EE435
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Assignment5_Thunyaporn.R

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```
#EE 435 Assignment 5 Thunyaporn 6004640451

#install.packages("quantmod")
#install.packages("fBasics")
#install.packages("sn")
#install.packages("PerformanceAnalytics")
#install.packages("car")
#install.packages("tseries")
#install.packages("forecast")
#install.packages("fGarch")

library(fBasics)

## Warning: package 'fBasics' was built under R version 4.0.5

## Loading required package: timeDate

## Loading required package: timeSeries

library(quantmod)

## Warning: package 'quantmod' was built under R version 4.0.5

## Loading required package: xts

## Loading required package: zoo

##
## Attaching package: 'zoo'

## The following object is masked from 'package:timeSeries':
##
##     time<-

## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric

## Loading required package: TTR

##
## Attaching package: 'TTR'
```

```
## The following object is masked from 'package:fBasics':
##
##     volatility

## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo

library(sn)

## Warning: package 'sn' was built under R version 4.0.5

## Loading required package: stats4

##
## Attaching package: 'sn'

## The following objects are masked from 'package:fBasics':
##
##     tr, vech

## The following object is masked from 'package:stats':
##
##     sd

library(PerformanceAnalytics)

## Warning: package 'PerformanceAnalytics' was built under R version 4.0.5

##
## Attaching package: 'PerformanceAnalytics'

## The following objects are masked from 'package:timeDate':
##
##     kurtosis, skewness

## The following object is masked from 'package:graphics':
##
##     legend

library(car)

## Warning: package 'car' was built under R version 4.0.5

## Loading required package: carData

##
## Attaching package: 'car'

## The following object is masked from 'package:fBasics':
##
##     densityPlot

library(tseries)
```

```
## Warning: package 'tseries' was built under R version 4.0.5
```

```
library(forecast)
```

```
## Warning: package 'forecast' was built under R version 4.0.5
```

```
library(fGarch)
```

```
## Warning: package 'fGarch' was built under R version 4.0.5
```

```
getSymbols("CAT",from="2006-01-03",to="2017-04-13")
```

```
## 'getSymbols' currently uses auto.assign=TRUE by default, but will
## use auto.assign=FALSE in 0.5-0. You will still be able to use
## 'loadSymbols' to automatically load data. getOption("getSymbols.env")
## and getOption("getSymbols.auto.assign") will still be checked for
## alternate defaults.
##
## This message is shown once per session and may be disabled by setting
## options("getSymbols.warning4.0"=FALSE). See ?getSymbols for details.
```

```
## [1] "CAT"
```

```
rt=diff(log(as.numeric(CAT[,6])))
Box.test(rt, lag=10, type = 'Ljung')
```

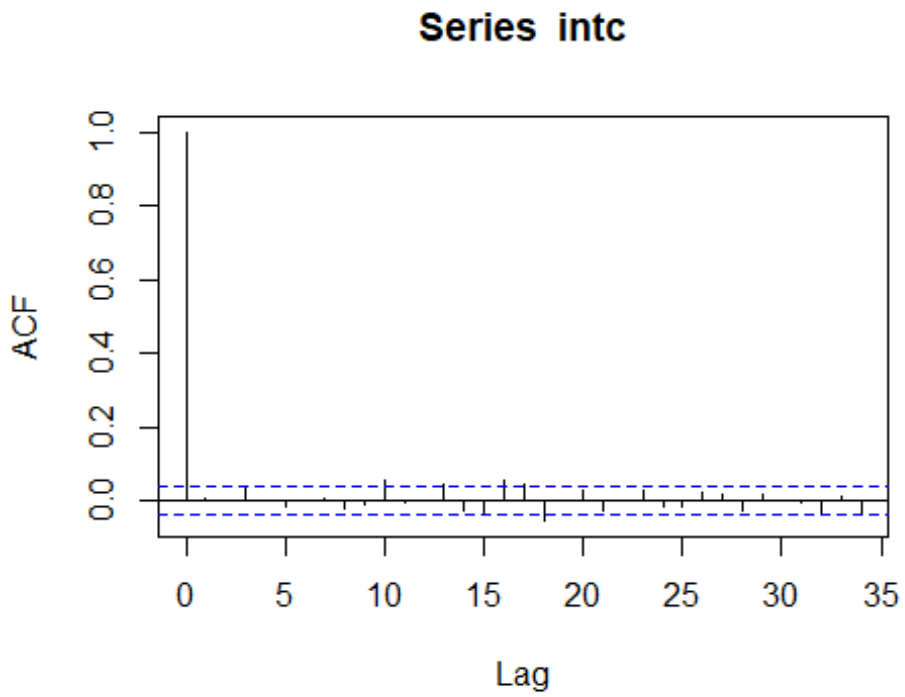
```
##
##  Box-Ljung test
##
## data:  rt
## X-squared = 16.291, df = 10, p-value = 0.09159
```

