$\boxed{\text{Q.7 in PS. 6}}$

$$y = y(K, L)$$

Given $y^{1+c\ln y} = AK^{\alpha}L^{\beta}$. Want to find $\frac{\partial y}{\partial K}$ and $\frac{\partial y}{\partial L}$.

$$y^{1+c\ln y} = AK^{\alpha}L^{\beta}$$ → take ln on both sides:

$$\ln y^{(1+c\ln y)} = \ln(AK^{\alpha}L^{\beta})$$

$$(1+c\ln y)\cdot\ln y = \ln A + \alpha\ln K + \beta\ln L$$

$$\ln y + c(\ln y)^2 = \ln A + \alpha\ln K + \beta\ln L. \quad —— (*)$$

From (*) Define an implicit function:

$$F \equiv \ln y + c(\ln y)^2 - \ln A - \alpha\ln K - \beta\ln L$$

Use implicit function rule:

① $$\frac{\partial y}{\partial K} = \frac{-F_K}{F_y} = -\frac{(-\alpha/K)}{\frac{1}{y} + \frac{2c\cdot\ln y}{y}} \Rightarrow \boxed{\frac{\partial y}{\partial K} = \frac{\alpha}{K}\cdot\left[\frac{y}{1+2c\ln y}\right]}$$

② $$\frac{\partial y}{\partial L} = -\frac{F_L}{F_y} = -\frac{(-\beta/L)}{\frac{1}{y} + \frac{2c\ln y}{y}} \Rightarrow \boxed{\frac{\partial y}{\partial L} = \frac{\beta}{L}\left[\frac{y}{1+2c\ln y}\right]}$$

} Ans

Alternative way: use total differentiation.

from (*), totally differentiate gives:

$$\frac{1}{y}dy + 2c\frac{\ln y}{y}dy + (\ln y)^2 \cancel{dc}^{0} = \cancel{d(\ln A)}^{0} + \frac{\alpha}{K}dK + \frac{\beta}{L}dL.$$ ($\because$ c & A are constants)

$$\left[\frac{1+2c\cdot\ln y}{y}\right]dy = \frac{\alpha}{K}dK + \frac{\beta}{L}dL$$

(Recall: $dy = \frac{\partial y}{\partial K}\cdot dK + \frac{\partial y}{\partial L}\cdot dL$.)

$$dy = \underbrace{\frac{\alpha}{K}\cdot\left[\frac{y}{1+2c\ln y}\right]}_{=\partial y/\partial K}dK + \underbrace{\frac{\beta}{L}\left[\frac{y}{1+2c\ln y}\right]}_{=\partial y/\partial L}\cdot dL$$

Thus; $\frac{\partial y}{\partial K} = \frac{\alpha\cdot y}{K\cdot[1+2c\ln y]}$ and $\frac{\partial y}{\partial L} = \frac{\beta\cdot y}{L[1+2c\ln y]}$

Q.10 in PS.6

$$p \cdot F'(L) - w = 0 \quad \text{—①}$$

$$pF(L) - wL - B = 0 \quad \text{—②}$$

① $\Rightarrow$ $p \cdot F''(L) \cdot dL + F'(L)\,dp = dw$ —③

② $\Rightarrow$ $\underbrace{p \cdot F'(L)}_{=w}dL + F(L)\,dp - w\,dL - L\,dw = dB$

$$\cancel{w\,dL} + F(L)\,dp - \cancel{w\,dL} - L\,dw = dB$$

$$F(L)\,dp - L\,dw = dB$$

$$\Rightarrow dp = \frac{dB + L\,dw}{F(L)} \quad \text{—④}$$

Note: $\boxed{L = L(w, B)}$

Sub dp in ①:

$$p \cdot F''(L)\,dL + F'(L) \cdot \frac{[dB + L\,dw]}{F(L)} = dw$$

$$pF''(L) \cdot F(L)\,dL + F'(L)\,dB + L \cdot F'(L)\,dw = F(L)\,dw$$

$$pF''(L) \cdot F(L) \cdot dL = [-LF'(L) + F(L)]\,dw - F'(L)\,dB$$

$$dL = \underbrace{\frac{[-LF'(L) + F(L)]}{p \cdot F''(L)F(L)}}_{= L_w = \frac{\partial L}{\partial w}}dw \underbrace{- \frac{F'(L)}{p \cdot F''(L) \cdot F(L)}}_{= L_B = \frac{\partial L}{\partial B}} \cdot dB$$

$\boxed{\text{Recall: } dL = L_w \cdot dw + L_B \cdot dB}$

Thus, $\boxed{\frac{\partial L}{\partial w} = \frac{F(L) - LF'(L)}{p \cdot F''(L) \cdot F(L)}}$; $\boxed{\frac{\partial L}{\partial B} = \frac{-F'(L)}{p \cdot F''(L) \cdot F(L)}}$

To find $\frac{\partial p}{\partial w}$ and $\frac{\partial p}{\partial B}$, recall that $P = P(w, B)$. So,

$$dp = \frac{\partial p}{\partial w} \cdot dw + \frac{\partial p}{\partial B} \cdot dB.$$

From ④, we have $dp = \underbrace{\frac{L}{F(L)}}_{P_w}dw + \underbrace{\frac{1}{F(L)}}_{P_B}dB.$

Thus, $\boxed{\frac{\partial p}{\partial w} = \frac{L}{F(L)}}$ and $\boxed{\frac{\partial p}{\partial B} = \frac{1}{F(L)}}$.