Quiz 1: Date: April 19, 2022 from 11.00-12.30

Question 1 (10 Points) Score.....................

Consider the one-period model of consumption and portfolio choice. Let an individual in this economy has the utility function as follow:

$$U(C) = ln(C)$$

Also, let $\frac{C_1}{C_0}$ is distributed as log-normal with mean equals μ_c and its variance is σ_c.

Please read and answer the following questions carefully and completely.

Score.....................

Question 1.1 (10 marks) Calculate the risk free rate R_f in terms of the individual's consumption, C_0 and C_1. Then,explain the relationship between the level of consumption and the risk free rate in this economy.

From $P_j U'(C_0) = \delta E[U'(C_1)X_j]$

$U'(C_0) = R_f \delta E[U'(C_1)]$

$\frac{1}{R_f} = \delta E\left[\frac{U'(C_1)}{U'(C_0)}\right]$

$\frac{1}{R_f} = \delta E\left[\frac{\frac{1}{C_1}}{\frac{1}{C_0}}\right]$

$\frac{1}{R_f} = \delta E\left[\frac{C_0}{C_1}\right]$

$\frac{dU(c)}{dc} = \frac{1}{c} \quad \therefore U'(c) = \frac{1}{c}$

$\frac{d^2U(c)}{d^2c} = -\frac{1}{c^2}$

$R(c) = \frac{-\frac{1}{c^2}}{-\frac{1}{c}} = \frac{1}{c}$

$R_r(c) = c_0 \frac{1}{c} = 1$

$\frac{dR_r(c)}{dc} = 0$ CRRA

$R_f = \frac{1}{\delta} \cdot \frac{C_1}{C_0}$

$\therefore$ Since it is CRRA utility function given the economy pay R_f as a return. when i is higher, we expected to have more growth in consumption

Score....................

ε

Question 1.2 (10 marks) Calculate the elasticity of intertemporal substitution in this setting. If in the next year, the interest rate is falling, Will the individual's consumption level increase or decrease? Why? To support your answer, use the concepts of income effect and substitution effect.

From

$$R_f = \frac{1}{\delta} \cdot \frac{c_1}{c_0}$$

take log both sides

$$\ln(R_f) = -\ln(\delta) + \ln\left(\frac{c_1}{c_0}\right)$$

$$\frac{\partial R_f}{\partial \frac{c_1}{c_0}} = 1 \cdot \frac{R_f}{\frac{c_1}{c_0}}$$

$$\frac{\frac{\partial c_1}{c_0}}{\frac{\cancel{c_1}}{\cancel{c_0}}} = 1 \cdot \frac{\frac{\cancel{c_1}}{\cancel{c_0}}}{\cancel{R_f}}$$

$$\frac{\frac{\partial R_f}{\cancel{R_f}}}{}$$

$$\varepsilon = \frac{\%\Delta \frac{c_1}{c_0}}{\%\Delta R_f} = \frac{R_f}{\frac{c_1}{c_0}} \frac{\partial \frac{c_1}{c_0}}{\partial R_f}$$

$$\therefore \frac{R_f}{\frac{c_1}{c_0}} \frac{\partial \frac{c_1}{c_0}}{\partial R_f} = 1$$

$\therefore$ There is no change whether interest rate is increase or decrease because the elasticity is equal to 1. Moreover, both income and is equal and offset each other

$E[m_{01}] = \frac{1}{R_f}$

Score.....................

Question 1.3 (10 marks) Solve for the pricing kernel P_i of any risky asset i in this economy. Then explain the meaning of this pricing kernel.

$P_i = E[m_{01} X_i]$

$m_{01} = \frac{\delta U'(c_1)}{U'(c_0)} = \delta\left(\frac{c_0}{c_1}\right)$

$\div P$

$1 = E[m_{01} R_i]$

$E[XY] = E[X] \cdot E[Y] + cov(X, Y)$

$= E[m_{01}] \cdot E[R_i] + cov(m_{01}, R_i)$

$\frac{1}{E[m_{01}]} = E[R_i] + \frac{cov(m_{01}, R_i)}{E[m_{01}]}$

$R_f = E[R_i] + \frac{cov(m_{01}, R_i)}{E[m_{01}]}$

$\therefore$ pricing kernel will shows difference in random labor income between investors.

$E[R_i] = R_f - \frac{cov(m_{01}, R_i)}{E[m_{01}]}$

$E[R_i] = R_f - \frac{cov(U'(c_1), R_i)}{E[U'(c_1)]}$

Score.....................

Question 1.4 (10 marks) Calculate Hansen-Jaganathan Bound and explain the meaning.

From : $\left|\frac{E[R_j]-R_f}{\sigma_{R_j}}\right| \leq \frac{\sigma_{m_{01}}}{E[m_{01}]} = \sigma_{m_{01}} R_f$

note $m_{01} = \delta\left(\frac{c_0}{c_1}\right)$

$= \delta e^{\ln\left(\frac{c_0}{c_1}\right)}$

$$\frac{\sigma_{m_{01}}}{E[m_{01}]} = \frac{\sqrt{Var[e^{\ln(\frac{c_0}{c_1})}]}}{E[e^{\ln(\frac{c_0}{c_1})}]}$$

no parameter in front of $\ln\left(\frac{c_0}{c_1}\right)$ since it is a log-function

$$= \frac{\sqrt{E[e^{2\ln(\frac{c_0}{c_1})}] - E[e^{\ln(\frac{c_0}{c_1})}]^2}}{E[e^{\ln(\frac{c_0}{c_1})}]}$$

$Var(x) = E[x^2] - E[x]^2$

$$= \sqrt{\frac{E[e^{2\ln(\frac{c_0}{c_1})}]}{E[e^{\ln(\frac{c_0}{c_1})}]^2} - 1}$$

$\therefore \left|\frac{E[R_p]-R_f}{\sigma_{R_p}}\right| \leq \sigma_c^2$

$e^{\mu + \frac{1}{2}\cdot\sigma^2}$

$$= \sqrt{\frac{e^{2\mu_c + 2\cdot\sigma_c^2}}{e^{2\mu_c + \sigma_c^2}} - 1}$$

It means that the sharpe ratio of this equation should be lower or equal to σ_c^2

$$= \sqrt{e^{\sigma_c^2} - 1}$$

$$= 1 + \sigma_c^2 - 1$$

$$\frac{\sigma_{m_{01}}}{E[m_{01}]} = \sigma_c^2$$