```
# According to Ljung box test, p-value = 0.09159 > 0.05, H0 is not rejected
at 95% confidence interval. It means there is no serial correlations in the
log return series rt.
#Answer 1a
```

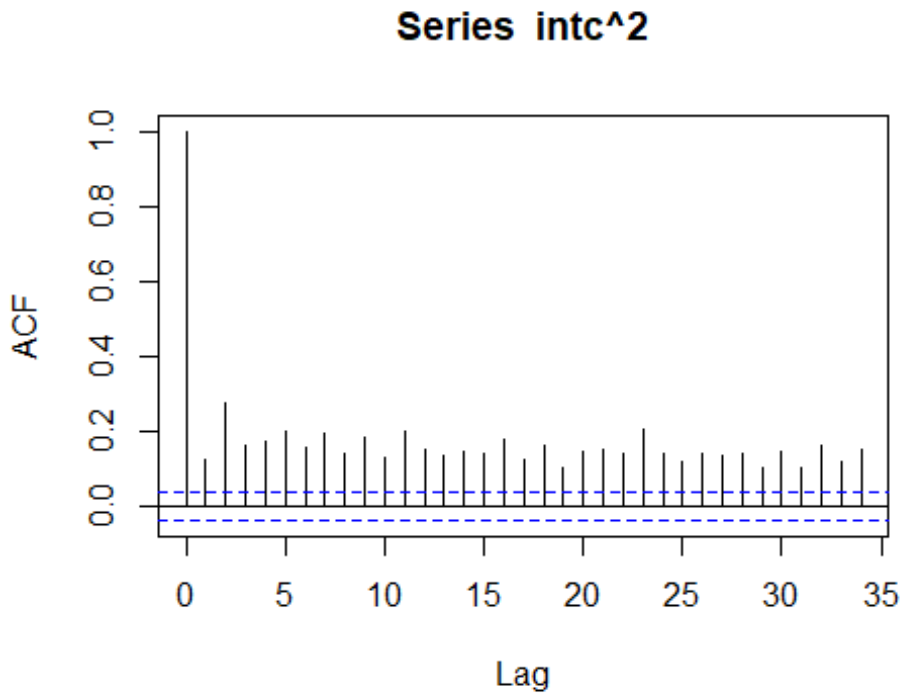
$H_0: \rho_1 = \rho_2 = \dots = \rho_m = 0$

$H_a: \exists \rho_i \neq 0$

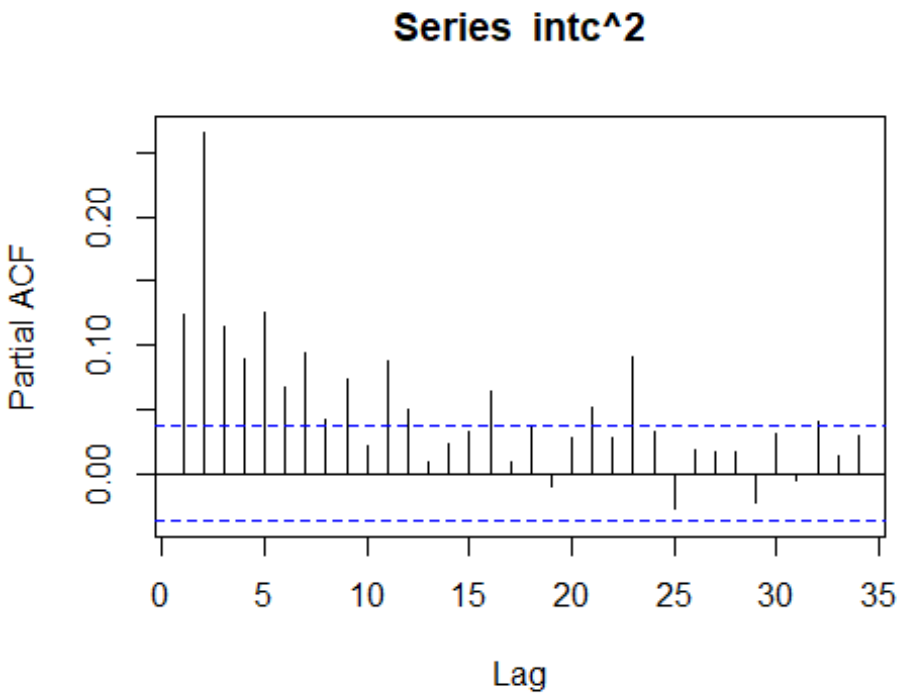
```
intc=rt
acf(intc)
```




```
acf(intc^2)
```




```
pacf(intc^2)
```




```
Box.test(intc^2, lag=10, type = 'Ljung')

##
##  Box-Ljung test
##
## data:  intc^2
## X-squared = 917.21, df = 10, p-value < 2.2e-16

# According to the Ljung Box test, p-value < 0.05, H0 is rejected at 95%
confidence interval. Hence, there is ARCH effect on rt.
#Answer 1b

m1=garchFit(~arma(1,0)+garch(1,1),data=rt,trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of
length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m1)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = rt, trace = F)
##
## Mean and Variance Equation:
```

H_0: There is no ARCH effect
H_a: There is ARCH effect

```
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x000000001f6b9588>
##  [data = rt]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu          ar1        omega       alpha1        beta1
## 4.8298e-04  1.6866e-02  4.4779e-06  4.9720e-02  9.3866e-01
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     4.830e-04   3.075e-04    1.571 0.116297
## ar1    1.687e-02   2.006e-02    0.841 0.400353
## omega  4.478e-06   1.278e-06    3.503 0.000461 ***
## alpha1 4.972e-02   8.191e-03    6.070 1.28e-09 ***
## beta1  9.387e-01   1.031e-02   91.048  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7378.56    normalized:  2.599916
##
## Description:
##  Wed Apr 28 21:12:57 2021 by user: CHATKEAWPAISAL
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  3298.441  0
##  Shapiro-Wilk Test  R    W      0.9663735 0
##  Ljung-Box Test     R    Q(10)  12.37554  0.2607088
##  Ljung-Box Test     R    Q(15)  14.79514  0.4662719
##  Ljung-Box Test     R    Q(20)  19.20107  0.5087928
##  Ljung-Box Test     R^2  Q(10)  0.980939  0.9998424
##  Ljung-Box Test     R^2  Q(15)  3.682825  0.9986048
##  Ljung-Box Test     R^2  Q(20)  6.9285    0.996913
##  LM Arch Test       R    TR^2   2.723165  0.9972029
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.196308 -5.185823 -5.196314 -5.192526
```

*maybe bc 1 was asked to put # in order to compile report so the graph of QQ-plot did not show in the report for all questions.

```
#plot(m1)
#13
#0
```

H_0: There is no ARCH effect
H_a: There is ARCH effect

According to Ljung box test, p-value > 0.05, H_0 is not rejected at 95% CI. This means that the model is adequate.

The fitted model is:

Mean equation: $4.830e\text{-}04 (1 - 1.687e\text{-}02) + 1.687e\text{-}02 r_{t-1}$
($3.075e\text{-}04$) ($2.006e\text{-}02$)

Volatility equation: $4.478e\text{-}06 + 4.972e\text{-}02 a^2_{t-1} + 9.387e\text{-}01 \sigma^2_{t-1}$
($1.278e\text{-}06$) ($8.194e\text{-}03$) ($2.031e\text{-}02$)

```
# Answer 1c

m2=garchFit(~garch(1,1),data=rt,cond.dist="std",trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of
length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m2)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = rt, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x000000001efe37d0>
##  [data = rt]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##         mu       omega      alpha1       beta1       shape
## 5.9780e-04  4.2035e-06  7.2376e-02  9.2033e-01  5.0958e+00
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     5.978e-04   2.702e-04    2.212  0.02695 *
## omega  4.203e-06   1.571e-06    2.675  0.00747 **
## alpha1 7.238e-02   1.374e-02    5.267 1.39e-07 ***
## beta1  9.203e-01   1.472e-02   62.503  < 2e-16 ***
## shape  5.096e+00   4.825e-01   10.561  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  7507.106    normalized:  2.64521
##
## Description:
##  Wed Apr 28 21:12:57 2021 by user: CHATKEAWPAISAL
##
##
```

