.625)$

$\hat{\beta}_0 = 0.5681319$

1.2 For each observation i, find $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$. $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$

$\hat{Y}_1 = 2.7143$
$\hat{u}_1 = 0.0857$

$\hat{Y}_2 = 3.0209$
$\hat{u}_2 = 0.3791$

$\hat{Y}_3 = 3.2252$
$\hat{u}_3 = -0.225$

$\hat{Y}_4 = 3.3275$
$\hat{u}_4 = 0.175$

$\hat{Y}_5 = 3.5319$
$\hat{u}_5 = 0.0681$

$\hat{Y}_6 = 3.1231$
$\hat{u}_6 = -0.1231$

$\hat{Y}_7 = 3.1231$
$\hat{u}_7 = -0.4231$

$\hat{Y}_8 = 3.6340$
$\hat{u}_8 = 0.066$

$\sum_{i=0}^{N} \hat{u}_i = 0$

$\sum \hat{u}_i = 0.0027$

rounding error

1.3 Find $var(\widehat{u_i}), var(\widehat{\beta_0}), var(\widehat{\beta_1})$

$var(\hat{u}_i) = \sigma^2 = \frac{\Sigma \hat{u}_i^2}{n-2}$

$var(\hat{u}_i) = \sigma^2 = 0.0857^2 + 0.3791^2 + (-0.225^2) + 0.175^2 + 0.0681^2 + (-0.1231^2) + (-0.4231^2) + 0.066^2$

$var(\hat{u}_i) = \sigma^2 = \frac{0.4355}{6} = 0.0726$

$var(\hat{\beta}_1) = \frac{\sigma^2}{\Sigma x_i^2} = \frac{\sigma^2}{\Sigma_i (x_i - \bar{x})^2}$

$var(\hat{\beta}_1) = \frac{0.0726}{511.875} = 0.0001$

$var(\hat{\beta}_0) = \frac{\Sigma X_i^2}{n \Sigma x_i^2} \sigma^2$

$var(\hat{\beta}_0) = \Sigma X_i^2 \cdot \frac{\sigma^2}{n \Sigma x_i^2}$

$var(\hat{\beta}_0) = 48{,}717 \cdot 0.0001$

$var(\hat{\beta}_0) = 48{,}717$

2. Data is listed in the table

X_i	Y_i	$(x_i - \bar{x})(y_i - \bar{y})$
10	0	$(-10)(-9.1) = 91$
12	2	$(-8)(-7.1) = 56.8$
14	5	$(-6)(-4.1) = 24.6$
16	6	$(-4)(-3.1) = 12.4$
18	7	$(-2)(-2.1) = 4.2$
22	10	$2\ (0.9) = 1.8$
24	10	$4\ (0.9) = 3.6$
26	15	$6\ (5.9) = 35.4$
28	16	$8\ (6.9) = 55.2$
30	20	$10\ (10.9) = 109$

$\bar{x} = \frac{200}{10}$

$\bar{x} = 20$

$\bar{y} = \frac{91}{10}$

$\bar{y} = 9.1$

$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = 394$

$\Sigma(x_i - \bar{x})^2 = 440$

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i, u_i \sim NIID(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

