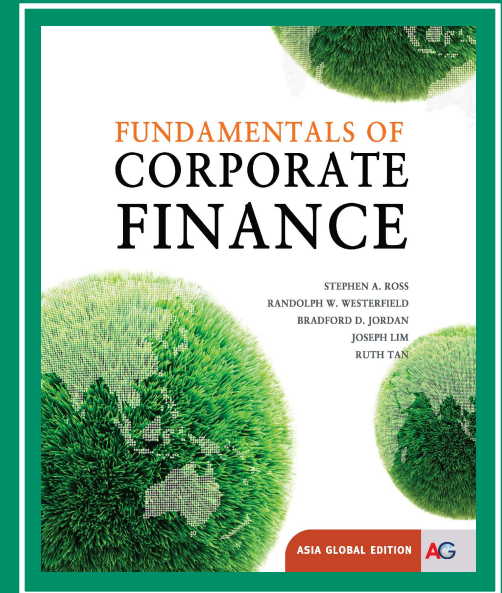


# Chapter 5



# Introduction to Valuation: The Time Value of Money



# Key Concepts and Skills

- Be able to compute the future value of an investment made today
- Be able to compute the present value of cash to be received at some future date
- Be able to compute the return on an investment
- Be able to compute the number of periods that equates a present value and a future value given an interest rate
- Be able to use a financial calculator and a spreadsheet to solve time value of money problems

# Chapter Outline

- Future Value and Compounding
- Present Value and Discounting
- More about Present and Future Values



# Basic Definitions

- Present Value – earlier money on a time line
- Future Value – later money on a time line
- Interest rate – “exchange rate” between earlier money and later money
  - Discount rate
  - Cost of capital
  - Opportunity cost of capital
  - Required return

# Future Values – Example 1

- Suppose you invest \$100 for one year at 5% per year. What is the future value in one year?
  - Interest =  $100(.05) = 5$
  - Value in one year = principal + interest =  $100 + 5 = 105$
  - Future Value (FV) =  $100(1 + .05) = 105$
- Suppose you leave the money in for another year. How much will you have two years from now?
  - FV =  $100(1.05)(1.05) = 100(1.05)^2 = 110.25$

# Future Values: General Formula

- $FV = PV(1 + r)^t$ 
  - FV = future value
  - PV = present value
  - r = period interest rate, expressed as a decimal
  - t = number of periods
- Future value interest factor =  $(1 + r)^t$

# Simple Interest versus Compound Interest

- Simple interest
  - FV with simple interest =  $100 + 5 + 5 = 110$
- Compound interest
  - FV with compound interest = 110.25
- What is the effect of compounding?
  - The extra 0.25 comes from the interest of  $.05(5) = 0.25$  earned on the first interest payment

# Calculator Keys

- Texas Instruments BA-II Plus
  - FV = future value
  - PV = present value
  - I/Y = period interest rate
    - P/Y must equal 1 for the I/Y to be the period rate
    - Interest is entered as a percent, not a decimal
  - N = number of periods
  - Remember to clear the memory (CLR TVM) after each problem
  - Other calculators are similar in format

# Future Values – Example 2

- Suppose you invest the \$100 from the previous example for 5 years. How much would you have?
  - FV with simple interest =  $100 + 5 + 5 + 5 + 5 + 5 = 125$
  - FV with compound interest =  $100(1.05)^5 = 127.63$ 
    - Using financial calculator: 5 N; 5 I/Y; 100 PV, CPT FV = -127.63
- The effect of compounding is small for a small number of periods but increases as the number of periods increases

# Future Values – Example 3

- Suppose you had a relative who deposited \$100 at 5.5% interest 200 years ago. How much would the investment be worth today?
  - FV with simple interest =  $100 + 200(100)(.055) = 1200$
  - FV with compound interest =  $100(1.055)^{200} = 4,471,898$ 
    - Using financial calculator: 200 N; 5.5 I/Y; -100 PV, CPT FV = 4,471,898
- What is the effect of compounding?
  - Compounding resulted in an additional \$4,470,698 to the value of the investment

# Future Value as a General Growth Formula

- Suppose your company expects to increase unit sales of widgets by 15% per year for the next 5 years. If you sell 3 million widgets in the current year, how many widgets do you expect to sell in the fifth year?
  - FV with compounding =  $3,000,000(1.15)^5 = 6,034,072$
  - Using financial calculator: 5 N; 15 I/Y; 3,000,000 PV; CPT FV = -6,034,072 units (remember the sign convention)

# Quick Quiz – Part I

- What is the difference between simple interest and compound interest?
- Suppose you have \$500 to invest and you believe that you can earn 8% per year over the next 15 years.
  - How much would you have at the end of 15 years using simple interest?
  - How much would you have at the end of 15 years using compound interest?