```
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  4056.502  0
##  Shapiro-Wilk Test  R    W      0.9639091 0
##  Ljung-Box Test     R    Q(10)  14.77968  0.140303
##  Ljung-Box Test     R    Q(15)  16.74279  0.3344718
##  Ljung-Box Test     R    Q(20)  20.39783  0.433304
##  Ljung-Box Test     R^2  Q(10)  2.953085  0.9825066
##  Ljung-Box Test     R^2  Q(15)  5.482428  0.9871938
##  Ljung-Box Test     R^2  Q(20)  9.458146  0.9769677
##  LM Arch Test       R    TR^2   4.273688  0.977976
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -5.286897 -5.276412 -5.286903 -5.283115
```

$H_0: \rho_1 = \dots = \rho_m$

$H_a: \exists \rho_i \neq 0$

p-value of ljungbox test > 0.05, H_0 is not rejected at 95% CI. Hence, there is no linear dependent in $\tilde{a}_t$. Therefore, the model is adequate

```
#plot(m2)
#13
#0
#Answer 1d

# The fitted model is:
#Answer 1e

predict(m2,5)
```

mean eqn: $\hat{r}_t = 5.978e\text{-}04$ (2.702e-04)

$\hat{\sigma}_t^2 = 4.203e\text{-}06 + 7.238e\text{-}02\, a_{t-1}^2 + 9.203e\text{-}01\, \sigma_{t-1}^2$ (1.571e-06) (1.374e-02) (1.432e-02)

```
##   meanForecast  meanError standardDeviation
## 1 0.0005977987 0.01515760        0.01515760
## 2 0.0005977987 0.01524072        0.01524072
## 3 0.0005977987 0.01532280        0.01532280
## 4 0.0005977987 0.01540384        0.01540384
## 5 0.0005977987 0.01548387        0.01548387

#Answer 1f

# 95% confidence interval:
#Answer 1g
```

1g) 1-step: (-1.107854, 1.010885)
2 : (-1.009879, 1.01088)
3 : (-1.009917, 1.010918)
4 : (-1.009954, 1.010955)
5 : (-1.009991, 1.010992)

```
da=read.table("m-kovw-5116.txt", header = T)
head(da)

##   PERMNO     date        ko    vwretd
## 1  11308 19520131  0.024390  0.017002
## 2  11308 19520229  0.009524 -0.025141
## 3  11308 19520331  0.016509  0.045870
## 4  11308 19520430 -0.016393 -0.049148
## 5  11308 19520529  0.028571  0.032847
## 6  11308 19520630  0.046296  0.039575

rt2=log(da$ko+1)
t.test(rt2)
```

```
##
##  One Sample t-test
##
## data:  rt2
## t = 4.9853, df = 779, p-value = 7.628e-07
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.00625636 0.01438347
## sample estimates:
##  mean of x
## 0.01031992

Box.test(rt2,lag=10,type = 'Ljung')

##
##  Box-Ljung test
##
## data:  rt2
## X-squared = 5.9201, df = 10, p-value = 0.8219

Box.test(rt2^2,lag=10,type='Ljung')

##
##  Box-Ljung test
##
## data:  rt2^2
## X-squared = 228.23, df = 10, p-value < 2.2e-16

#Answer 2a

m3=garchFit(~arma(1,0)+garch(1,1),data=rt2,trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of
length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m3)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = rt2, trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x00000000216da830>
##  [data = rt2]
##
## Conditional Distribution:
##  norm
```

2a) $H_0: E(r_t) = 0$
$H_a: E(r_t) \neq 0$
From t-test, p-value < 0.05, we reject H_0 at 95% CI.
Hence, expected return of KO log return is not 0.

$H_0: \rho_1 = \ldots = \rho_m$
$H_a: \exists \rho_i \neq 0$
p-value of Ljung box test > 0.05, we do not reject H_0 at 95% CI. Hence, there is no serial correlation.

H_0: There is no ARCH effect
H_a: There is ARCH effect
p-value of Ljung box test < 0.05, we reject H_0 at 95% CI. Hence, there is ARCH effect.

```
##
## Coefficient(s):
##          mu          ar1        omega       alpha1        beta1
##  0.01124544  -0.02633742   0.00018112   0.09535029   0.84861593
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      1.125e-02   1.897e-03    5.929 3.05e-09 ***
## ar1    -2.634e-02   3.881e-02   -0.679  0.49740
## omega   1.811e-04   5.852e-05    3.095  0.00197 **
## alpha1  9.535e-02   1.915e-02    4.978 6.42e-07 ***
## beta1   8.486e-01   2.766e-02   30.675  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1170.664    normalized:  1.500852
##
## Description:
##  Wed Apr 28 21:12:58 2021 by user: CHATKEAWPAISAL
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  92.91946  0
##  Shapiro-Wilk Test  R    W      0.9857081 6.655604e-07
##  Ljung-Box Test     R    Q(10)  9.306169  0.5033144
##  Ljung-Box Test     R    Q(15)  22.9901   0.0843502
##  Ljung-Box Test     R    Q(20)  27.44814  0.1231201
##  Ljung-Box Test     R^2  Q(10)  12.63377  0.2448749
##  Ljung-Box Test     R^2  Q(15)  13.62088  0.5544545
##  Ljung-Box Test     R^2  Q(20)  15.19817  0.7649584
##  LM Arch Test       R    TR^2   10.65102  0.5590389
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -2.988883 -2.959016 -2.988965 -2.977396
```

2b) p-value of Ljung box test > 0.05 so we do not reject H_0 at 95% CI. Hence, the model is adequate.

The fitted model is: $\hat{r}_t = 1.125e\text{-}02\ (1+0.634e\text{-}02) - 2.634e\text{-}02$ (3.887e-02)

$\hat{\sigma}_t^2$: $1.811e\text{-}04 + 9.535e\text{-}02\, a_{t-1}^2 + 8.486e\text{-}01\, \hat{\sigma}_{t-1}^2$ (1.897e-03) (1.852e-05) (1.915e-02) (2.766e-02)