$\hat{\beta}_1 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2}$

$\hat{\beta}_1 = \frac{394}{440}$

$\hat{\beta}_1 = 0.8955$

$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$

$\hat{\beta}_0 = 9.1 - 0.8955(20)$

$\hat{\beta}_0 = -8.8091$

2.2 Find the value of $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$ | $\hat{u}_i = Y_i - \hat{Y}_i$

$\hat{Y}_1 = 0.1455$
$\hat{u}_1 = -0.1455$

$\hat{Y}_2 = 1.9364$
$\hat{u}_2 = 0.0636$

$\hat{Y}_3 = 3.7273$
$\hat{u}_3 = 1.2727$

$\hat{Y}_4 = 5.5182$
$\hat{u}_4 = 0.4818$

$\hat{Y}_5 = 7.3091$
$\hat{u}_5 = -0.3091$

$\hat{Y}_6 = 10.8909$
$\hat{u}_6 = -0.8909$

$\hat{Y}_7 = 12.6818$
$\hat{u}_7 = -2.6818$

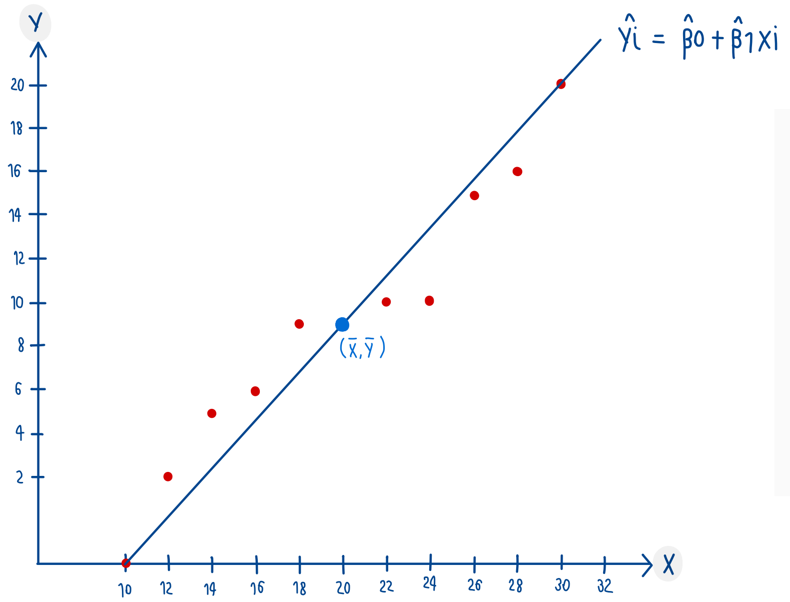
$\hat{Y}_8 = 14.4727$
$\hat{u}_8 = 0.5273$

$\hat{Y}_9 = 16.2636$
$\hat{u}_9 = -0.2636$

$\hat{Y}_{10} = 18.0545$
$\hat{u}_{10} = 1.9455$

From the calculated data,
$\sum_i \hat{u}_i = 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?



From 2.1, $\bar{X} = 20$, $\bar{Y} = 9.1$

$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$

$\hat{Y}_i = -8.8091 + 0.8955(20)$

$\hat{Y}_i = 9.1$

$\therefore$ The line passes through $\bar{X}, \bar{Y}$

2.4 If X_i =16, what is the predicted Y?

$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i = -8.8091 + 0.8955(16) = 5.5189$

2.5 Find $var(\widehat{u}_i), var(\widehat{\beta}_0), var(\widehat{\beta}_1)$

$var(\hat{u}_i) = \sigma^2 = \dfrac{\sum \hat{u}_i^2}{n-2}$

$var(\hat{u}_i) = \sigma^2 = (-0.1455)^2 + 0.0636^2 + 1.2727^2 + 0.4818^2 + (-0.3091)^2 + (-0.8909)^2 + (-2.6818)^2 + 0.5273^2 + (-0.2636)^2$

$var(\hat{u}_i) = \sigma^2 = \dfrac{14.0909}{10-2} = 1.7614$

$var(\hat{\beta}_1) = \dfrac{\sigma^2}{\sum x_i^2} = \dfrac{1.7614}{440} = 0.004$

$var(\hat{\beta}_0) = \dfrac{\sum X_i^2}{n \sum x_i^2} \sigma^2$

$var(\hat{\beta}_0) = \dfrac{4400 \cdot 0.004}{10}$

$var(\hat{\beta}_0) = 1.7614$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i \,,$$

Where $u_i \sim NIID(0, \sigma^2)$. Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$$\hat{\beta}_i = \frac{\sum_i (y_i - \bar{y})(X_i - \bar{X})}{\sum_i (x_i - \bar{x})^2} \quad , \quad \text{then lets } A = X_i - \bar{X} \text{ and } K = \frac{X_i - \bar{X}}{\sum_i (x_i - \bar{X})^2}$$

$$\hat{\beta}_i = \sum_i (y_i - \bar{y}) K_i$$

$$\hat{\beta}_i = \sum_i (\beta_1 X_1 + u_i - \beta_1 \bar{X}) K_i$$

$$= \sum_i (\beta_1 X_1 - \beta_1 \bar{X}) K_i + \sum_i u_i K_i$$

$$= \beta_1 \sum_i (x_1 - \bar{x}) K_i + \sum_i u_i K_i$$

$$= \beta_1 \sum_i A \cdot \frac{A}{\sum_i A^2} + \sum_i u_i K_i$$

$$= \beta_1 \cdot \cancel{\frac{\sum_i A^2}{\sum_i A^2}} + \sum_i u_i K_i$$

$$E(\hat{\beta}_1) = E[\beta_1 + \sum_i u_i K_i]$$

$$= \beta_1 + \underbrace{E(\textstyle\sum_i u_i K_i)}_{\text{SLR4 to make it to be zero}} \longleftarrow$$

SLR4 : $E(u_i | X_i) = 0$

this assumption takes the value of X as given (or fixed) so, we can treat X as a constant. therefore, the error has expected value of zero given any value of X variables.

function of X_i

$$E(\hat{\beta}_1) = \sum_i K_i \, E(u_i)$$

zero

$$\text{so, } E(\hat{\beta}_1) = \beta_1$$

$\therefore$ From the proof, we know that $\hat{\beta}_1$ is an unbiased estimator under assumption SLR 1-4