# Present Values

- How much do I have to invest today to have some amount in the future?
  - $FV = PV(1 + r)^t$
  - Rearrange to solve for  $PV = FV / (1 + r)^t$
- When we talk about discounting, we mean finding the present value of some future amount.
- When we talk about the “value” of something, we are talking about the present value unless we specifically indicate that we want the future value.

# Present Value – One Period Example

- Suppose you need \$10,000 in one year for the down payment on a new car. If you can earn 7% annually, how much do you need to invest today?
  - $PV = 10,000 / (1.07)^1 = 9,345.79$
  - Using financial calculator: 1 N; 7 I/Y; 10,000 FV, CPT PV = -9,345.79

# Present Values – Example 2

- You want to begin saving for your daughter's college education and you estimate that she will need \$150,000 in 17 years. If you feel confident that you can earn 8% per year, how much do you need to invest today?
  - $PV = 150,000 / (1.08)^{17} = 40,540.34$
  - Using financial calculator: 17 N; 8 I/Y; 150,000 FV, CPT PV = -40,540.34 (remember the sign convention)

# Present Values – Example 3

- Your parents set up a trust fund for you 10 years ago that is now worth \$19,671.51. If the fund earned 7% per year, how much did your parents invest?
  - $PV = 19,671.51 / (1.07)^{10} = 9,999.998 = 10,000$
  - Using financial calculator: 10 N; 7 I/Y; 19,671.51 FV, CPT  
PV = -10,000 (remember the sign convention)

# Present Value – Important Relationship I

- For a given interest rate – the longer the time period, the lower the present value
  - What is the present value of \$500 to be received in 5 years? 10 years? The discount rate is 10%

- 5 years:  $PV = 500 / (1.10) = 310.46$

Using financial calculator: 5 N; 10 I/Y; 500 FV, CPT

$PV = -310.46$

- 10 years:  $PV = 500 / (1.10) = 192.77$

Using financial calculator: 10 N; 10 I/Y; 500 FV, CPT

$PV = -192.77$

# Present Value – Important Relationship II

- For a given time period – the higher the interest rate, the smaller the present value
  - What is the present value of \$500 received in 5 years if the interest rate is 10%? 15%?
    - Rate = 10%:  $PV = 500 / (1.10)^5 = 310.46$   
Using financial calculator: 5 N; 10 I/Y; 500 FV,  
CPT PV = -310.46
    - Rate = 15%:  $PV = 500 / (1.15)^5 = 248.59$   
Using financial calculator: 5 N; 15 I/Y; 500 FV,  
CPT PV = -248.59

# Quick Quiz – Part II

- What is the relationship between present value and future value?
- Suppose you need \$15,000 in 3 years. If you can earn 6% annually, how much do you need to invest today?
- If you could invest the money at 8%, would you have to invest more or less than at 6%?

# The Basic PV Equation - Refresher

- $PV = FV / (1 + r)^t$
- There are four parts to this equation
  - PV, FV, r and t
  - If we know any three, we can solve for the fourth
- If you are using a financial calculator, be sure to remember the sign convention or you will receive an error (or a nonsensical answer) when solving for r or t

# Discount Rate

- Often we will want to know the implied rate of interest of an investment
- Rearrange the basic PV equation and solve for r
  - $FV = PV(1 + r)^t$
  - $r = (FV / PV)^{1/t} - 1$

# Discount Rate – Example 1

- You are looking at an investment that will pay \$1,200 in 5 years if you invest \$1,000 today. What is the implied rate of interest?
  - $r = (1,200 / 1,000)^{1/5} - 1 = .03714 = 3.714\%$
  - Using financial calculator (the sign convention matters!!!): 5 N; -1,000 PV (you pay 1,000 today); 1,200 FV (you receive 1,200 in 5 years); CPT I/Y = 3.714%

# Discount Rate – Example 2

- Suppose you are offered an investment that will allow you to double your money in 6 years. You have \$10,000 to invest. What is the implied rate of interest?
  - $r = (20,000 / 10,000) - 1 = .1225 = 12.25\%$
  - Using financial calculator: 6 N; -10,000 PV; 20,000 FV; CPT I/Y = 12.25%

# Discount Rate – Example 3

- Suppose you have a 1-year old son and you want to provide \$75,000 in 17 years towards his college education. You currently have \$5,000 to invest. What interest rate must you earn to have the \$75,000 when you need it?
  - $r = (75,000 / 5,000)^{1/17} - 1 = .1727 = 17.27\%$
  - Using financial calculator: 17 N; -5,000 PV; 75,000 FV; CPT I/Y = 17.27%

# Quick Quiz – Part III

- What are some situations in which you might want to know the implied rate of interest?
- You are offered the following investments:
  - You can invest \$500 today and receive \$600 in 5 years. The investment is low risk.
  - You can invest the \$500 in a bank account paying 4%.
  - What is the implied rate of interest for the first choice, and which investment should you choose?