```
#Answer 2b
m4=garchFit(~arma(1,0)+garch(1,1),data=rt2,cond.dist='std',trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of
length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m4)
```

```
##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 1), data = rt2, cond.dist =
"std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 1)
## <environment: 0x000000001fb82478>
##  [data = rt2]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##          mu          ar1        omega       alpha1        beta1
shape
##  0.01124020  -0.01887601   0.00017395   0.09642927   0.85044151
7.47877780
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##          Estimate  Std. Error  t value Pr(>|t|)
## mu      1.124e-02   1.810e-03    6.211 5.27e-10 ***
## ar1    -1.888e-02   3.691e-02   -0.511  0.60904
## omega   1.739e-04   6.596e-05    2.637  0.00836 **
## alpha1  9.643e-02   2.338e-02    4.124 3.72e-05 ***
## beta1   8.504e-01   3.267e-02   26.028  < 2e-16 ***
## shape   7.479e+00   1.840e+00    4.066 4.79e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1184.863    normalized:  1.519055
##
## Description:
##  Wed Apr 28 21:12:58 2021 by user: CHATKEAWPAISAL
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  93.6433   0
##  Shapiro-Wilk Test  R    W      0.9857385 6.832848e-07
##  Ljung-Box Test     R    Q(10)  8.966733  0.5352637
##  Ljung-Box Test     R    Q(15)  22.44818  0.09657967
```

```
##  Ljung-Box Test     R    Q(20)  26.86769  0.1390276
##  Ljung-Box Test     R^2  Q(10)  12.48941  0.2536355
##  Ljung-Box Test     R^2  Q(15)  13.37442  0.5734021
##  Ljung-Box Test     R^2  Q(20)  14.90709  0.7816987
##  LM Arch Test       R    TR^2   10.48089  0.5738501
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.022725 -2.986885 -3.022843 -3.008941
```

2c) p-value of Ljung box test > 0.05, H_0 is not reject at 95% C.I. The model is adequate

The fitted model is:

$\hat{r}_t = 1.124e\text{-}02 (1 + 1.888e\text{-}02) - 1.888e\text{-}02 r_{t-1}$
$(1.810e\text{-}03) \qquad (3.691e\text{-}02)$

```
#Answer 2c
```

$\hat{\sigma}^2_t = 1.739e\text{-}04 + 9.643e\text{-}02 \hat{a}^2_{t-1} + 8.504e\text{-}01 \hat{\sigma}^2_{t-1}$
$(6.396e\text{-}05) \quad (2.338e\text{-}02) \quad (3.267e\text{-}02)$

```
m5=garchFit(~garch(1,1),data=rt2,trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of
length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m5)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = rt2, trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x0000000021632c90>
##  [data = rt2]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##         mu       omega      alpha1       beta1
## 0.01098417  0.00018497  0.09479925  0.84780406
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.098e-02   1.846e-03    5.950 2.68e-09 ***
## omega  1.850e-04   5.899e-05    3.135  0.00172 **
## alpha1 9.480e-02   1.912e-02    4.958 7.11e-07 ***
## beta1  8.478e-01   2.787e-02   30.416  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Log Likelihood:
##  1170.393    normalized:  1.500504
##
## Description:
##  Wed Apr 28 21:12:58 2021 by user: CHATKEAWPAISAL
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  95.07163  0
##  Shapiro-Wilk Test  R    W      0.9856773 6.481596e-07
##  Ljung-Box Test     R    Q(10)  8.125181  0.6166108
##  Ljung-Box Test     R    Q(15)  21.27199  0.128362
##  Ljung-Box Test     R    Q(20)  25.62765  0.1784646
##  Ljung-Box Test     R^2  Q(10)  12.90586  0.228983
##  Ljung-Box Test     R^2  Q(15)  13.87463  0.5350581
##  Ljung-Box Test     R^2  Q(20)  15.35522  0.755734
##  LM Arch Test       R    TR^2   10.96004  0.532346
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -2.990752 -2.966858 -2.990804 -2.981562
```

2d) p-value of Ljung box test > 0.05. H_0 is not reject at 95% CI. The model is adequate

The fitted model is:

$\hat{r}_t = 1.098e\text{-}02$
$(1.846e\text{-}03)$

$\hat{\sigma}^2_t = 1.850e\text{-}04 + 9.480e\text{-}02\, a^2_{t-1} + 8.478e\text{-}01\, \hat{\sigma}^2_{t-1}$
$(3.899e\text{-}05) \quad (1.972e\text{-}02) \quad (2.787e\text{-}02)$

#Answer 2d

```
m6=garchFit(~garch(1,1),data=rt2,cond.dist='std',trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of
length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m6)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 1), data = rt2, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 1)
## <environment: 0x000000001ef4b040>
##  [data = rt2]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##         mu       omega      alpha1       beta1       shape
```

```
## 0.01105016  0.00017528  0.09632874  0.85006800  7.48604505
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     1.105e-02   1.757e-03    6.291 3.16e-10 ***
## omega  1.753e-04   6.627e-05    2.645  0.00817 **
## alpha1 9.633e-02   2.337e-02    4.123 3.75e-05 ***
## beta1  8.501e-01   3.277e-02   25.941  < 2e-16 ***
## shape  7.486e+00   1.840e+00    4.069 4.72e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1184.68    normalized:  1.518821
##
## Description:
##  Wed Apr 28 21:12:58 2021 by user: CHATKEAWPAISAL
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  95.31715  0
##  Shapiro-Wilk Test  R    W      0.9857263 6.761141e-07
##  Ljung-Box Test     R    Q(10)  8.228765  0.6065024
##  Ljung-Box Test     R    Q(15)  21.34759  0.1260864
##  Ljung-Box Test     R    Q(20)  25.67699  0.1767469
##  Ljung-Box Test     R^2  Q(10)  12.61146  0.2462139
##  Ljung-Box Test     R^2  Q(15)  13.4693   0.5660982
##  Ljung-Box Test     R^2  Q(20)  14.93694  0.7800047
##  LM Arch Test       R    TR^2   10.62989  0.560875
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.024822 -2.994954 -3.024903 -3.013334
```

2e) p-value of Ljung box test > 0.05. H_0 is not reject at 95% C.I. The model is adequate

The fitted model is:

$$\hat{r}_t = 1.105e\text{-}02 \quad (1.757e\text{-}03)$$

$$\hat{\sigma}_t^2 = 1.753e\text{-}04 + 9.633e\text{-}02\, a_{t-1}^2 + 8.501e\text{-}01 \quad (6.629e\text{-}05) \quad (2.337e\text{-}02) \quad (3.277e\text{-}02)$$

#Answer 2e

#Answer 2f : I select model in e) due to the lowest AIC.

