Exercise 1: from $\rho = \phi(i, y, R)$

Suppose that money multiplier does not vary too much with *i*, prove that $\phi_1 > 0$, $\phi_2 > 0$, and $\phi_3 > 0$

Answer :

from $L(\rho, i, y) = \lambda(\rho, i)\, m(i)\, R(1-\tau)$
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(1) Differentiate w.r.t. *i* (assuming *y* and *R* are fixed):

$$L_\rho \frac{\partial \rho}{\partial i} + L_i = m(i)R(1-\tau)\left(\lambda_\rho \frac{\partial \rho}{\partial i} + \lambda_i\right) + \lambda(\rho, i)R(1-\tau)m'(i)$$

$$\left(L_\rho - m(i)R(1-\tau)\lambda_\rho\right)\frac{\partial \rho}{\partial i} = \lambda(\rho, i)R(1-\tau)m'(i) - L_i + \lambda_i m(i)R(1-\tau)$$

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$$\text{If } m'(i) \approx 0, \text{then } \frac{\partial \rho}{\partial i} > 0$$

(2) Differentiate w.r.t. *y* (assuming *i* and *R* are fixed):

$$L_\rho \frac{\partial \rho}{\partial y} + L_y = \lambda_\rho \frac{\partial \rho}{\partial y} m(i)R(1-\tau)$$

$$L_y = \frac{\partial \rho}{\partial y}\left(\lambda_\rho m(i)R(1-\tau) - L_\rho\right)$$

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$$\frac{\partial \rho}{\partial y} > 0$$

(3) Differentiate w.r.t. *R* (assuming *i* and *y* are fixed):

$$L_\rho \frac{\partial \rho}{\partial R} = \lambda m(1-\tau) + mR(1-\tau)\lambda_\rho \frac{\partial \rho}{\partial R}$$

$$\left(mR(1-\tau)\lambda_\rho - L_\rho\right)\frac{\partial \rho}{\partial R} = -\lambda m(1-\tau)$$

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$$\frac{\partial \rho}{\partial R} < 0$$