# Finding the Number of Periods

- Start with the basic equation and solve for t (remember your logs)
  - $FV = PV(1 + r)^t$
  - $t = \ln(FV / PV) / \ln(1 + r)$
- You can use the financial keys on the calculator as well; just remember the sign convention.

# Number of Periods – Example 1

- You want to purchase a new car, and you are willing to pay \$20,000. If you can invest at 10% per year and you currently have \$15,000, how long will it be before you have enough money to pay cash for the car?
  - $t = \ln(20,000 / 15,000) / \ln(1 + 0.1) = 3.02$  years
  - Using financial calculator: 10 I/Y; -15,000 PV; 20,000 FV; CPT N = 3.02 years

# Number of Periods – Example 2

- Suppose you want to buy a new house. You currently have \$15,000, and you figure you need to have a 10% down payment plus an additional 5% of the loan amount for closing costs. Assume the type of house you want will cost about \$150,000 and you can earn 7.5% per year. How long will it be before you have enough money for the down payment and closing costs?

# Number of Periods – Example 2 Continued

- How much do you need to have in the future?
  - Down payment =  $.1(150,000) = 15,000$
  - Closing costs =  $.05(150,000 - 15,000) = 6,750$
  - Total needed =  $15,000 + 6,750 = 21,750$
- Compute the number of periods
  - $t = \ln(21,750 / 15,000) / \ln(1.075) = 5.14$  years
  - Using financial calculator: -15,000 PV; 21,750 FV; 7.5 I/Y; CPT N = 5.14 years

# Handy Rule of Thumb

- Rule of 72 can estimate how long it takes to double a sum of money
  - Time to double money =  $72 / (\text{interest rate per year})$
- If interest rate = 9% per year, it will take 8 years to double the money
  - Time to double money =  $72 / 9\% = 8 \text{ years}$
- If the time taken to double the money is 8 years, the interest rate is 9% per year
  - Interest rate per year =  $72 / 8 \text{ years} = 9\%$

# Quick Quiz – Part IV

- When might you want to compute the number of periods?
- Suppose you want to buy some new furniture for your family room. You currently have \$500, and the furniture you want costs \$600. If you can earn 6%, how long will you have to wait if you don't add any additional money?

# Spreadsheet Example

- Use the following formulas for Time Value of Money (TVM) calculations
  - $FV(\text{rate}, \text{nper}, \text{pmt}, \text{pv})$
  - $PV(\text{rate}, \text{nper}, \text{pmt}, \text{fv})$
  - $RATE(\text{nper}, \text{pmt}, \text{pv}, \text{fv})$
  - $NPER(\text{rate}, \text{pmt}, \text{pv}, \text{fv})$
- The formula icon is very useful when you can't remember the exact formula
- Click on the Excel icon to open a spreadsheet containing the four formulas.



Microsoft Excel  
Worksheet

# Work the Web Example

- Many financial calculators are available online
- Click on the web surfer to go to Investopedia's web site and work the following example:
  - You need \$50,000 in 10 years. If you can earn 6% interest, how much do you need to invest today?
  - You should get \$27,919.74

# Table 5.4

## I. Symbols:

PV = Present value, what future cash flows are worth today

$FV_t$  = Future value, what cash flows are worth in the future

$r$  = Interest rate, rate of return, or discount rate per period—typically, but not always, one year

$t$  = Number of periods—typically, but not always, the number of years

$C$  = Cash amount

## II. Future Value of $C$ Invested at $r$ Percent for $t$ Periods:

$$FV_t = C \times (1 + r)^t$$

The term  $(1 + r)^t$  is called the *future value factor*.

## III. Present Value of $C$ to Be Received in $t$ Periods at $r$ Percent per Period:

$$PV = C / (1 + r)^t$$

The term  $1 / (1 + r)^t$  is called the *present value factor*.

## IV. The Basic Present Value Equation Giving the Relationship between Present and Future Value is:

$$PV = FV_t / (1 + r)^t$$

# Comprehensive Problem

- You have \$10,000 to invest for five years.
- How much additional interest will you earn if the investment provides a 5% annual return versus if it provides a 4.5% annual return?
- How long will it take your \$10,000 to double in value if it earns 5% annually?
- What is the annual implied rate of interest if \$1,000 grows into \$4,000 in 20 years?



End of Chapter