```
getSymbols("^GSPC",from="2005-01-2",to="2021-03-31")
```

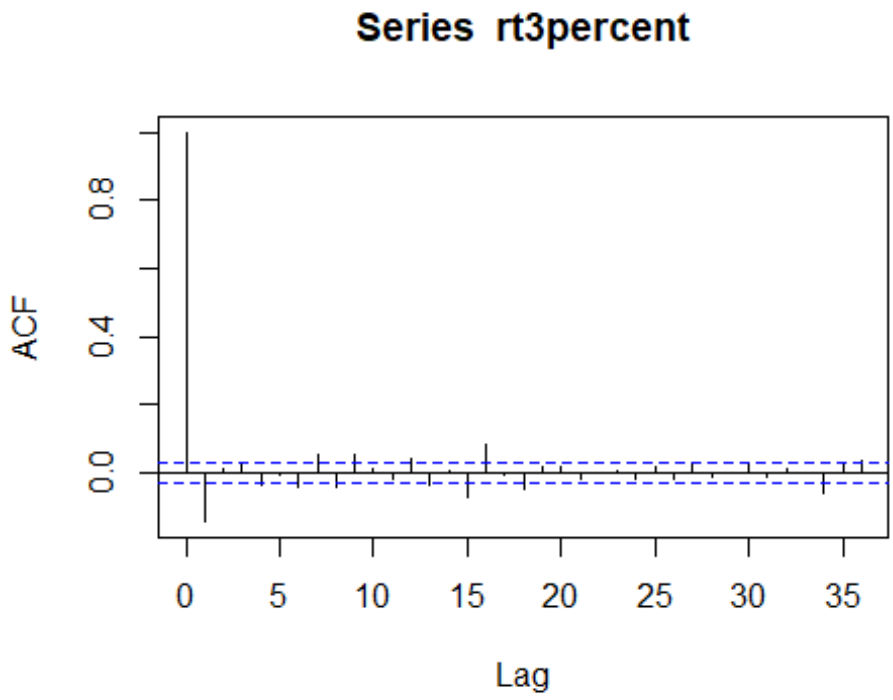
```
## [1] "^GSPC"
```

```
rt3=diff(log(as.numeric(GSPC[,6])))
rt3percent=rt3*100
t.test(rt3percent)
```

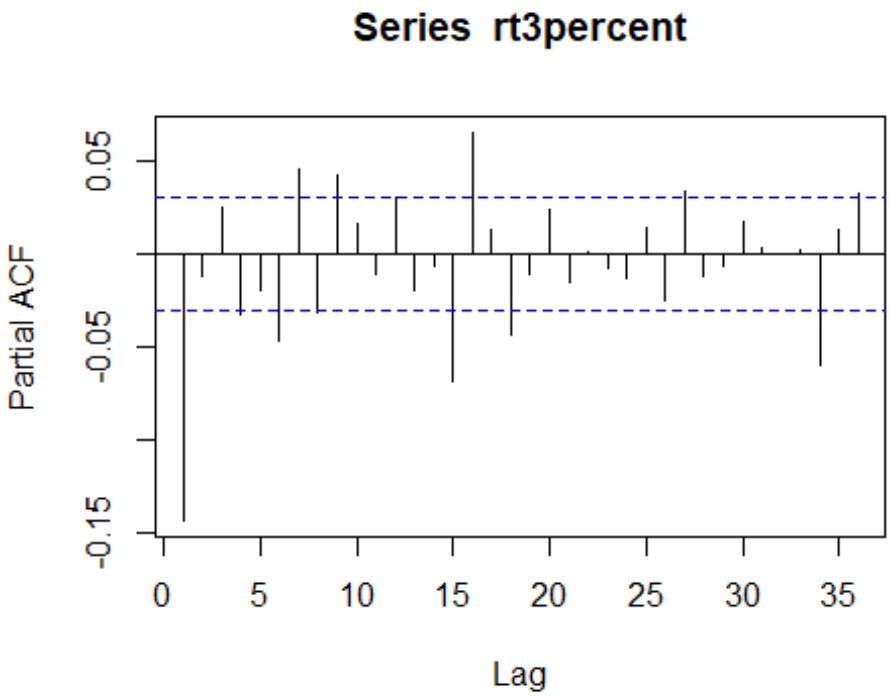
```
##
##  One Sample t-test
##
## data:  rt3percent
## t = 1.4961, df = 4086, p-value = 0.1347
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  -0.009051939  0.067374643
## sample estimates:
##  mean of x
## 0.02916135

Box.test(rt3percent,lag=10,type='Ljung')

##
##  Box-Ljung test
##
## data:  rt3percent
## X-squared = 131.85, df = 10, p-value < 2.2e-16

#Answer 3a
```

p-value of t-test > 0.05 at 95% CI. The expected value of r_t is 0.
p-value of Ljung box test < 0.05 at 95% CI. There is serial correlation in r_t

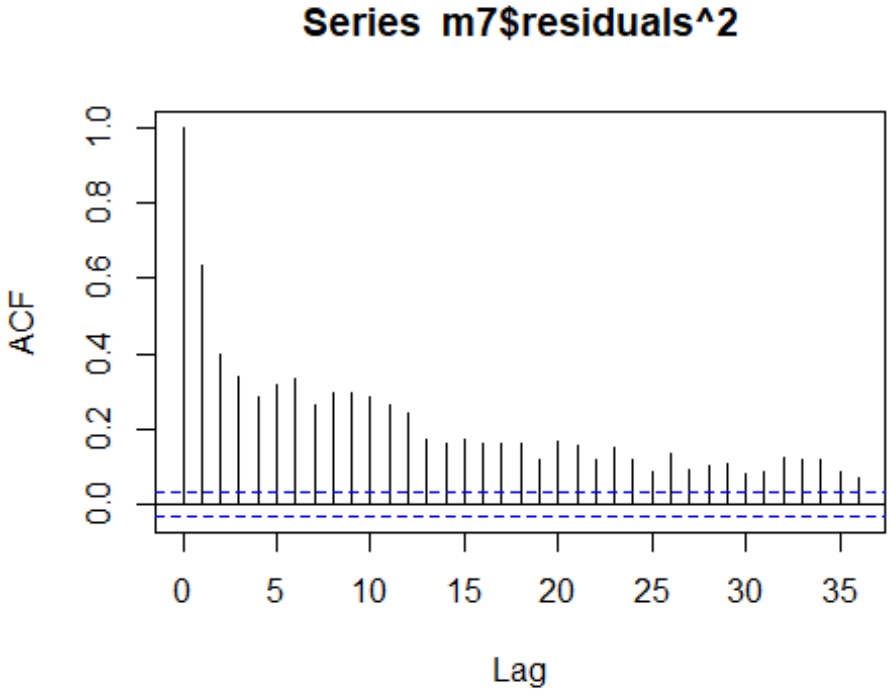
```
acf(rt3percent)
```




```
pacf(rt3percent)
```




```
m7=arima(rt3percent,order=c(0,1,0))
acf(m7$residuals^2)
```



```
Box.test(m7$residuals^2,lag=10,type='Ljung')

##
##  Box-Ljung test
##
## data:  m7$residuals^2
## X-squared = 5312, df = 10, p-value < 2.2e-16

m8=garchFit(~arma(0,1)+garch(1,1),data=rt3percent,trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of
length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m8)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(0, 1) + garch(1, 1), data = rt3percent,
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(0, 1) + garch(1, 1)
## <environment: 0x0000000020fa82d8>
##  [data = rt3percent]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##        mu        ma1      omega     alpha1      beta1
##  0.068986  -0.077582   0.027170   0.141814   0.837406
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.068986    0.010800    6.387 1.69e-10 ***
## ma1    -0.077582    0.017503   -4.433 9.31e-06 ***
## omega   0.027170    0.003504    7.754 8.88e-15 ***
## alpha1  0.141814    0.012017   11.801  < 2e-16 ***
## beta1   0.837406    0.011956   70.039  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  -5392.874    normalized:  -1.319519
```

```
##
## Description:
##  Wed Apr 28 21:12:59 2021 by user: CHATKEAWPAISAL
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  1134.026  0
##  Shapiro-Wilk Test  R    W      0.9717603 0
##  Ljung-Box Test     R    Q(10)  17.7885   0.05863789
##  Ljung-Box Test     R    Q(15)  26.08443  0.03714455
##  Ljung-Box Test     R    Q(20)  31.72122  0.04636094
##  Ljung-Box Test     R^2  Q(10)  16.02024  0.09905437
##  Ljung-Box Test     R^2  Q(15)  18.27301  0.2485849
##  Ljung-Box Test     R^2  Q(20)  19.66181  0.4792555
##  LM Arch Test       R    TR^2   16.80097  0.1572388
##
## Information Criterion Statistics:
##      AIC      BIC      SIC     HQIC
## 2.641485 2.649211 2.641482 2.644220
```

3b) From ACF and PACF, the model is MA since there is no effect of exponential decay. The optimal lag is 1 lag.

p-value of Ljung box test > 0.05. H_0 is not reject at 95% C.I. The model is adequate

The fitted model is:

$$\hat{r}_t = \underset{(0.010800)}{0.068986}(1 - \underset{(0.077582)}{0.077582}) - \underset{(0.077303)}{0.077582}\, r_{t-1}$$

$$\hat{\sigma}^2_t = \underset{(0.003504)}{0.027730} + \underset{(0.072037)}{0.141814}\, a^2_{t-1} + \underset{(0.079756)}{0.837406}\, \hat{\sigma}^2_{t-1}$$

```
#plot(m8)
#13
#0
#Answer 3b

m9=garchFit(~arma(0,1)+garch(1,1),data=rt3percent,cond.dist="std",trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of
length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m9)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(0, 1) + garch(1, 1), data = rt3percent,
##     cond.dist = "std", trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(0, 1) + garch(1, 1)
## <environment: 0x000000001eecaaa8>
##  [data = rt3percent]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
```

```
##        mu         ma1      omega     alpha1      beta1      shape
##  0.085537  -0.073712   0.016925   0.141240   0.856667   5.075507
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.085537    0.009733    8.789  < 2e-16 ***
## ma1    -0.073712    0.016039   -4.596 4.31e-06 ***
## omega   0.016925    0.003621    4.673 2.96e-06 ***
## alpha1  0.141240    0.014598    9.675  < 2e-16 ***
## beta1   0.856667    0.012848   66.675  < 2e-16 ***
## shape   5.075507    0.427751   11.866  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  -5257.344    normalized:  -1.286358
##
## Description:
##  Wed Apr 28 21:13:00 2021 by user: CHATKEAWPAISAL
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  1431.666  0
##  Shapiro-Wilk Test  R    W      0.969611  0
##  Ljung-Box Test     R    Q(10)  17.95781  0.05567943
##  Ljung-Box Test     R    Q(15)  26.2946   0.0350378
##  Ljung-Box Test     R    Q(20)  31.68531  0.04676904
##  Ljung-Box Test     R^2  Q(10)  13.12031  0.217023
##  Ljung-Box Test     R^2  Q(15)  17.77068  0.2749137
##  Ljung-Box Test     R^2  Q(20)  20.51701  0.4260342
##  LM Arch Test       R    TR^2   14.78812  0.2532284
##
## Information Criterion Statistics:
##      AIC      BIC      SIC     HQIC
## 2.575652 2.584923 2.575647 2.578935
```

3c) p-value of Ljung box test > 0.05. H_0 is not reject at 95% C.I. The model is adequate

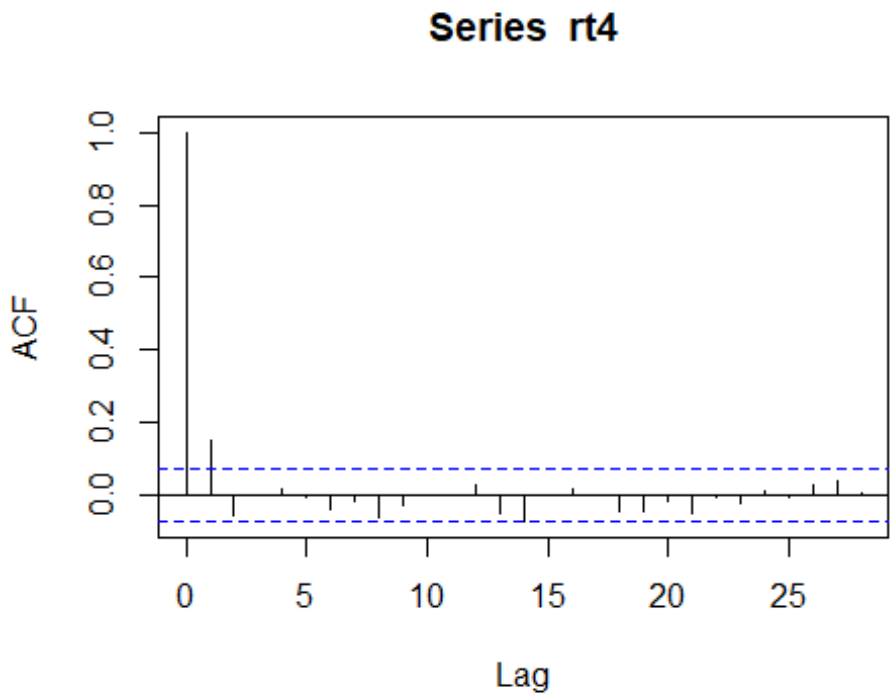
The fitted model is:

$$\hat{r}_t = \underset{(0.009783)}{0.085537}(1-\underset{(0.016039)}{0.073712}) - 0.073712 r_{t-1}$$

$$\hat{\sigma}^2_t = \underset{(0.003621)}{0.016925} + \underset{(0.014598)}{0.141240}\, a^2_{t-1} + \underset{(0.012848)}{0.856667}\, \hat{\sigma}^2_{t-1}$$

```
#Answer 3c

predict(m9,5)

##   meanForecast meanError standardDeviation
## 1   0.11545471 0.9040206         0.9040206
## 2   0.08553707 0.9148267         0.9123965
## 3   0.08553707 0.9231321         0.9206790
## 4   0.08553707 0.9313464         0.9288705
## 5   0.08553707 0.9394718         0.9369735
```

```
#Answer 3d

da4=read.table("m-deciles.txt",header=T)
rt4=log(da4$CAP9RET+1)
acf(rt4)
```




```
t.test(rt4)

##
##  One Sample t-test
##
## data:  rt4
## t = 5.1808, df = 719, p-value = 2.873e-07
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.005946562 0.013203545
## sample estimates:
##   mean of x
## 0.009575054

Box.test(rt4,lag=10,type='Ljung')

##
##  Box-Ljung test
##
## data:  rt4
## X-squared = 24.257, df = 10, p-value = 0.006946
```

4a) From t-test, p-value < 0.05. H_0 is rejected at 95% C.I. Expected return of CRSP is not 0.
$H_0: E(\epsilon_t) = 0$
$H_a: E(\epsilon_t) \neq 0$

p-value of Ljung box test < 0.05. H_0 is rejected at 95% C.I. Hence, there is serial correlation in the log returns
$H_0: \rho_1 = \ldots = \rho_m$
$H_a: \exists \rho_i \neq 0$

```
#Answer 4a

Box.test(rt4^2,lag=10,type='Ljung')

##
##  Box-Ljung test
##
## data:  rt4^2
## X-squared = 19.824, df = 10, p-value = 0.03096
```

H_0: There is no ARCH effect
H_a: There is ARCH effect
p-value of Ljung box test < 0.05. H_0 is rejected at 95% C.I. Hence, there is ARCH effect.

```
#Answer 4b

m10=garchFit(~arma(1,0)+garch(1,0),data=rt4,trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of
length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m10)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 0), data = rt4, trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 0)
## <environment: 0x000000001fdb6688>
##  [data = rt4]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##      mu      ar1    omega   alpha1
## 0.01053  0.14707  0.00200  0.18152
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     0.0105300   0.0019275    5.463 4.68e-08 ***
## ar1    0.1470670   0.0424301    3.466 0.000528 ***
## omega  0.0020000   0.0001482   13.493  < 2e-16 ***
## alpha1 0.1815182   0.0651142    2.788 0.005309 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

4c) The fitted model is:

$\hat{r}_t = 0.0105300 + 0.1430670\,r_{t-1}$
(0.0019275) (0.0424301)

$\hat{\sigma}_t^2 = 0.0020 + 0.1815182\,a_{t-1}^2$
(0.0001482) (0.0651142)

```
## Log Likelihood:
##  1158.628    normalized:  1.609206
##
## Description:
##  Wed Apr 28 21:13:01 2021 by user: CHATKEAWPAISAL
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  647.6357  0
##  Shapiro-Wilk Test  R    W      0.962804  1.502752e-12
##  Ljung-Box Test     R    Q(10)  8.582995  0.572082
##  Ljung-Box Test     R    Q(15)  12.95811  0.6055337
##  Ljung-Box Test     R    Q(20)  15.77362  0.7305655
##  Ljung-Box Test     R^2  Q(10)  8.153476  0.6138486
##  Ljung-Box Test     R^2  Q(15)  12.29206  0.6568012
##  Ljung-Box Test     R^2  Q(20)  13.57787  0.851238
##  LM Arch Test       R    TR^2   8.24593   0.7656286
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.207301 -3.181861 -3.207363 -3.197480
```

p-value of Ljung box test > 0.05. H_0 is not rejected at 95% C.I. Hence, there is no serial correlation in the log returns. The model is adequate.

$H_0: \rho_1 = \dots = \rho_m$
$H_a: \exists \rho \neq 0$

```
#Answer 4c

m11=garchFit(~arma(1,0)+garch(1,0),data=rt4,cond.dist="std",trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of
length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m11)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~arma(1, 0) + garch(1, 0), data = rt4, cond.dist =
"std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ arma(1, 0) + garch(1, 0)
## <environment: 0x0000000020c5ee58>
##  [data = rt4]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
```

4d) The fitted model is:

$\hat{r}_t = 0.0116789\,(1 - 0.1076982) + 0.1076982\,r_{t-1}$
(0.0017710) (0.0403169)

$\hat{a}_t^2 = 0.0019203 + 0.1900830\,a_{t-1}^2$
(0.0001818) (0.0713692)

```
##         mu         ar1       omega      alpha1       shape
## 0.0116189  0.1076982  0.0019203  0.1900830  6.4225253
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     0.0116189   0.0017710    6.561 5.35e-11 ***
## ar1    0.1076982   0.0403169    2.671  0.00756 **
## omega  0.0019203   0.0001818   10.564  < 2e-16 ***
## alpha1 0.1900830   0.0713692    2.663  0.00774 **
## shape  6.4225253   1.3115905    4.897 9.74e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1187.516    normalized:  1.649328
##
## Description:
##  Wed Apr 28 21:13:01 2021 by user: CHATKEAWPAISAL
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  680.4834  0
##  Shapiro-Wilk Test  R    W      0.9612945 7.49198e-13
##  Ljung-Box Test     R    Q(10)  9.486455  0.4866411
##  Ljung-Box Test     R    Q(15)  13.8214   0.5391158
##  Ljung-Box Test     R    Q(20)  17.0087   0.6524086
##  Ljung-Box Test     R^2  Q(10)  7.567444  0.671006
##  Ljung-Box Test     R^2  Q(15)  11.42176  0.7221637
##  Ljung-Box Test     R^2  Q(20)  12.79422  0.8860373
##  LM Arch Test       R    TR^2   7.723697  0.8063327
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.284768 -3.252967 -3.284863 -3.272491
```

```
#Answer 4d
```

p-value of Ljung box test > 0.05. H_0 is not rejected at 95% CI. Hence, there is no serial correlation in the log returns. The model is adequate.

$H_0: \rho_1 = \dots = \rho_m$
$H_a: \exists \rho \neq 0$

```
m12=garchFit(~garch(1,0),data=rt4,trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of
length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m12)

##
## Title:
```

```
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 0), data = rt4, trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 0)
## <environment: 0x000000001eeffd18>
##  [data = rt4]
##
## Conditional Distribution:
##  norm
##
## Coefficient(s):
##       mu     omega    alpha1
## 0.012346  0.002016  0.194126
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.012346    0.001923    6.421 1.35e-10 ***
## omega   0.002016    0.000145   13.900  < 2e-16 ***
## alpha1  0.194126    0.062209    3.121  0.00181 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1152.289    normalized:  1.600401
##
## Description:
##  Wed Apr 28 21:13:01 2021 by user: CHATKEAWPAISAL
##
##
## Standardised Residuals Tests:
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  746.8024  0
##  Shapiro-Wilk Test  R    W      0.9570702 1.171061e-13
##  Ljung-Box Test     R    Q(10)  18.72223  0.04393591
##  Ljung-Box Test     R    Q(15)  23.27998  0.07837365
##  Ljung-Box Test     R    Q(20)  27.61004  0.1189566
##  Ljung-Box Test     R^2  Q(10)  7.012055  0.7243064
##  Ljung-Box Test     R^2  Q(15)  10.3604   0.7964784
##  Ljung-Box Test     R^2  Q(20)  11.97825  0.9168225
##  LM Arch Test       R    TR^2   7.047101  0.8544853
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.192468 -3.173388 -3.192503 -3.185102
```

p-value of Ljung box test > 0.05. H_0 is not rejected at 95% C.I. Hence, there is no serial correlation in the log returns. The model is adequate.

$H_0: \rho_1 = \dots = \rho_m$
$H_a: \exists \rho \neq 0$

The fitted model is: $\hat{r}_t = 0.012346$ (0.001923)

$\hat{\sigma}^2_t = 0.002016 + 0.194126$ (0.000145) (0.067209)

```
#Answer 4e
m13=garchFit(~garch(1,0),data=rt4,cond.dist="std",trace=F)

## Warning: Using formula(x) is deprecated when x is a character vector of
length > 1.
##   Consider formula(paste(x, collapse = " ")) instead.

summary(m13)

##
## Title:
##  GARCH Modelling
##
## Call:
##  garchFit(formula = ~garch(1, 0), data = rt4, cond.dist = "std",
##     trace = F)
##
## Mean and Variance Equation:
##  data ~ garch(1, 0)
## <environment: 0x000000001fe2a920>
##  [data = rt4]
##
## Conditional Distribution:
##  std
##
## Coefficient(s):
##       mu     omega    alpha1     shape
## 0.013356  0.001928  0.204163  6.220222
##
## Std. Errors:
##  based on Hessian
##
## Error Analysis:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.013356    0.001685    7.927 2.22e-15 ***
## omega   0.001928    0.000185   10.421  < 2e-16 ***
## alpha1  0.204163    0.072375    2.821  0.00479 **
## shape   6.220223    1.236608    5.030 4.90e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Log Likelihood:
##  1183.349    normalized:  1.64354
##
## Description:
##  Wed Apr 28 21:13:01 2021 by user: CHATKEAWPAISAL
##
##
## Standardised Residuals Tests:
```

```
##                                 Statistic p-Value
##  Jarque-Bera Test   R    Chi^2  746.3878  0
##  Shapiro-Wilk Test  R    W      0.9574271 1.363165e-13
##  Ljung-Box Test     R    Q(10)  18.51406  0.04688705
##  Ljung-Box Test     R    Q(15)  22.99926  0.08415548
##  Ljung-Box Test     R    Q(20)  27.3947   0.1245202
##  Ljung-Box Test     R^2  Q(10)  6.667871  0.7563837
##  Ljung-Box Test     R^2  Q(15)  10.0157   0.8187514
##  Ljung-Box Test     R^2  Q(20)  11.63085  0.928196
##  LM Arch Test       R    TR^2   6.819315  0.8693193
##
## Information Criterion Statistics:
##       AIC       BIC       SIC      HQIC
## -3.275969 -3.250529 -3.276031 -3.266148

#Answer 4f

#Answer 4g
```

4f) p-value of Ljung box test > 0.05. H_0 is not rejected at 95% CI. Hence, there is no serial correlation in the log returns $H_0: \rho_1 = \dots = \rho_m$ $H_a: \exists \rho \neq 0$ The model is adequate.

The fitted model is:

$$\hat{r}_t = \underset{(0.001683)}{0.013356}$$

$$\hat{\sigma}^2_t = \underset{(0.000185)}{0.001928} + \underset{(0.072375)}{0.204163}\, a^2_{t-1}$$

4g) I select the model in question 4d.
AR(1)- ARCH(1) model with standardized student-t innovations because it has the lowest AIC value.